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Price Competition in a Duopoly Characterized by Positional Effects

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Abstract

We examine the price decisions in a vertically differentiated duopoly where the decision to buy a good depends not only upon the intrinsic utility from consuming it but also upon the social attributes (prestige, uniqueness etc) associated with its consumption. These social attributes are especially important in vertically differentiated markets. We show that when these attributes are not very strong, if their intensity increases, the profits of both firms increase. However, when these attributes are very important, if their intensity increases, the profits of the firm that offers a lower quality variant increase whereas the profits of the firm that offers the higher quality variant decrease. Our results have implications on the amount of persuasive advertising firms should conduct in such markets.

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1 Introduction

Although ignored to a large extent, social factors are many times important when consumers make their purchasing decisions. People like to buy a product not only for its physical characteristics, but also for the social attributes (prestige, uniqueness, etc.) associated with its consumption. Such social factors are important especially for vertically differentiated products. However, in the standard models of vertical differentiation (Gabszewicz and Thisse (1979), Shaked and Sutton (1982)) the social aspects are completely missing. A typical example of a market that exclusivity plays a significant role is the market of smartphones. Although there is a wide number of firms that produce such devices of similar quality, there are many consumers who prefer to buy an iPhone or a Blackberry as they are convinced that the specific gadgets will confer status to them. The Apple paradigm is also applied in the market for tablet PCs where iPad, amongst all other brand names, is acknowledged as the product that confers status to its purchasers.

Here, we examine how the social factors affect the pricing of vertically differentiated firms. We adopt Deltas and Zacharias (2012)¹ approach and analyze a duopoly market in which consumers' purchase choices are affected both by the products' quality and price, and by the buying behavior of their fellows. In particular, each consumer's utility is affected positively by the number of people who have bought a lower quality model of the good or have made no purchase at all. In a similar way, each consumer's utility is negatively affected by the number of consumers who have bought a higher quality model. We show that the impact of the intensity of the social factors that affect consumption is not uniform across firms of different quality. When the social factors are not intense, both firms benefit if their intensity increases. However, when the social factors are sufficiently strong, only the low quality firm benefits if their intensity increases.

A number of papers examine the effects of the existence of social factors. The literature of the effects of the social factors on consumption originates from Veblen (1899). Leibenstein (1950) showed that the demand for a good may be a decreasing function of the number of people purchasing it. This "snob" effect is attributed to peoples' need for exclusiveness. If, on the other hand, the demand increases as the number of people who purchase it increase, we have the "bandwagon"

¹Deltas and Zacharias (2012) analyze how the existence of social factors affect the pricing and quality decisions of a monopolist.

effect. Here, both effects coexist: A consumer who buys a specific model enjoys a positive network externality as the number of the consumers who buy less advanced models (or do not buy at all) increases. Conversely, a consumer suffers a negative network externality as the number of the consumers who buy more advanced models increases.

Bagwell and Bernheim (1996) show that “luxury” brands though not intrinsically superior, are sold at higher prices than “budget” brands as the former are signals of wealth. Deltas et al. (2011) provide evidence that the state of the art PCs are priced more (after controlling for the most important technical characteristics of PCs) and this is consistent with consumers willing to pay a premium to buy the most advanced PCs in the market. Within months, when these PCs cease to be in the frontier of the industry, their prices decrease.

Basu (1987) illustrates a model where aggregate excess demand for a good or a service is used as an index of status and increases consumers’ valuation of the particular commodity. For example, a long waiting list at a doctor’s practice can be considered as a signal of the quality of his services and his model explains why it may not be optimal for the doctor to increase his fees. Also, there are famous schools that have long waiting lists and children should be registered before even they are born. Acceptance in such schools signals social status and establishes higher class networks.

Since income is usually not observable, argue Bagwell and Bernheim (1996), members of a higher class, in their attempt to dissociate themselves from lower classes, are often trying to advertise their wealth through highly conspicuous consumption. Automobile industries use expensive promotional campaigns to enhance the belief that buying the new model of a sports car will boost social status. Similar campaigns are also used by firms selling luxury goods like women’s purses, haute couture clothes and cosmetics. Most of the times firms use the marketing strategy of celebrity endorsement and recruit famous and socially accepted people that most consumers look up to, to promote their products.

Not surprisingly, such conspicuous behavior is not only met in wealthy people who want to show off their abundance. It is also observed that there are people who, although considered of middle or even lower class, purchase “status-offering” goods so as to feel or show that they belong to an upper class. Grilo et al. (2001), use the term “vanity” to describe “snob” effects and examine a horizontally differentiated duopoly showing that its presence results in higher prices and relaxes

price competition. They also compare their results to those when the market is characterized by an opposing effect, “conformity”. This term is used to describe the mimicking behavior of people in their attempt to be accepted in certain social groups. “Conformity”, when strong enough, can lead to market capturing by only a single firm. This kind of consumer behavior is associated to network effects. When those effects are at work, the utility gained by purchasing a good is also affected by the overall number of consumers who own it, only in a positive way.

Literature has widely used network effects to incorporate products’ social attributes into vertical differentiation models. Jing (2007) demonstrates that when such effects are present, a monopolist has incentives to exercise price discrimination through producing two considerably differentiated goods. The lower quality product is used to mainly lure consumers in the firm’s network, while the higher quality product to extract profits. Ghazzai (2008), although through a different approach, results in price discrimination optimality as well. He presents a monopoly where there are two types of consumers who have preferences for low and high quality respectively. Consumers who have bought a higher quality product gain extra utility which is proportional to the degree of differentiation of the products. The inverse occurs when consumers buy the lower quality product and the paper concludes that the monopolist should employ a multiproduct strategy.

Lambertini and Orsini have examined how positional effects affect both a monopoly (Lambertini and Orsini 2002) and a duopoly (Lambertini and Orsini 2005). In the second paper, they examine how the snob effect relaxes price competition in a vertically differentiated duopoly that operates in an exogenously fully covered market. In this market, one firm offers a positional good, while the other firm offers a non-positional good. This paper is an immediate extension of the one where the authors study the monopoly case, in the sense that the utility of the consumers who buy the non-positional good is not affected by the number of the consumers that buy the positional one. Departing from their model, we analyze a duopoly where both firms offer positional goods and the market coverage is endogenously defined. The endogenous market coverage assumption can produce very different equilibrium outcomes than those when the market is exogenously determined, as showed by Cheng et al. (2011). Allowing the size of the market to be more flexible, offers incentives to firms to produce multiple qualities in order to motivate more consumers to make a purchase. However, the emergence of multiproduct firms leads to fiercer price competition.

Positional product attributes are usually communicated to consumers through advertising. Pro-

moting those products is more associated to “persuasive” advertising rather than “informative” advertising, since firms aim in focusing in specific social benefits that consumers will have if they own the product. The literature of persuasive advertising includes Krahmer (2006) who introduces the concept of “image” advertising, as the type of advertising that promotes the image of a product to consumers. In his paper, a firm examines whether to enter in a market where advertising is used as a signal of product differentiation. Consumers that are interested in communicating a favorable image to their fellows, which are considered as the “premium“ segment, choose to buy a product that its image is highly advertised. The inverse holds for “budget” consumers that do not have such image concerns. In equilibrium, as “image” advertising increases consumers’ willingness to pay for the specific product, both the incumbent and the entrant find it optimal to overinvest in advertising, which results in entry deterrence.

Here, we show that when the social factors are not intense, both firms have an incentive to use persuasive advertising as both benefit when their intensity increases. However, when the social factors are sufficiently strong, it only the low quality firm that has an incentive to advertise as the profits of the high quality firm decrease as the intensity of the social factors increases.

In the next section, we describe the model and present the equilibrium prices, quantities and profits for the two firms. In section 3 we analyze how the results change as and the intensity of the positional effects change. The last section concludes with a few remarks on various possible extensions.

2 Model Outline

We employ the standard vertical differentiation model presented by Shaked and Sutton (1982) as extended by Deltas and Zacharias (2012). There are two single-product firms selling different variants of the same good, which is characterized by its level of quality $s_i, i = 1, 2$. The product of firm 1 is considered of lower quality compared to the one of firm 2 ($s_2 > s_1$).

Consumers are uniformly distributed over $[0, 1]$ with density normalized to 1 and make indivisible and mutually exclusive purchases. Their marginal willingness to pay for quality is denoted by θ_i , which is likewise uniformly distributed across $[0, 1]$. Consumers’ purchase choices are no longer affected by the products’ quality θ and price only, but also by the buying behavior of the other

consumes. In particular, each consumer's net surplus is affected positively by the number of people who have bought a lower quality model of the good or have made no purchase at all. Furthermore, each consumer's net surplus is negatively affected by the number of consumers who have bought a higher quality model.

More extensively, the utility U_i of the consumer who chooses not to make a purchase exhibits negative network externalities with respect to the number of people that have bought either of the firms' products. Assuming that the type of the marginal consumer who is indifferent between buying the lower quality variant and not buying a product at all is θ_1 , the above reasoning can be described by:

$$U_i = -\alpha(1 - F(\theta_1)), \quad (1)$$

where, $1 - F(\theta_1)$ indicates the mass of consumers who have bought either variant of the product. The positive parameter α captivates the intensity of the positional or status effect and is the same for all agents.

Furthermore, the utility derived from buying firm 1's variant is positively affected by the number of consumers who have chosen not to buy the good, while, at the same time, is negatively affected by the number of consumers who have bought the higher quality variant. Accordingly, those who have bought firm 2's product, feeling superior to consumers that own either the lower quality model or no product at all, gain extra utility which is proportional to the number of those consumers. Let θ_2 be the type of the marginal consumer who is indifferent between buying the lower quality variant and buying the higher quality variant at prices p_1 and p_2 respectively.

A consumer i who buys the variant of quality s_2 has utility

$$U_i = \theta_i s_2 + \alpha F(\theta_2) - p_2. \quad (2)$$

A consumer i who buys the variant of quality s_1 has utility

$$U_i = \theta_i s_1 - \alpha(1 - F(\theta_2)) + \alpha F(\theta_1) - p_1. \quad (3)$$

Since market coverage is endogenously determined, solving for the two types of the indifferent consumers will define firms' market shares and, ultimately their profits. We examine the case where each firm is using a specific technology that allows it to produce only a certain fixed quality of its variant. In addition, firms face constant marginal production costs $c_i, i = 1, 2$ with $c_2 > c_1$ for

which it holds that $s_i > c_i$. Fixed costs are assumed to be zero. The above described setup allows us to present the profit functions of the two rival firms:

$$\pi_1 = (p_1 - c_1)(\theta_2 - \theta_1) \quad (4)$$

$$\pi_2 = (p_2 - c_2)(1 - \theta_2) \quad (5)$$

In the standard vertical differentiation model of Shaked and Sutton (1982) there are no status effects (i.e. $\alpha = 0$), and both firms produce positive qualities. As we want our results to be comparable to that model, we focus on parameters values of the qualities s_i and the unit costs c_i of the two variants that result in positive equilibrium prices, profits and market shares for sufficiently small values of the status parameter a . In addition, as we focus on a duopoly market, we require that the quality and cost parameter values are such that both firms produce positive quantities.² More specifically, to guarantee that the high quality firm has a positive market share, we assume:

Condition 1 $s_2 - s_1 > c_2 - c_1$.

To guarantee that the low quality firm has a positive market share, when α is sufficiently small we assume:

Condition 2 $\frac{s_1}{s_2 - s_1} > \frac{c_1}{c_2 - c_1}$.

To ensure that the low quality firm has a positive price, when α is sufficiently small we assume:

Condition 3 $s_2 + c_2 > 2s_1$.

As we require the equilibrium quantity of the low quality firm to be positive for all values of α , we also require that:

Condition 4 $4s_1(s_2 + c_2 - 2s_1) > c_1^2$.

The following proposition describes the product market equilibrium:

²If only one firm produces, then we have a monopoly that offers only one variant, a case discussed in Deltas and Zacharias (2012).

Proposition 1 *In the presence of positional externalities, the equilibrium prices of the two firms are:*

$$p_1^* = \frac{2s_1s_2c_1 - (s_1 + \alpha)(s_1^2 - \alpha^2) + s_1(s_1 + \alpha)(s_2 + c_2 - \alpha)}{s_1(4s_2 - s_1) + \alpha^2}$$

and

$$p_2^* = \frac{s_2[c_1(s_1 - \alpha) + 2s_1(s_2 + c_2 + \alpha)] - (2s_2 + \alpha)(s_1^2 - \alpha^2)}{s_1(4s_2 - s_1) + \alpha^2}.$$

The firms earn profits:

$$\pi_1^* = \frac{s_2}{s_1(s_2 - s_1) + \alpha^2} \left[\frac{s_1(s_1 + \alpha)(s_2 + c_2 - \alpha) - (s_1 - c_1 + \alpha)(s_1^2 - \alpha^2) - 2s_1s_2c_1}{s_1(4s_2 - s_1) + \alpha^2} \right]^2$$

and

$$\pi_2^* = \frac{s_1}{s_1(s_2 - s_1) + \alpha^2} \left[\frac{s_2c_1(s_1 - \alpha) + 2s_1s_2(s_2 - c_2 + \alpha) - (2s_2 - c_2 + \alpha)(s_1^2 - \alpha^2)}{s_1(4s_2 - s_1) + \alpha^2} \right]^2$$

and the respective market shares are:

$$q_1^* = \frac{s_2}{s_1(s_2 - s_1) + \alpha^2} \frac{\alpha s_1(s_2 + c_2 - 2s_1) + s_1^2(s_2 - s_1 + c_1 + c_2) + \alpha^2(\alpha - c_1) - 2s_1s_2c_1}{s_1(4s_2 - s_1) + \alpha^2}$$

and

$$q_2^* = \frac{s_1}{s_1(s_2 - s_1) + \alpha^2} \frac{s_1s_2(2s_2 - 2c_2 + c_1 + 2\alpha) + (2s_2 - c_2 + \alpha)(s_1^2 - \alpha^2) - \alpha s_2c_1}{s_1(4s_2 - s_1) + \alpha^2}$$

Proof. The value of θ_1 for the consumer who is indifferent between buying firm 1's product and not buying at all is obtained by:

$$U_1 = U_0 \Rightarrow \theta_1 s_1 - \alpha(1 - F(\theta_2)) + \alpha F(\theta_1) - p_1 = -\alpha(1 - F(\theta_1))$$

Taking into account that θ_i is uniformly distributed across $[0, 1]$ and solving for θ_1 we have:

$$\theta_1 = \frac{p_1 - \alpha\theta_2}{s_1} \tag{6}$$

Respectively, the value of θ_2 for the consumer who is indifferent between buying firm 1's and firm 2's product is obtained by:

$$U_1 = U_2 \Rightarrow \theta_2 s_1 - \alpha(1 - F(\theta_2)) + \alpha F(\theta_1) - p_1 = \theta_2 s_2 + \alpha F(\theta_2) - p_2$$

Solving for θ_2 yields:

$$\theta_2 = \frac{p_2 - p_1 - \alpha(1 - \theta_1)}{s_2 - s_1} \quad (7)$$

Solving jointly (6) and (7) we have the values of θ_1 and θ_2 :

$$\theta_1 = \frac{p_1(s_2 - s_1) - \alpha(p_2 - p_1 - \alpha)}{s_1 s_2 - s_1^2 + \alpha^2},$$

and

$$\theta_2 = \frac{\alpha p_1 + s_1(p_2 - p_1 - \alpha)}{s_1 s_2 - s_1^2 + \alpha^2}$$

Substituting the above values into the profit functions of the two firms we have:

$$\pi_1 = \frac{(p_1 - c_1)[(s_1 + \alpha)(p_2 - \alpha) - s_2 p_1]}{s_1 s_2 - s_1^2 + \alpha^2},$$

and

$$\pi_2 = \frac{(p_2 - c_1)[s_1(s_2 - p_2 + \alpha) - (s_1 - \alpha)(s_1 + \alpha - p_1)]}{s_1 s_2 - s_1^2 + \alpha^2}$$

The first order conditions of firms' maximization problems with respect to their prices are:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{(s_1 + \alpha)(p_2 - \alpha) + s_2 c_1}{2s_2}$$

and

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \frac{s_1(s_2 + c_2 + \alpha) - (s_1 - \alpha)(s_1 - p_1 + \alpha)}{2s_1}$$

Solving jointly the above we have the equilibrium prices:

$$p_1^* = \frac{2s_1 s_2 c_1 - (s_1 + \alpha)(s_1^2 - \alpha^2) + s_1(s_1 + \alpha)(s_2 + c_2 - \alpha)}{s_1(4s_2 - s_1) + \alpha^2}, \quad (8)$$

and

$$p_2^* = \frac{s_2[c_1(s_1 - \alpha) + 2s_1(s_2 + c_2 + \alpha)] - (2s_2 + \alpha)(s_1^2 - \alpha^2)}{s_1(4s_2 - s_1) + \alpha^2}. \quad (9)$$

Substituting optimal prices into profits we get profits as functions of qualities and the status factor

α :

$$\pi_1^* = \frac{s_2}{s_1(s_2 - s_1) + \alpha^2} \left[\frac{s_1(s_1 + \alpha)(s_2 + c_2 - \alpha) - (s_1 - c_1 + \alpha)(s_1^2 - \alpha^2) - 2s_1 s_2 c_1}{s_1(4s_2 - s_1) + \alpha^2} \right]^2, \quad (10)$$

and

$$\pi_2^* = \frac{s_1}{s_1(s_2 - s_1) + \alpha^2} \left[\frac{s_2 c_1(s_1 - \alpha) + 2s_1 s_2(s_2 - c_2 + \alpha) - (2s_2 - c_2 + \alpha)(s_1^2 - \alpha^2)}{s_1(4s_2 - s_1) + \alpha^2} \right]^2. \quad (11)$$

The firms' market shares are respectively:

$$q_1^* = \theta_2 - \theta_1 = \frac{s_2}{s_1(s_2 - s_1) + \alpha^2} \frac{s_1(s_1 + \alpha)(s_2 + c_2 - 2s_1) + (s_1^2 - \alpha^2)(\alpha - c_1) - 2s_1s_2c_1}{s_1(4s_2 - s_1) + \alpha^2}, \quad (12)$$

and

$$q_2^* = 1 - \theta_2 = \frac{s_1}{s_1(s_2 - s_1) + \alpha^2} \frac{(2s_1s_2 - s_1^2 + \alpha^2)(s_2 + \alpha - c_2) - s_1s_2(s_1 - c_1) + \alpha s_2(\alpha - c_1)}{s_1(4s_2 - s_1) + \alpha^2} \quad (13)$$

As it is apparent by the above equilibrium values, both firms enjoy positive profits in equilibrium. Due to condition 3, the respective prices are also positive, which can be verified if we rewrite them as:

$$p_1^* = \frac{\alpha^3 + \alpha s_1(s_2 + c_2 - 2s_1) + s_1[s_1(s_2 - s_1) + s_1c_2 + 2s_2c_1]}{s_1(4s_2 - s_1) + \alpha^2}$$

and

$$p_2^* = \frac{(2s_2 + \alpha)[s_1(s_2 - s_1) + \alpha^2] + \alpha s_2(s_1 - c_1) + s_1s_2(2c_2 + c_1)}{s_1(4s_2 - s_1) + \alpha^2}$$

To show that the market share of the lower quality firm is positive, it suffices to show that the expression in the numerator of the second fraction in (12) is positive. For $\alpha = 0$ the equilibrium market share is:

$$q_1^*|_{\alpha=0} = \frac{s_2[s_1(s_2 - s_1) + s_1c_1 + s_1c_2 - 2s_2c_1]}{s_1(4s_2 - s_1)(s_2 - s_1)}$$

Lets write s_2 as $s_2 = s_1 + s$ and c_2 as $c_2 = c_1 + c$. By substituting s_2 and c_2 into the expression in the numerator we get:

$$s_1(s_1s + s_1c - 2c_1s)$$

which, by condition 2 is positive. Hence, $q_1^*|_{\alpha=0} > 0$. Rearranging the terms in the numerator of (12) we have:

$$\alpha^3 - \alpha^2c_1 + \alpha s_1(s_2 + c_2 - 2s_1) + s_1[s_1(s_2 - s_1) + s_1c_1 + s_1c_2 - 2s_2c_1] \quad (12.a)$$

The last term, as shown above and due to $s_2 > s_1$, is positive. If we divide by $\alpha > 0$ the three first parameters we have $\alpha^2 - \alpha c_1 + s_1(s_2 + c_2 - 2s_1)$. Solving $\alpha^2 - \alpha c_1 + s_1(s_2 + c_2 - 2s_1) = 0$ for α we get:

$$\left\{ \begin{array}{l} \alpha = \frac{c_1}{2} + \frac{\sqrt{c_1^2 - 4s_1(s_2 + c_2 - 2s_1)}}{2} \\ \alpha = \frac{c_1}{2} - \frac{\sqrt{c_1^2 - 4s_1(s_2 + c_2 - 2s_1)}}{2} \end{array} \right.$$

Due to condition 4 and the fact that $q_1^*|_{\alpha=0} > 0$, the equation we solved for α is always positive and consequently is (12.a) which ensures that q_1^* is positive for every $\alpha \geq 0$.

The market share of the higher quality firm can be rewritten as:

$$q_2^* = \frac{s_1}{s_1(s_2 - s_1) + \alpha^2} \frac{\alpha^3 + a^2(2s_2 - c_2) + \alpha[s_1(s_2 - s_1) + s_2(s_1 - c_1)] + s_1[(s_2 - s_1)(s_2 - c_2) + s_2(s_2 - s_1 - c_2 + c_1)]}{s_1(4s_2 - s_1) + \alpha^2}$$

which, given condition 1 and the initial assumptions of the model; i.e. $s_i > c_i$, $s_2 > s_1$, is positive.

■

In the next section we examine how the equilibrium prices, quantities and profits change as α increases.

3 Comparative statics

The incorporation of the positional externality into the model, not only affects pricing decisions but induces firms to examine how their profits change as α increases. This is important as it affects their desirable amount of persuasive advertising for their product, in order to increase status awareness in consumers' minds. That can be identified by conducting comparative statics to determine how the positional parameter α affects the equilibrium values.

As the expressions are highly nonlinear, at this stage, we focus on $\alpha = 0$ or α sufficiently high. As all the expressions are continuous functions on α , by examining the sign of the derivative at $\alpha = 0$ we are also able to comment on the sign of the derivative at α sufficiently small but positive.

We first show the effects of the change in a on prices. We show that the prices for both products increase as the positional parameter α increases both when a is zero and when a is sufficiently high.

Proposition 2 *The higher quality variant's price is increasing in α . The lower quality variant's price is increasing for sufficiently small α and for α high enough.*

Proof. The partial derivative of the lower quality firm's price with respect to α is:

$$\frac{\partial p_1^*}{\partial \alpha} = \frac{\alpha^4 + \alpha^2 s_1 (11s_2 - s_1 - c_2) - 2\alpha s_1 [s_1 (s_2 - s_1) + c_2 (2s_2 + s_1)] + s_1^2 (4s_2 - s_1) (s_2 + c_2 - 2s_1)}{[s_1 (4s_2 - s_1) + \alpha^2]^2} \quad (14)$$

For $\alpha = 0$ the derivative becomes:

$$\left. \frac{\partial p_1^*}{\partial \alpha} \right|_{\alpha=0} = \frac{s_2 + c_2 - 2s_1}{4s_2 - s_1}$$

which, due to condition 3, is positive.

As α increases, since the term with α in the greatest power, both in the numerator and denominator, is α^4 , $\frac{\partial p_1^*}{\partial \alpha}$ approaches 1.

The derivative of p_2^* with respect to α is:

$$\frac{\partial p_2^*}{\partial \alpha} = \frac{\alpha^4 + \alpha^2 [2s_1 (5s_2 - s_1) + s_2 c_1] + 2\alpha s_1 s_2 (6s_2 - c_1 - 2c_2) + s_1 (4s_2 - s_1) [s_1 (s_2 - s_1) + s_2 (s_1 - c_1)]}{[4s_1 (s_2 - s_1) + \alpha^2]^2} \quad (15)$$

Given that $s_2 > s_1$ and $s_i > c_i$, the above derivative is positive for every $\alpha \geq 0$. ■

We now examine how the profits of the two firms change as α increases. For α sufficiently small the profits for both firms increase, as α increases. However, for sufficiently high values of α , the profits of the firm that produces the lower quality variant are increasing in α , whereas those of the higher quality producing firm are decreasing in α . We have the following Proposition:

Proposition 3 *For α sufficiently small, both firms' equilibrium profits increase with α . For α high enough, the equilibrium profits of the firm that produces the lower quality variant are increasing in α , whereas those of the higher quality producing firm are decreasing in α .*

Proof. The derivative of the profits of the firm that produces the lower quality good with

respect to α is:

$$\begin{aligned} \frac{\partial \pi_1^*}{\partial \alpha} = & \frac{2s_2}{[s_1(s_2 - s_1) + \alpha^2]^2 [4s_1(s_2 - s_1)s_2 + \alpha^2]^3} \{ \alpha^3 - \alpha^2 c_1 + \alpha s_1(s_2 + c_2 - 2s_1) \\ & - s_1[2s_2 c_1 - s_1(s_2 + c_2 - s_1)] \} \{ \alpha^5 c_1 + \alpha^4 s_1(7s_2 + s_1 - 2c_2) \\ & + \alpha^3 s_1[2s_2 c_1 - s_1(c_1 - 3s_1 + 3s_2 + 3c_2)] + \alpha^2 s_1^2 [s_2(11s_2 - c_2) - s_1(12s_2 - s_1 - c_2)] \\ & + \alpha s_1^2 [c_1(4s_2^2 + 3s_1^2) + s_1 s_2(9s_1 + 6s_2 - 6c_2 - 2c_1)] \\ & + s_1^3 [4s_2^2(s_2 + c_2) - s_1^2(2s_1 - c_2) + s_1 s_2(11s_1 - 13s_2 - 5c_2)] \} \end{aligned} \quad (16)$$

For $\alpha = 0$ the above derivative becomes:

$$\left. \frac{\partial \pi_1^*}{\partial \alpha} \right|_{\alpha=0} = \frac{2s_2 [s_1(s_2 - s_1) + s_1 c_1 + s_1 c_2 - 2s_2 c_1] (s_2 + c_2 - 2s_1)}{s_1(s_2 - s_1)(4s_2 - s_1)^2}$$

which, by conditions 2 and 3, is positive.

Due to the fact that (16) is highly non-linear, there are no unambiguous arguments that can grant us its sign, but as α increases, since the terms with α in the highest power, both in the numerator and denominator, are positive ($2\alpha^8 s_2 c_1$ and α^{10} respectively) the derivative of π_1^* with respect to α is positive.

The derivative of the profits of the higher quality firm with respect to α is:

$$\begin{aligned} \frac{\partial \pi_2^*}{\partial \alpha} = & \frac{2s_1}{[s_1(s_2 - s_1) + \alpha^2]^2 [4s_1(s_2 - s_1)s_2 + \alpha^2]^3} \{ \alpha^3 + \alpha^2(2s_2 - c_2) + \alpha[s_1(s_2 - s_1) + s_2(s_1 - c_1)] \\ & + s_1[(2s_2 - c_2)(s_2 - s_1) - s_2(c_2 - c_1)] \} \{ -\alpha^5(2s_2 - c_2) + \alpha^4(2s_1 s_2 + 2s_2 c_1 - s_1^2) \\ & + \alpha^3[s_2(2c_2 + 4s_1 - 3c_1 + 2s_2) - 2s_1 c_2] + \alpha^2 s_1[s_2^2 c_1 + 2s_1^3 + s_1 s_2(10s_2 - 12s_1 - c_1)] \\ & + \alpha s_1^2[2s_2^2(2s_2 - 3c_1 + 2c_2) + s_1^2 c_2 - s_1 s_2(2s_2 + 2s_1 + 2c_2 - 3c_1)] \\ & + s_1^3[s_1 s_2^2(5c_1 + 8s_2 - 14s_1) + s_1^3(7s_2 - s_1) - 4s_1^2 s_2 c_1] \} \end{aligned} \quad (17)$$

For $\alpha = 0$ the above derivative becomes:

$$\left. \frac{\partial \pi_2^*}{\partial \alpha} \right|_{\alpha=0} = \frac{2[(2s_1 - c_2)(s_2 - s_1) - s_2(c_2 - c_1)][s_1(s_2 - s_1) + s_2(s_1 - c_1)]}{s_1(s_2 - s_1)(4s_2 - s_1)^2}$$

The second square bracket of the numerator and the denominator are positive. Substituting $s_2 = s_1 + s$ and $c_2 = c_1 + c$ into the first square bracket of the numerator and rearranging the terms we have:

$$(s_1 + 2s)(s - c) + s(s_1 - c_1)$$

which, by condition 1, is positive and thus $\frac{\partial \pi_2^*}{\partial \alpha} \Big|_{\alpha=0} > 0$.

Again, we cannot reach a solid conclusion about the sign of (17). However, for large enough values of α , since the term with the highest power of α in the numerator and denominator are negative $(-\alpha^5(2s_2 - c_2))$ and positive (α^{10}) respectively, (17) becomes negative. ■

For small values of α the market shares for both firms increase as α increases. However, the opposite happens as α increases, when α is sufficiently high. We show the effects of an increase in α on market shares in the following Proposition.

Proposition 4 *The equilibrium market shares of both firms increase as α increases for small values of α and decrease as the positional externality becomes strong enough.*

Proof. Differentiating the market share of the lower quality firm with respect to α we get:

$$\begin{aligned} \frac{\partial q_1^*}{\partial \alpha} = & \frac{s_2}{[s_1(s_2 - s_1) + \alpha^2]^2 [4s_1(s_2 - s_1)s_2 + \alpha^2]^2} \{-\alpha^6 + 2\alpha^5 c_1 + \alpha^4 s_1(2s_2 + 4s_1 - 3c_2) \\ & + 4\alpha^3 s_1[s_1(s_1 - s_2 - c_1 - c_2) + 2s_2 c_1] + \alpha^2 s_1[s_2(7s_2 - 5c_2) - s_1(3s_2 + s_1 - 2c_2)] \\ & + 2\alpha s_1[6s_2^2 c_1 - 2s_1(2s_1 - c_1 - 2c_2) - s_1 s_2(5s_2 - 7s_1 + 4c_1 + 5c_2)] \\ & + s_1^2[s_1^3(c_2 - 2s_1) + s_1 s_2(4s_2 c_2 - 13s_1 s_2 + 4s_2^2 + 11s_1^2 - 5s_1 c_2)]\} \end{aligned} \quad (18)$$

Substituting $\alpha = 0$ into the above expression we have:

$$\frac{\partial q_1^*}{\partial \alpha} \Big|_{\alpha=0} = \frac{s_2(s_2 + c_2 - 2s_1)}{s_1(s_2 - s_1)(4s_2 - s_1)}$$

which is positive by condition 3.

The sign of the denominator in (18) is positive and since the term with α in the highest power in the numerator is negative $(-\alpha^6 s_2)$, the sign of the derivative changes into negative as α increases.

The derivative of q_2 with respect to α is:

$$\begin{aligned} \frac{\partial q_2^*}{\partial \alpha} = & \frac{s_1}{[s_1(s_2 - s_1) + \alpha^2]^2 [4s_1(s_2 - s_1)s_2 + \alpha^2]^2} \{-\alpha^6 + 2\alpha^5(2s_2 - c_2) - \alpha^4(s_1 s_2 - s_1^2 - 3s_2 c_1) \\ & + 4\alpha^3 s_1[s_2(2s_1 - 2s_2 - c_1 + 2c_2) - s_1 c_2] + \alpha^2 s_1[s_1^3 + 5s_2^2 - s_1 s_2(2s_2 + 6s_1 - 2c_1)] \\ & + 2\alpha s_1^2[s_1^2 c_2 - s_2^2(2s_2 + 5c_1 + 6c_2) + 2s_1 s_2(2s_2 - s_1 + c_1 - 2c_2)] \\ & + s_1^2[s_1^3(7s_2 - s_1) + 4s_2^3(2s_1 - c_1) - s_1 s_2(14s_1 s_2 + s_1 c_1 - 5s_2 c_1)]\} \end{aligned} \quad (19)$$

For $\alpha = 0$ the above derivative is positive:

$$\left. \frac{\partial q_2^*}{\partial \alpha} \right|_{\alpha=0} = \frac{s_1(s_2 - s_1) + s_2(s_1 - c_1)}{s_1(s_2 - s_1)(4s_2 - s_1)}$$

Following the above reasoning, due to the fact that the denominator of (19) is positive and the term with α in the highest power is negative ($-\alpha^6 s_1$), when α is high enough, it holds that $\frac{\partial q_2^*}{\partial \alpha} < 0$. ■

The analysis shows that the effects of the positional parameter α depend on its size. For small values of the positional parameter α , as positional effects become stronger, both firms find it optimal to increase their prices. As at the same time their market shares also increase, both firms enjoy higher profits. In such case, both have an incentive to increase their advertising.

However, this is not the case for the firm that produces the high quality variant for sufficiently high α : When the positional parameter is high enough, owing any variant of the product becomes more important than owing the high quality product. The high quality firm sets a very high price. As a result, consumers that would buy the higher quality variant, cannot afford it anymore and switch to the inferior variant that, although does not offer them the same level of status and luxury, it is more affordable. This behavior leads to a decrease in the market share and profits of the high quality firm. The fact that both prices are high, affects negatively the lower quality firm's market share as well, but this is partially counterbalanced by the fact that the lower quality firm gains the consumers that cannot bear the cost of the superior version anymore. As a result, the profits of the lower quality firm keep being increasing. In such case, it is optimal for the low quality firm to increase its amount of advertising only.

4 Concluding Remarks

In vertically differentiated markets, consumers often care both on the intrinsic characteristics and the social attributes of the product they buy. It is for this reason that firms in their ads often point out the status that their products offer to those who purchase them. We analyze the implications of the social attributes in a vertically differentiated duopoly market. In particular, we assume that consumers suffer a negative externality by the number of consumers who have bought a higher -than theirs- quality model and enjoy a positive externality by the number of people who have bought a lower -than theirs- quality model or have made no purchase at all.

We show that in such a setting, when the intensity of the social attributes is not very strong, it is optimal for both firms to stimulate status awareness to the consumers. This increases both the prices consumers are willing to pay for their products and the number of those who are willing to buy any variant of the product. As a result, the profits of both firms increase. However, when social attributes are very strong, it is only the firm that produces the product of the lower quality that benefits from an increase in the intensity of these factors. The price of the high quality firm increases but as a result less consumers are willing to buy the high variant. In such case, the profits of the high quality firm decrease. However, this is not the case for the firm that offers the low quality product. Although its price increase, the demand for its product decreases by less and as a result its profits increase.

Of course social attributes can be modeled in other ways as well. In our setting, the parameter α that captures the intensity of the positional effects is constant. However, it might well be a function of the degree of the luxuriousness s of each variant. That would make sense if we think for example that people who haven't made any purchase, envy those who have bought the lower quality variant, since they do not have it, but they envy the most those who have bought the higher quality variant.

5 References

- Bagwell, L.; Bernheim, B.D. , 1996. Veblen effects in a theory of conspicuous consumption. *American Economic Review*, vol. 86, pages 349-373.
- Basu K., 1987. Monopoly, Quality Uncertainty and Status Goods. *International Journal of Industrial Organization*, vol. 5, pages 435-446.
- Cheng Y.-L.; Peng S.-K.; Tabuchi T., 2011. Multiproduct Duopoly with Vertical Differentiation. *The B.E. Journal of Theoretical Economic*, vol. 11, pages 1-27.
- Deltas G.; Stengos T.; Zacharias E., 2011. Product Line Pricing in a Vertically Differentiated Oligopoly, *Canadian Journal of Economics*, vol. 44, pages 907-929.
- Deltas G.; Zacharias E., 2012. Product Offerings and Pricing in a Vertical Differentiated Market with Positional or Status Effects. Working paper.

Gabszewicz J.J.; Thisse J.-F., 1979. Competition, Quality and Income. *Journal of Economic Theory*, vol. 20, pages 340-359.

Ghazzai H.; 2008. Multi-product Strategies and Relative Preferences for Quality. *Economics Bulletin*, vol. 4, No 34, pages 1-10.

Grilo I.; Shy O.; Thisse J.-F., 2001. Price competition when consumer behavior is characterized by conformity or vanity. *Journal of Public Economics* vol. 80, pages 385-408.

Jing B.; 2007. Network Externalities and Market Segmentation in a Monopoly. *Economic Letters*, vol. 95, pages 7-13.

Krahmer D.; 2006. Advertising and Conspicuous Consumption. *Journal of Institutional and Theoretical Economics*, vol. 162, pages 661-682.

Lambertini L.; Orsini R., 2002. Vertically Differentiated Monopoly with a Positional Good. *Australian Economic Papers*, vol. 41, pages 151-163.

Lambertini L.; Orsini R., 2005. Positional effects, product quality and regulation in duopoly. *Louvain Economic Review*, vol. 71, pages 367-381

Leibenstein H., 1950. Bandwagon, Snob and Veblen Effects in the Theory of Consumers' Demand. *Quarterly Journal of Economics*, vol. 64, pages 183-207.

Shaked A.; Sutton J.; 1982. Relaxing Price Competition Through Product Differentiation. *Review of Economic Studies*, vol. 49, No. 1, pages 3-13.

Veblen T.; 1899 *The Theory of the Leisure Class: An Economic Study of Institutions*, Unwin Books: London; reprinted 1994, Dover Publications: New York.