Vertical Probabilistic Selling under Competition: 
The Role of Consumer Anticipated Regret

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Vertical Probabilistic Selling under Competition: 
the Role of Consumer Anticipated Regret*

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ABSTRACT

This paper studies probabilistic selling with vertically differentiated products when firms compete and consumers anticipate the potential post-purchase regret raised by possibly obtaining the inferior products. Intuitively, anticipated regret hurts the attractiveness of probabilistic selling. However, we find that probabilistic selling can be more profitable, and more likely to arise with anticipated regret than without it. This is due to the “reverse quality discrimination” (perceived quality of the random product becomes decreasing in consumer type at the competition margin), which increases the perceived differentiation, and may still maintain sufficient attractiveness of the random product for infra-marginal consumers. Meanwhile, it may hurt the competitor.

Keywords: reverse quality discrimination, probabilistic selling, vertical differentiation, anticipated regret, competition

* We gratefully acknowledge financial support from the NET Institute (www.netinst.org) through the 2016 summer grant program.
1. Introduction

Probabilistic selling is an innovative business practice of selling a random product that mixes several products whose exact identity or attributes are (partially) hidden until the purchase has been made. The probabilistic selling strategy was introduced by some online travel sites (e.g., Hotwire and Priceline) and since then has become popular (e.g. Expedia’s Special Cars, Travelocity’s Top Secret Hotels, Bookit’s Mystery Hotel, Booking’s Hidden Hotel, Ctrip’s Secret Hotels, etc.) in the industry.\(^1\) The probabilistic selling has received much attention recently, and researchers have mostly focused on its *horizontal* features (Fay and Xie 2008, 2010, Jerath et al. 2010), e.g., product colors or hotel locations.

In many situations, however, a seller mixes products with different *qualities* in probabilistic selling. For example, a hotel can create a “random hotel” by mixing its suite rooms with ordinary rooms on Hotwire. A car rental company can use full-size cars and compact cars to create a “random car” whose identity is uncovered upon pick-up. A cruise company can mix ocean-view staterooms with different square footage to create a “secret stateroom.” In all of these examples, products involved in probabilistic selling are *vertically* differentiated, implying that after purchase it is possible for a consumer to eventually obtain a lower quality product than her expectation.

Once the random product turns out to be a low quality one, unfortunately, the consumer usually cannot return it for a refund, due to the prevalent “no return” policy.\(^2\) As a result, she may regret her purchase of the random product.\(^3\) Specifically, there are two possible situations: first, she might say “I should not make any purchase”; second, she might say “I should purchase the most favorable transparent product”. We refer to the former as *purchase regret* and the later as *selection regret*. If the consumer can anticipate such a potential post-purchase regret before
buying, she will incorporate it in her purchase decisions. Since both purchase regret and selection regret reduce the attractiveness of random products to consumers, this constitutes an important drawback to random products, and intuitively, should discourage the use of probabilistic selling. However, it seems that this self-deficiency of probabilistic selling has been largely ignored from the previous studies, which might call into question the earlier insights (e.g., merits) of probabilistic selling in the literature. In this study, we explore consumers’ anticipated regret and its role in a competitive market consisted of a vertical random product and its transparent rival. Our results suggest that, rather than being a curse, consumers’ anticipated regret can be a blessing for the random product provider. Probabilistic selling can be more profitable, and more likely to arise with anticipated regret than without it. Moreover, it may hurt its competitor’s profit.

In our setting, we incorporate some other important features of probabilistic selling in a vertical market. Industrial evidence suggests that firms with high quality normally do not participate in probabilistic selling, because their image may get hurt by the discounted prices associated with random products (See Özer and Phillips, 2012). This concern has become more salient recently since the identity of products involved in probabilistic selling might be revealed online to consumers (e.g. Hotwire). Accordingly, in our two-firm model, one firm (Firm H) provides a product with high quality (product H) and sells it transparently. The other firm (Firm R) provides two products with lower and different qualities (products M and L) and can mix them to create any possible random products, in addition to transparent products M and L.

We start with the benchmark case in which consumers have no regret. We show that Firm R offers the random product only when the quality of product H is intermediate. When product H’s quality is too high, Firm R offers product M because the product differentiation is large
enough and it can extract more surplus from the consumers who value quality without worrying much about competition. When product $H$’s quality is too low, Firm $R$ will only offer product $L$ to maximize differentiation from product $H$. Therefore, when the quality of product $H$ is intermediate, a random product that mixes $M$ and $L$ should be offered to better balance surplus extraction and product differentiation by adjusting the probability of obtaining $L$.

We then explore the case in which consumers can anticipate the potential-post purchase regret. Intuitively, because of the anticipated regret, the random product looks less appealing to consumers, implying that the probabilistic selling strategy should be unattractive to both consumers and firms. However, our results suggest that the consumers’ anticipated regret can actually incentivize the firm to adopt probabilistic selling, depending on the relative magnitude of consumers’ sensitivity to purchase regret and selection regret. Specifically, we show that purchase regret makes the perceived quality of the random product follow the nature order (increase in taste for quality), while selection regret makes this order reversed (non-increasing in taste for quality). We call the latter situation as “reverse quality discrimination” and our results show that, when consumers are more sensitive to selection regret, a creative strategy “reverse quality discrimination” can be used by the random product provider: the anticipated regret deteriorates the perceived quality of the random product to a greater extent for higher type consumers. Such reverse discrimination can increase perceived product differentiation for consumers at competition margin, and maintain the attractiveness of the random product for infra-marginal consumers. That is, under this situation, probabilistic selling can both increase perceived product differentiation and better extract surplus. Thus, it is indeed more encouraged than in the benchmark case when consumers have no regret. Moreover, probabilistic selling may hurt its competitor’s profit. However, when consumers are more sensitive to purchase regret, we
show that probabilistic selling with anticipated regret might yield lower profits than in the benchmark case when consumers have no regret because the effect of “reverse quality discrimination” vanishes.

Furthermore, even when consumers are extremely averse to selection regret, the random product should still be provided because of the benefits from the “reverse quality discrimination.” In fact, as long as the selection regret sensitivity is sufficiently large, probabilistic selling always arises. As the selection regret sensitivity increases, the random product provider increases the quality of the random product towards $M$, which makes the competing firms’ actual quality levels closer to each other, but never reaches $M$. This is because the benefit of the “reverse quality discrimination” will disappear once a random product is degenerated to a transparent one (i.e., the random product’s quality is increased to $M$ for sure).

2. Literature Review

Our research connects to the recent growing literature of probabilistic selling. Some studies explore how a firm uses its existing distinct products in probabilistic selling (Fay and Xie 2010, 2015) while others examine the use of probabilistic selling in competitive markets (Fay 2008, Shapiro and Shi 2008, Jerath et al. 2010, Chen et al. 2014). However, they all focus on the horizontal feature of random products. Interestingly, the most recent literature on probabilistic selling seems to shift the focus on the vertical feature of random products (Huang and Yu 2014, Zhang et al. 2015). Although the literature has explored the use of probabilistic selling strategy under different situations, the potential post-purchase regret is largely ignored, which might challenge the merits of this strategy. To the best of our knowledge, we are among the first to explore anticipated regret in probabilistic selling. More importantly, we discover an interesting mechanism associated with probabilistic selling to increase the profit of the random product
provider and might hurt its competitor in the vertical market: the reverse quality discrimination, which is new to the literature. Thus, our study shows that, rather than being a curse, consumers’ anticipated regret can actually be a blessing to the probabilistic selling strategy.

Our work builds off the literature relating to consumers’ anticipated regret. Zeelenberg et al. (1996) suggest that consumers may anticipate the possible post-purchase regret and incorporate it when making a choice before purchase. Filiz-Ozbay and Ozbay (2007) investigate the role of anticipated regret in bidding behavior. Jiang et al. (2015) study the effect of anticipated regret on product innovation. Nasiry and Popescu (2012) focus on how anticipated regret affects advance purchase decision. Our paper focuses on the role of anticipated regret in the adoption of probabilistic selling strategy.

In addition, random products usually consist of unsold units. The previous literature often explains random products as an inventory management strategy subject to capacity constraints. Interestingly, data show that the unsold units/services in the travel industry have been overwhelming in the recent years (e.g. more than 40% of hotel rooms in North America and more than 50% in China were vacant each year from 2006 to 2014). Thus, some research (Shapiro and Shi 2008, Fay and Xie 2010, Huang and Yu 2014) as well as this paper offer different rationales for probabilistic selling in the absence of capacity constraints.

Finally, our work is also related to the growing literature of behavioral economics and its implications in marketing research (Cui et al. 2007, Chen et al. 2010, Chen and Cui 2013, Chen and Turut 2013, Chioveanu and Zhou 2013, Narasimhan and Turut 2013, Huang and Yu 2014, Jiang et al. 2014, Guo 2015, Guo and Jiang 2015, Jiang et al. 2015). Our paper provides contributions to the literature in three ways. First, we are among the first to explore the implications of anticipated regret in vertical probabilistic selling strategy and study how this self-
deficiency of probabilistic selling affects the use of this strategy. Second, we identify a new mechanism associated with vertical probabilistic selling: the “reverse quality discrimination,” which can simultaneously increase product differentiation and extract consumer surplus. Third, our paper provides potential behavioral implications of the “no return” policy in probabilistic selling: making some consumers feel regret yields higher profit to the random product provider.

The paper is organized as follows. Section 3 sets up the model for consumer anticipated post-purchase regret. Section 4 analyzes the benchmark case in which consumers have no regret; Sections 5 explores the situation in which consumers can anticipate the regret. Section 6 discusses the implications of our main results, and Section 7 concludes. Notations are summarized in Table 1.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Symbol</th>
<th>No Regret</th>
<th>With Anticipated Regret</th>
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</thead>
<tbody>
<tr>
<td>Regret Intensity on buying</td>
<td></td>
<td>NA</td>
<td>$\gamma_0$</td>
</tr>
<tr>
<td>Regret Intensity on not buying $H$</td>
<td></td>
<td>NA</td>
<td>$\gamma_1$</td>
</tr>
<tr>
<td>Quality of Product $H, M, R, L$</td>
<td></td>
<td>$q_H, q_M, q_R, q_L = 1$</td>
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</tr>
<tr>
<td>Profits of Firms $H$ and $R$</td>
<td></td>
<td>$\pi_H, \pi_R$</td>
<td></td>
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<tr>
<td>Mixing Probability of Product $R$</td>
<td></td>
<td>$\alpha_R$</td>
<td></td>
</tr>
<tr>
<td>Cutoff between No-Purchase and Purchase from Firm $R$</td>
<td>$\theta_0$</td>
<td></td>
<td>$\hat{\theta}_0$</td>
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<tr>
<td>Cutoff between Purchase from Firms $H$ and $R$</td>
<td>$\theta_R$</td>
<td></td>
<td>$\hat{\theta}_R$</td>
</tr>
<tr>
<td>Cutoff between Regret or Not Regret on not Buying $H$</td>
<td>NA</td>
<td></td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>Cutoff between Regret on Buying and on not Buying $H$</td>
<td>NA</td>
<td></td>
<td>$\begin{cases} NA &amp; \text{if } \theta_1 \geq p_R \ \theta_2 &amp; \text{if } \theta_1 &lt; p_R \end{cases}$</td>
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<tr>
<td>Perceived Quality of Product $R$</td>
<td></td>
<td>$q_R$</td>
<td>$q_{R_{pd}}$</td>
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<td>Equilibrium Quality of Product $R$</td>
<td>$q^*_R$</td>
<td></td>
<td>$\hat{q}^*_R$</td>
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<tr>
<td>Equilibrium Prices</td>
<td>$p^<em>_H, p^</em>_R$</td>
<td></td>
<td>$\hat{p}^<em>_H, \hat{p}^</em>_R$</td>
</tr>
<tr>
<td>Equilibrium Profits</td>
<td>$\pi^<em>_H, \pi^</em>_R$</td>
<td></td>
<td>$\hat{\pi}^<em>_H, \hat{\pi}^</em>_R$</td>
</tr>
<tr>
<td>Equilibrium Mixing Probability</td>
<td>$\alpha^*_R$</td>
<td></td>
<td>$\hat{\alpha}^*_R$</td>
</tr>
</tbody>
</table>
3. Model

There are two firms, $H$ and $R$, each selling one product. Firm $H$ offers a transparent product $H$ with quality level $q_H$. Firm $R$ can offer a transparent product $M$ with quality level $q_M$, or a transparent product $L$ with quality level $q_L$, or a random product $R$ that mixes products $M$ and $L$ with a self-select probability. Here $0 < q_L < q_M < q_H$. Without loss of generality, we normalize $q_L$ to one. Denote by $\alpha_R \in [0,1]$ the probability that consumers obtain $L$ when purchasing random product $R$. Thus, the expected quality of random product $R$ is given by

$$q_R(\alpha_R) = \alpha_R + (1 - \alpha_R) \cdot q_M.$$  

Note that transparent product $L$ or $M$ can be viewed as the special case of random product $R$ respectively when $q_R = 1$ or $q_R = q_M$. Since we focus on offering random product as a marketing/sale strategy, given the products are already produced, we abstract away from production stage, and assume that the marginal cost of production is zero.\(^6\)

There is a unit mass of consumers and each consumer at most buys one product. A consumer’s quality taste $\theta$ is uniformly distributed on a unit interval $[0,1]$. For a consumer with taste $\theta$ buying product $i$ ($i = H, M, L, \text{ or } R$) at price $p_i$, she enjoys a standard utility

$$u = \theta \cdot q_i - p_i \ (i = H, M, L, R)$$

when she has no regret. However, when she can anticipate the regret, her utility is given by

$$u = \begin{cases} 
\theta \cdot q_i - p_i & \text{if } i = H, M, L \\
\theta \cdot q_i - p_i - \text{regret} & \text{if } i = R
\end{cases}.$$  

Note that none of the transparent products ($i = H, M, L$) would result in any disutility from regret, because buying them does not involve any uncertainty. But for random product $R$, the utility function has a disutility term (denoted by $\text{regret}$) that represents the potential post-purchase regret.
The timeline of the game is as follows. At Date 0, Firm $R$ chooses whether to offer a random product and the corresponding expected quality $q_R$ if it chooses to offer $R$. At Date 1, both firms compete in prices simultaneously. At Date 2, consumers make their purchase decisions. Throughout the whole game, we assume complete information. That is, when firms set their prices, they are fully aware of each other’s product (expected) quality; when consumers make choices, they observe the prices for all the products. In addition, they also observe the quality for the transparent products and rationally deduce the expected quality of the random product, if any. We will determine the subgame-perfect equilibrium of the game in the next section.

4. Benchmark: No Regret

First, we consider a benchmark case in which consumers have no regret associated with the random product.

Since it is a sequential-move, complete information game, we solve it by backward induction. Starting from consumers’ product choices in the last stage, given quality-price pairs $(q_i, p_i)$ ($i = R, H$), we can obtain two cutoff tastes for quality: one for the consumer who is indifferent between no buy and buying product $R$; and the other for the consumer who is indifferent between buying $R$ and buying $H$. They are, respectively,

$$
\theta_0 = \frac{p_R}{q_R}, \quad \theta_R = \frac{p_H - p_R}{q_H - q_R}.
$$

The corresponding demand for Firm $R$ and Firm $H$ are given by $(\theta_R - \theta_0)$ and $(1 - \theta_R)$. Thus, the profits of Firm $R$ and Firm $H$ are

$$
\pi_R = p_R \cdot (\theta_R - \theta_0), \quad \pi_H = p_H \cdot (1 - \theta_R).
$$
We can solve their equilibrium prices as a function of any quality pair \((q_R, q_H)\). Then in the first stage, Firm \(R\) will choose its optimal \(q_R\) from the range \([1, q_M]\). The equilibrium for no regret case is given by the following proposition.

**Proposition 1 (No Regret):** With no regret,

(i) When \(q_H \leq \frac{7}{4}(\frac{7}{4}q_M \leq q_H)\), Firm \(R\) sells product \(L\) (\(M\)).

(ii) When \(\frac{7}{4} < q_H < \frac{7}{4}q_M\), Firm \(R\) offers a random product.

This proposition suggests that, when consumers have no regret associated with the random product, the necessary and sufficient condition for Firm \(R\) to use probabilistic selling is that the quality differentiation between Firm \(H\) and Firm \(R\) is neither too large nor too small, i.e., \(\frac{7}{4} < q_H < \frac{7}{4}q_M\). If \(q_H\) is too low, Firm \(R\) will offer product \(L\) in order to maximize differentiation. Alternatively, if \(q_H\) is too high, Firm \(R\) will offer product \(M\) to extract more surplus from consumers without worrying much about competition. The random product will emerge only when \(q_H\) is neither too high nor too low as compared to \(q_L = 1\) and \(q_M\). Indeed, given \(q_H\), there exists an optimal quality level \((\frac{4}{7}q_H)\) that Firm \(R\) would like to achieve in order to balance product differentiation and surplus extraction. When such an ideal quality falls within \([1, q_M]\), the random product will be offered; when it falls outside of the interval, only the transparent product, either \(L\) or \(M\), will be provided.\(^7\) By adopting probabilistic selling, Firm \(R\) chooses the mixing probability \((\alpha_R)\) to control its product design (the expected quality \(q_R\)).

An immediate result following from the proposition is how the optimal mixing probability \(\alpha_R^*\) changes when qualities vary.

**Corollary 1:** With no regret, the optimal mixing probability \(\alpha_R^*\) is non-decreasing (non-increasing) in \(q_M\ (q_H)\).
This corollary regards the design of the random product, operationalized by the mixing probability, when consumers have no regret. In particular, when the random product is offered, i.e., $\frac{7}{4} < q_H < \frac{7}{4} q_M$, the mixing probability increases with $q_M$. That is, Firm $R$ would like to make it more likely for consumers to eventually obtain product $L$ as $q_M$ improves. This is because as $q_M$ improves, in order to keep expected quality exactly equal to the ideal quality level $\frac{4}{7} q_H$, Firm $R$ needs to downwardly adjust the expected quality of the random product. By contrast, as $q_H$ increases, Firm $R$ wants to upwardly adjust the expected quality by making product $M$ more likely to occur in the random product.

5. Equilibrium Analysis of Anticipated Regret

We now turn to the case when consumers can anticipate the regret associated with a random product. Intuitively, when consumers can anticipate regret from a random product, Firm $R$ should be less likely to use probabilistic selling strategy compared with the benchmark case in which regret is not involved, because the random product will become less attractive to consumers when it is associated with regret. However, our findings challenge this intuition.

5.1. Definition of Anticipated Regret

We model the anticipated regret following the literature (Syam et al. 2008, Nasiry and Popescu 2012, Jiang et al. 2015). Specifically, we adopt a linear term for anticipated regret, which is defined as follows.

Definition (Anticipated Regret):

$$\text{regret} = \alpha_R \cdot \max \left\{ \gamma_0 \cdot \left[ 0 - (\theta - p_R) \right]^+, \gamma_1 \cdot \left[ (\theta q_H - p_H) - (\theta - p_R) \right]^+ \right\}, \quad (2)$$
where \( \gamma_i > 0 \) \((i = 0, 1)\) is a measure of consumer regret intensity, respectively, on buying behavior and on not buying transparent product \( H \). And \([x]^+ \equiv \max\{0, x\}\).

In the above definition, the regret occurs only if a consumer receives product \( L \) after purchasing the random product. Indeed, this is a natural result from consumer’s rational choice. That is, even if we include a similar regret term for product \( M \), we have shown that that term will disappear \textit{endogenously} (see Claim 1 in Appendix): consumers choosing the random product will never feel regret if obtaining product \( M \) under random product \( R \). It is because, if they feel regret even when receiving the best possible outcome under the random product: product \( M \), they would never choose product \( R \) in the first place. Thus, for simplicity, in our definition we do not incorporate the redundant regret term for the possible situation when \( R \) turns out to be \( M \text{ ex post} \).

It is possible that consumers regret on (i) buying behavior itself (e.g., after getting a bad hotel from a random product purchase, she may think “if I had known I was going to get the low quality hotel, I would have made no purchase and saved my money”); (ii) not buying the transparent product alternative (e.g., after getting a bad hotel from a random product purchase, she may think “if I had known I was going to get the low quality hotel, I would have gone for a transparent hotel, even though this would cost more”). Our definition of regret above allows for both types of regrets, with possibly different regret intensities: \( \gamma_0 \) for the former and \( \gamma_1 \) for the latter. More importantly, as we shall see, these two benchmarks and two intensities will play different roles in vertical probabilistic selling.

5.2. Perceived Quality and Consumer Segmentation

According to (2), the first regret term in the bracket will be positive if and only if \( \theta < p_R \); and the second term will be positive if and only if
\[ \theta > \theta_1 \equiv \frac{p_H - p_R}{q_H - 1}. \]  

(3)

It is possible to have \( \theta_1 < p_R \), in which case the consumer will regret both on buying itself and on not buying product \( H \). It is easy to calculate, for this case, the first regret term dominates the second one if and only if

\[ \theta < \theta_2 = \frac{\gamma_0 \cdot p_R + \gamma_1 \cdot (p_H - p_R)}{\gamma_0 + \gamma_1 \cdot (q_H - 1)}. \]  

(4)

Accordingly, under anticipated regret, we can define the \textit{perceived} quality of product \( R \) for a consumer with taste \( \theta \) as

\[ q_{Rpd}(\theta) \equiv q_R - \frac{\text{regret}(\theta)}{\theta}, \]

which becomes

- \( \text{when } \theta_1 \geq p_R, q_{Rpd}(\theta) = \left\{ \begin{array}{ll} q_R - \alpha_R \cdot \gamma_0 \cdot \left( \frac{p_R}{\theta} - 1 \right) & \text{if } \theta < p_R \\ q_R & \text{if } p_R \leq \theta \leq \theta_1 \\ q_R - \alpha_R \cdot \gamma_1 \cdot \left( q_H - 1 - \frac{p_H - p_R}{\theta} \right) & \text{if } \theta_1 < \theta \end{array} \right. \)

- \( \text{when } \theta_1 < p_R, q_{Rpd}(\theta) = \left\{ \begin{array}{ll} q_R - \alpha_R \cdot \gamma_0 \cdot \left( \frac{p_R}{\theta} - 1 \right) & \text{if } \theta < \theta_2 \\ q_R - \alpha_R \cdot \gamma_1 \cdot \left( q_H - 1 - \frac{p_H - p_R}{\theta} \right) & \text{if } \theta \geq \theta_2 \end{array} \right. \)

By introducing \( q_{Rpd}(\theta) \), we make a consumer’s utility under anticipated regret congruent to that with no regret as \( u_R(\theta) = \theta \cdot q_{Rpd}(\theta) - p_R. \)
Figure 1: Consumers’ Perceived Quality under Anticipated Regret

Figure 1 illustrates that, when random product $R$ is offered, different consumers perceive its quality level differently due to their heterogeneous tastes, which is summarized in the following lemma.

**Lemma 1 (Perceived Quality):** Consumers’ perceived quality of product $R$ is inverted U/V-shaped:

\[
q_{R_{pd}}(\theta) = \begin{cases} 
> 0 & \text{if } \theta < p_R \\
= 0 & \text{if } p_R \leq \theta \leq \theta_1 \\
< 0 & \text{if } \theta_1 < \theta 
\end{cases}
\]

\[
q_{R_{pd}}(\theta) = \begin{cases} 
> 0 & \text{if } \theta < \theta_2 \\
< 0 & \text{if } \theta \geq \theta_2 
\end{cases}
\]

where $\theta_1$ and $\theta_2$ are, respectively, given by (3) and (4).

Lemma 1 demonstrates that (i) the anticipated regret deteriorates the perceived quality, as intuition suggested; (ii) however, it impacts heterogeneous consumers to different extents: it deteriorates more to the two ends of consumers than to the medium ones.

For consumers with relatively low tastes (i.e., $\theta < p_R$), they are not bothered by the option of buying transparent product $H$ because they cannot afford $H$ anyway. Instead, they will regret on buying behavior itself when the random product $R$ turns out to be the low quality...
product $L$, but will regret less on that as $\theta$ increases. So their perceived quality of random product $R$ is downwardly adjusted by $\alpha_R \cdot \gamma_0 \cdot \left(\frac{p_R}{\theta} - 1\right)$. Such downward adjustment shrinks toward zero as $\theta$ increases toward $p_R$, which implies their perceived quality is increasing in $\theta$.

For consumers with relatively high tastes (i.e., $\theta > \theta_1$), they will not regret on buying even when $R$ turns out to be $L$, because their tastes for the product are sufficiently high. Nevertheless, they will regret not buying transparent product $H$ when $R$ turns out to be the less desirable outcome $L$. Consequently, their perceived quality of product $R$ will be downwardly adjusted by the potential post-purchase regret $\alpha_R \cdot \gamma_1 \cdot \left(q_H - 1 - \frac{p_H - p_R}{\theta}\right)$. In contrast to that for consumers with relatively low tastes, the downward adjustment is strictly increasing in $\theta$, which yields a decreasing perceived quality for relatively high taste consumers.

As we shall see, the intuitive natural ordering of the deteriorated perceived quality for low taste consumers conforms to the intuition that anticipated regret will reduce the attractiveness of the random product, which results a negative effect on Firm $R$’s profit. Nevertheless, the “reverse ordering” of the deteriorated perceived quality for relatively high taste consumers will work in favor of, not against, Firm $R$, because it amplifies the product differentiation between $R$ and $H$.

Next, we determine who will buy nothing, who will choose random product $R$, and who will buy product $H$. The cutoff quality taste for consumers who are indifferent between buying product $R$ and buying nothing is determined by $\theta \cdot \left[q_R - \alpha_R \cdot \gamma_0 \cdot \left(\frac{p_R}{\theta} - 1\right)\right] = 0$. That is,

$$\theta_0 = \frac{(1 + \gamma_0 \cdot \alpha_R) \cdot p_R}{\gamma_0 \cdot \alpha_R + q_R}. \quad (5)$$
The consumer choosing between product $R$ and product $H$ is guided by a cutoff quality taste that is obtained by $\theta \cdot [q_R - \alpha_R \cdot y_1 \cdot (q_H - 1 - \frac{p_H - p_R}{\theta})] = \theta \cdot q_H - p_H$. That is,

$$\hat{\theta}_R \equiv \frac{(1 + y_1 \cdot \alpha_R) \cdot (p_H - p_R)}{y_1 \cdot \alpha_R \cdot (q_H - 1) + q_H - q_R}. \quad (6)$$

Note that when $y_i = 0 \ (i = 1,2)$, these cutoffs are reduced to those when consumers have no regret in (1), i.e., $\hat{\theta}_0 = p_R / q_R = \theta_0$, and $\hat{\theta}_R = (p_H - p_R) / (q_H - q_R) = \theta_R$. Thus, the no regret benchmark case in Section 4 can be considered as a special case nested in this section.

The above results can be summarized in the lemma below.

**Lemma 2 (Consumer Segmentation):** Consumers with $\theta \in [\hat{\theta}_0, \hat{\theta}_R)$ will choose product $R$, while those with $\theta \in [\hat{\theta}_R, 1]$ will choose product $H$, where $\hat{\theta}_0$ and $\hat{\theta}_R$ are, respectively, given by (5) and (6).

Lemma 2 illustrates that consumers with sufficiently high tastes $[\hat{\theta}_R, 1]$ buy product $H$, those with intermediate tastes $[\hat{\theta}_0, \hat{\theta}_R)$ buy product $R$, and those with low tastes $[0, \hat{\theta}_0)$ leave the market without any purchase. Moreover, it can be readily verified that $\hat{\theta}_R > \theta_1$, and $\hat{\theta}_R > \theta_2$ whenever $\theta_1 < p_R$. That is, the marginal consumer who is indifferent between product $R$ and product $H$ must lie to the right side of the peak of the inverted-U/V shaped perceived quality. Therefore, the “battle field” between Firm $H$ and Firm $R$ is the high end interval $[\max\{\theta_1, \theta_2\}, 1]$, where $q_{Rpd}(\theta)$ is less than $q_M$, and more importantly, is strictly decreasing in $\theta$. Since $q_H$ is constant for all consumers in this interval, the gap between the perceived quality of random product $R$ and the quality of transparent product $H$ at the battle field becomes larger than that between $M$ and $H$, and moreover, increases as $\theta$ increases. Hence, Firm $R$’s probabilistic selling creates a “moat defensive” to Firm $H$. 
5.3. Equilibrium Characterization

The consumer segment in Lemma 1 implies that the profits of the two firms are

\[ \hat{\pi}_R = p_R \cdot (\hat{\theta}_R - \hat{\theta}_0), \quad \hat{\pi}_H = p_H \cdot (1 - \hat{\theta}_R). \]

Firm H chooses \( p_H \) to maximize \( \pi_H \) and Firm R chooses \( p_R \) to maximize \( \pi_R \). After substituting the optimal prices as a function of \( q_R \) and \( q_H \), Firm R chooses its optimal \( q_R \). The equilibrium outcomes are summarized in Proposition 2.

**Proposition 2 (Anticipation of Regret):** With anticipated regret,

(i) When \( q_H \leq \frac{7}{4} ( q_H \geq \frac{7}{4} q_M ) \), there exists a unique threshold level \( \Gamma_1(\gamma_1) \), and \( \Gamma_1(\gamma_1) \leq \gamma_1 \), such that for any \( \gamma_0 < \Gamma_1(\gamma_1) \), Firm R offers a random product; otherwise, Firm R only sells transparent product L (M).

(ii) When \( \frac{7}{4} < q_H < \frac{7}{4} q_M \), there exists a unique threshold level \( \Gamma_2(\gamma_1) \), and \( \Gamma_2(\gamma_1) \geq \gamma_1 \), such that for any \( \gamma_0 < \Gamma_2(\gamma_1) \), Firm R offers a random product; otherwise, Firm R only sells a transparent product.

Moreover, \( \Gamma_1(\gamma_1) \) and \( \Gamma_2(\gamma_1) \) are increasing in \( \gamma_1 \).

As compared with Firm R’s profit with no regret,

\[ \hat{\pi}_R^* \geq \pi_R^* \text{ if and only if } \gamma_1 \geq \gamma_0. \] (7)

This proposition establishes two results. First, with anticipated regret, as long as \( \gamma_0 \) is sufficiently small as compared to \( \gamma_1 \), i.e., \( \gamma_0 \) is below certain cutoff as an increasing function of \( \gamma_1 \), random product will always be offered. Intuitively, when consumers anticipate the potential post-purchase regret from buying the random product, the probabilistic selling strategy seems less attractive, which should be less likely to emerge in equilibrium. However, we find the opposite (anticipated regret can encourage probabilistic selling) is possible, too. Specifically,
when $\gamma_1 > \gamma_0$, in Case (i), if $\gamma_0 < \Gamma_1(\gamma_1)$, random product is always offered while it is never offered in Case (i) of Proposition 1; in Case (ii), since $\gamma_0 \leq \gamma_1 \leq \Gamma_2(\gamma_1)$, so random product is always offered, the same as Case (ii) of Proposition 1. Compared with Proposition 1, Firm $R$ is more likely to offer random product when $\gamma_1 > \gamma_0$. In contrast, when $\gamma_1 \leq \gamma_0$, in Case (i), since $\gamma_0 \geq \gamma_1 \geq \Gamma_1(\gamma_1)$, random product is never offered, the same as Case (i) of Proposition 1; in Case (ii), random product is offered only when $\gamma_0 < \Gamma_2(\gamma_1)$ while it is always offered in Case (ii) of Proposition 1. Compared with Proposition 1, Firm $R$ is less likely to offer random product when $\gamma_1 \leq \gamma_0$.

Second, Firm $R$’s profit under anticipated regret will be higher than that under no regret if and only if the regret intensity on buying itself is smaller than that on not buying $H$, i.e., $\gamma_0 < \gamma_1$. Recall from Lemmas 1 and 2 that, the consumers’ perceived quality is inverted-U/V shaped. Hence, the random product hurts consumers more on two ends: those with low taste due to their regret on buying itself, and those with high taste because of their regret on not buying $H$. The former, whose regret is affected by $\gamma_0$, perceives the quality in a natural order (increasing in $\theta$). The deteriorated perceived quality discourages them from purchasing, which forces Firm $R$ to lower its price to retain those consumers, and hence tends to reduce the profitability of probabilistic selling. The latter, whose regret hinges on $\gamma_1$, perceives the quality in a “reverse order” (decreasing in $\theta$). We refer to this as to “reverse quality discrimination.” These high end “regret consumers” ($\theta \in [\max\{\theta_1, \theta_2\}, 1]$) are at the competition “battlefield” between the two firms. Such a lowered perceived quality on the random product increases the product differentiation between random product $R$ and transparent product $H$, which leaves some room for Firm $R$ to increase its price to take advantage of the increased product differentiation at the margin, and so tends to increase the profitability of probabilistic selling. Consequently, whether
probabilistic selling with anticipated regret will be more profitable than that without regret depends on whether the latter effect dominates the former one.

We can define a uniform threshold

\[ \Gamma(\gamma_1) \equiv \begin{cases} 
\Gamma_1(\gamma_1), & \text{if } q_H \leq \frac{7}{4} \text{ or } q_H \geq \frac{7}{4} q_M \\
\Gamma_2(\gamma_1), & \text{if } \frac{7}{4} < q_H < \frac{7}{4} q_M 
\end{cases} . \]

From the proposition we know that keeping \( \gamma_1 \) unchanged, when \( \gamma_0 \) increases to a threshold \( \Gamma(\gamma_1) \), Firm R stops providing random products. The next question is if we keep \( \gamma_0 \) unchanged, what would happen as the \( \gamma_1 \) increases? The answer is summarized in the following corollary.

**Corollary 2 (Limiting Property):**

\( \Gamma(\gamma_1) \) is increasing in \( \gamma_1 \). For any \( \gamma_0 \geq 0 \), define \( \Gamma^{-1}(\gamma_0) \) as the inverse function of \( \gamma_0 = \Gamma(\gamma_1) \) if \( \gamma_0 \geq \Gamma(0) \); otherwise, define \( \Gamma^{-1}(\gamma_0) = 0 \). If \( \gamma_1 \leq \Gamma^{-1}(\gamma_0) \), Firm R sells transparent products; otherwise, it always sells random products. We have the following limiting results:

\[
\begin{align*}
\lim_{\gamma_0 \to \infty} \hat{q}_R^* &= q_M, \quad q_M - \hat{q}_R^* = O\left(\gamma_1^{-\frac{1}{2}}\right). \\
\lim_{\gamma_1 \to \infty} \hat{\pi}_R^* &= \frac{q_M \cdot (q_H + q_M - 1) \cdot (q_H - 1)}{(4q_H + 3q_M - 4)^2}. \\
\lim_{\gamma_0 \to \infty} \hat{q}_R^* &= \begin{cases} q_M, & \text{if } q_M \leq \frac{16(q_H - 1)q_H}{16q_H - 7} \\
1, & \text{otherwise} \end{cases}.
\end{align*}
\]

\[
\lim_{\gamma_0 \to \infty} \hat{\pi}_R^* = \begin{cases} q_H(q_H - q_M)q_M, & \text{if } q_M \leq \frac{16(q_H - 1)q_H}{16q_H - 7} \\
q_H(q_H - 1) & \text{(4q_H - 1)^2, otherwise} \end{cases}.
\]
This corollary delivers an important message: keeping $\gamma_0$ unchanged, even as the regret intensity $\gamma_1$ goes to infinity, Firm $R$ still provides the random product. This is because the only way to implement the “reverse quality discrimination” is to adopt the probabilistic selling. In particular, as $\gamma_1$ becomes larger, Firm $R$ chooses lower mixing probability (but never down to zero) to reduce the chance that consumers eventually obtain product $L$ from the random product. This keeps the product random so that it can lower the perceived quality for those consumers at the competition margin. That is, the provision of random products reduces the competition between firms because consumers at the competition margin view products in the market have greater perceived product differentiation, even though the actual quality levels are not that different.

5.4. Comparative Statics

As suggested by Proposition 2, the profit of Firm $R$ is increasing in $\gamma_1$, and decreasing in $\gamma_0$. We further analyze how regret intensity $\gamma_1$ and $\gamma_0$ affect the expected quality of product $R$ ($\hat{q}_R^*$), and the profit of Firm $H$ ($\hat{\pi}_H^*$).

Keeping $q_R$ and $\gamma_1$ unchanged, increasing $\gamma_0$ decreases the attractiveness of product $R$ to the low-end consumers. Therefore, Firm $R$ has to lower price and increase $q_R$, which intensifies the competition with Firm $H$. Both Firm’s profits decrease. In fact, we have the following Lemma.

**Lemma 3 (Impact of $\gamma_0$):**

*Keeping $\gamma_1$ unchanged, the prices, demands and profits of both firms are decreasing in $\gamma_0$.***
When $\gamma_0 \geq \Gamma(\gamma_1)$, Firm $R$ chooses not to sell random products. Therefore, as $\gamma_0$ goes large, $q_R$ may increase to $q_M$ or decrease to $q_L$, depending on which deterministic product gives larger profit. See Figure 2.

**Figure 2. The equilibrium product quality of Firm $R$ as $\gamma_0$ varies**

![Graph showing equilibrium product quality of Firm R](image)

(a) $q_H = 3, q_M = 2$  
(b) $q_H = 1.5, q_M = 1.2$

Note. Blue line $\gamma_1 = 0$, orange line $\gamma_1 = 2$.

Keeping $\gamma_2$ unchanged, we do not have nice property the profit of Firm $H$ as Lemma when varying $\gamma_1$. Keeping $q_R$ and $\gamma_2$ unchanged, increasing $\gamma_1$ softens competition between Firm $R$ and Firm $H$. This benefits both Firm $R$ and Firm $H$. However, Firm $R$ can adjust $q_R$ to further increase its profit. When Firm $R$ increases $q_R$, the competition could be intensified to hurt of the profit of Firm $H$, as shown in Figure 4.

**Figure 3. $\hat{q}_R^*$ and $\pi_H$ as $\gamma_1$ varies when $q_H = 3, q_M = 2$**

![Graph showing $q_R^*$ and $\pi_H$](image)
Note. Blue line $\gamma_0 = 0$, orange line $\gamma_0 = 2$.

**Figure 4.** $q_R^*$ and $\pi_H$ as $\gamma_1$ varies when $q_H = 1.5, q_M = 1.2$

We know that as $\gamma_1 \to \infty$, $q_R \to q_M$. When $\gamma_1$ is large enough, as $\gamma_1$ increases, Firm $R$ needs to increase $q_R$ to keep product $R$ attractive to consumers but never reaches $q_M$, otherwise it loses the advantage of reverse quality discrimination. In Figure 3, product $H$’s quality is low, so Firm $R$ offers product L when $\gamma_1$ is low. As $\gamma_1$ increases, Firm $R$ keeps increasing $q_R$. In contrast, Figure 2 shows when product $H$’s quality is high, Firm $R$ offers $q_M$ or $q_R > q_L$ when $\gamma_1 = 0$. The competition is intense when $\gamma_1 = 0$. As $\gamma_1$ increases, firm $R$ may want to decrease $q_R$ first to further soften competition. Eventually, when $\gamma_1$ large enough, $q_R$ increases in $\gamma$.

6. Implications and Discussions

Our main results indicate overall that the probabilistic selling strategy can obtain higher profits, even though the possibility of obtaining an unfavorable outcome can yield post-purchase regret and consumers can anticipate such potential regret before purchase. Specifically, our results suggest that, when consumers are more sensitive to selection regret (“I should have purchased the best transparent product”) over purchase regret (“I should not have made any purchase”), consumers’ anticipated post-purchase regret can be a blessing, rather than being a curse. In
practice, the probabilistic selling strategy was introduced by Hotwire and Priceline, and prevails more in the travel industry than in others. Dellaert et al. (1998) show that people normally decide whether or not to travel several months before sort out travel logistics (e.g. reserving hotels). Thus, once an unfavorable outcome presents, they feel more regret in their travel logistics selection over the decision on whether or not to travel because people’s regret is more sensitive to the available alternatives (e.g., available hotel options) right before the revelation of the random outcome (e.g., purchasing a random hotel) than to the decision several months ago (e.g., deciding whether to travel) (see Zeelenberg 1999). Therefore, our results might explain why probabilistic selling strategy has be popular in the travel industry.

In addition, we show that greater selection regret sensitivity can always yield higher profit to the random product provider. Thus, a random product provider can find potential ways to increase consumers’ selection regret sensitivity: random product providers often share posts with consumers on TripAdvisor (a main third-party online travel forum) about the potential risk associated with the purchase of random products, and the “no return” policy are highlighted right beside the purchase button on Hotwire.

Last but not least, according to our results, no matter whether regret is considered or not, there exists an “ideal” quality for the random product to benefit its provider in the vertical market. However, a firm may find that its current production capabilities do not match up with the “ideal” quality. This is normally because the infrastructure/fixed investment costs (e.g. staterooms of a cruise ship) in the travel industry are normally sunk and hard to renovate. Interestingly, our results show that a random product can achieve this “ideal” quality as long as it is between the two extreme qualities being produced already. Importantly, offering a random product with an expected quality at this “ideal” level is more profitable than offering a
transparent product of this “ideal” quality. For example, suppose that a cruise ship has two
types of staterooms, 150 sq. ft. and 450 sq. ft., but that the “ideal” quality is 300 sq. ft.
Renovating the cruise ship to make all the rooms 300 sq. ft. would be an extremely costly
endeavor. But, mixing the two room types to create a random stateroom is much easier to do.
And our results suggest that whenever the “ideal” quality is 300 sq. ft., selling probabilistic
staterooms with an expected square footage of 300 sq. ft. would be more profitable than having
a cruise ship with all the staterooms exactly 300 sq. ft. for certain.

7. Conclusion and Limitations

Random products often consist of alternatives that differ in quality, and consumers may obtain
less desirable ones whose identity is revealed only after purchase. Because they cannot return
their purchase due to the “no return” policy, they might feel regret buying them. In this paper, we
study the role of consumers’ potential post-purchase regret in the use of probabilistic selling
strategy, when they can anticipate the regret before purchase. We find that the role of anticipated
regret is not straightforward. Specifically, the anticipated regret can be either a blessing or a
curse to the random product provider, depending on whether consumers are more sensitive to
selection regret or not. If consumers are more sensitive to selection regret, we identify a new
mechanism associated with vertical probabilistic selling: “reverse quality discrimination” which
increases random product provider’s profit by extracting more consumer surplus, and meanwhile
may hurt its competitor’s profit. Thus, anticipated regret is a blessing under this situation.
However, if consumers are less sensitive to selection regret, the effect of “reverse quality
discrimination” vanishes and we show that anticipated regret becomes a curse to the random
product provider.
This paper is only the first attempt to study the interactions between consumer anticipated regret and the vertical probabilistic selling. There are many omitted factors in this paper that leave several possible and interesting avenues for future research. First, we focus on the situation in which the marginal cost of production is negligible, because the infrastructure/fixed investment costs (e.g., hotel rooms) are significant but often sunk in the travel industry in which probabilistic selling is mainly applied, while the maintenance costs are marginal and less salient. Future studies can explore the situation with non-negligible marginal cost of production. By continuity, our main results are expected to remain valid when the marginal cost is close to zero. However, with large marginal cost, firms might choose a different product line. Note that Johnson and Myatt (2003) considered product line design with a focus on the discussions of marginal cost and quality competition. They show that a firm might provide multiple products with different quality levels under certain conditions. In the context of probabilistic selling, a different product line will affect the alternatives that consumers may forgo when selecting products, thus in turn influence her regret benchmarks. The possibility of multiple regret benchmarks will make the analysis much more involved. And general product line design with general cost function is beyond the scope of this paper.

Second, another important issue related to large marginal cost is credible quality commitment in vertical probabilistic selling: when the marginal cost is correlated with quality, the random product provider would like consumers to think that buying a random product can give them a higher quality product at a low price. But at the same time, the provider may be tempted to “cheat” by mixing with lower cost/quality products more frequently. Although nowadays most of the random product providers have delegated the probabilistic selling through online travel intermediaries (e.g., Hotwire), this still cannot guarantee that there is no collusion
between the random product providers and intermediaries because the latter might need to keep
the former’s competition in check (some studies have explored the provision of transparent
products on online intermediaries, e.g., Chen et al. 2002, Iyer and Pazgal 2003, Iyer and
Padmanabhan 2006, Dukes and Liu 2016). Future research could focus on these issues related to
marginal costs.

Third, we assume that the high quality firm does not use probabilistic selling due to
image concern. Future research might explore the incentive for a high quality multiproduct seller
in using probabilistic selling strategy. Intuitively, the high quality seller, who mainly serves high
valuation consumers, would like to choose to offer only the high quality product transparently if
the marginal cost is minimal, because the reverse quality discrimination associated with
probabilistic selling implies that high valuation consumers dislike random products more than
low valuation consumers.

Fourth, it might be interesting to discuss how anticipated regret differs from risk
aversion. The similarity of these two concepts is that both reduce the expected quality of the
random product, and the loss becomes larger as the quality difference of the component products
increases. However, one key difference is that anticipated regret is built off the best forgone
alternative in regret benchmarks, while risk aversion’s materialization, instead, is defined over
the random product’s own uncertainty, which could be independent of foregone alternatives.
This might bring a new angle to explore probabilistic selling strategy in a vertical market.
Last, we have focused on the vertical dimension of probabilistic selling strategy. It is well known
in the literature that random products involve many horizontal features (e.g., hotel location). It
might be interesting to explore a random product provider’s product line design when both
horizontal and vertical features are considered (some studies in the literature have explored
Reference:


APPENDIX

Claim 1 (Never Regret if Getting $M$ from $R$): Whenever random product $R$ is chosen,

$$\theta q_M - p_R \geq \max\{0, \theta q_H - p_H\}.$$

Proof of Claim 1. Since $u = \theta q_R - p_R - \text{regret}$ and \(\text{regret} \geq 0\), we must have $u \leq \theta q_R - p_R$.

Thus, in the presence of transparent product alternative and no buy option, product $R$ will be chosen only if $\theta q_R - p_R \geq \max\{0, \theta q_H - p_H\}$. It follows that $\theta q_M - p_R \geq \theta q_R - p_R \geq \max\{0, \theta q_H - p_H\}$.

Claim 1 guarantees that, whenever random product $R$ is chosen, the term

$$(1 - \alpha_R) \cdot \max\{\gamma_0 \cdot [0 - (\theta q_M - p_R)]^+, \gamma_1 \cdot [(\theta q_H - p_H) - (\theta q_M - p_R)]^+\} = 0.$$

So our definition of $\text{regret} = \alpha_R \cdot \max\{\gamma_0 \cdot [0 - (\theta - p_R)]^+, \gamma_1 \cdot [(\theta q_H - p_H) - (\theta - p_R)]^+\}$ is justified.

Proof of Proposition 1. In the last stage, given $q_R$, we can solve the equilibrium prices in the pricing stage as a function of $q_R$:

$$p_R(q_R) = \frac{q_R \cdot (q_H - q_R)}{4q_H - q_R}, \quad p_H(q_R) = \frac{2q_H \cdot (q_H - q_R)}{4q_H - q_R}.$$

So Firm $R$ and Firm $H$’s profits are

$$\pi_R(q_R) = \frac{q_H \cdot q_R \cdot (q_H - q_R)}{(4q_H - q_R)^2}, \quad \pi_H(q_R) = \frac{4q_H^2 \cdot (q_H - q_R)}{(4q_H - q_R)^2}.$$

In the first stage, Firm $R$ chooses $q_R \in [1, q_M]$ to maximize $\pi_R$; and

$$\frac{d\pi_R}{dq_R} = \frac{q_H \cdot (4q_H - 7q_R)}{(4q_H - q_R)^3}.$$
Hence, we have an interior solution \( q_R^* = \frac{4}{7} q_H \) if and only if \( 1 < \frac{4}{7} q_H < q_M \); otherwise we will have a corner solution \( q_R^* = 1 \) when \( \frac{4}{7} q_H \leq 1 < q_M \), or a corner solution \( q_R^* = q_M \) when \( 1 < q_M \leq \frac{4}{7} q_H \). The equilibrium mixing probability \( \alpha^* \) and profits then follow.

**Proof of Corollary 1.** It follows from direct differentiations. ■

**Proof of Lemma 1.** It directly follows from (4). ■

**Proof of Lemma 2.** The analysis is in the main text near (5) and (6). ■

**Proof of Proposition 2.** Define

\[
A(q_R) \equiv \frac{(q_H - q_R) \cdot (q_M - 1) + \gamma_1 \cdot (q_M - q_R) \cdot (q_H - 1)}{q_M - 1 + \gamma_1 \cdot (q_M - q_R)} > 0, \tag{A.1}
\]

and

\[
B(q_R) \equiv \frac{q_R \cdot (q_M - 1) + \gamma_0 \cdot (q_M - q_R)}{q_M - 1 + \gamma_0 \cdot (q_M - q_R)} > 0. \tag{A.2}
\]

Using notations above, from (5), we have \( \hat{\theta}_0 = \frac{p_R}{B(q_R)} \); from (6), we have \( \hat{\theta}_R = \frac{p_H - p_R}{A(q_R)} \). If follows that

\[
\hat{\alpha}_R = p_R \cdot (\hat{\theta}_R - \hat{\theta}_0) = p_R \cdot \left( \frac{p_H - p_R}{A} - \frac{p_R}{B} \right),
\]

and

\[
\hat{\alpha}_H = p_H \cdot (1 - \hat{\theta}_R) = p_H \cdot \left( 1 - \frac{p_H - p_R}{A} \right).
\]

The first-order conditions for \( \frac{\partial \hat{\alpha}_R}{\partial p_R} = 0 \) and \( \frac{\partial \hat{\alpha}_H}{\partial p_H} = 0 \) can be simplified to \( p_R = \frac{p_H - p_R}{2(A+B)} \) and \( p_H = \frac{p_R + A}{2} \). We can solve for \( p_R = \frac{A \cdot B}{4A + 3B} \) and \( p_H = \frac{2A(A+B)}{4A + 3B} \). Plugging them in to \( \hat{\alpha}_R \), we get

\[
\hat{\alpha}_R(q_R) = \frac{A \cdot B \cdot (A + B)}{(4A + 3B)^2}. \tag{A.3}
\]
To determine the optimal quality level $\hat{q}_R$, we differentiate Firm R’s profit function with respect to $q_R$

$$\frac{d\hat{q}_R}{dq_R} = \frac{B^2 \cdot (2A + 3B) \cdot (q_M - 1)^2 \cdot (1 + \gamma_0)}{(4A + 3B)^3 \cdot [q_M - 1 + \gamma_0 \cdot (q_M - q_R)]^2} \cdot f(q_R),$$

where

$$f(q_R) \equiv \frac{A^2 \cdot (4A + 5B) - 1 + \gamma_1}{B^2 \cdot (2A + 3B)} - \frac{1 + \gamma_1}{1 + \gamma_0} \cdot \frac{[q_M - 1 + \gamma_0 \cdot (q_M - q_R)]^2}{[q_M - 1 + \gamma_1 \cdot (q_M - q_R)]^2}. \quad (A.4)$$

(Equilibrium Characterization) We prove this part in the following 3 steps.

**Step 1: The signs of $f(1)$ and $f(q_M)$.

From (A.4), $f(1) = \frac{1}{4} \left[ 13 - \frac{4(1+\gamma_0)}{1+\gamma_1} - 18q_H + 8q_H^2 - \frac{9}{1+2q_H} \right]$. It follows that

$$f(1) \geq 0 \text{ if and only if } \gamma_0 \leq L(\gamma_1),$$

where $L(\gamma_1) \equiv \gamma_1 + (1 + \gamma_1) \cdot \frac{(4q_H-7)q_H^2}{2q_H+1}$.

Similarly, $f(q_M) = -\frac{1+\gamma_1}{1+\gamma_0} + \frac{(q_M-q_H)^2(q_H+q_M)}{q_M(2q_H+q_M)}$. We can show that

$$f(q_M) \geq 0 \text{ if and only if } \gamma_0 \geq M(\gamma_1),$$

where $M(\gamma_1) \equiv \gamma_1 + (1 + \gamma_1) \cdot \frac{(7q_H-4q_H^2)q_H^2}{(q_H-q_M)^2(4q_H+q_M)}$.

Clearly, $L(\gamma_1) \leq \gamma_1$ if and only if $q_H \leq \frac{7}{4}q_M$, and $M(\gamma_1) \leq \gamma_1$ if and only if $q_H \leq \frac{7}{4}q_M$.

**Step 2: Property of $f'(q_R)$.

From (A.4), if we denote $\varphi(q_R) \equiv \frac{A^2 \cdot (4A + 5B)}{B^2 \cdot (2A + 3B)}$ and $\mu(q_R) \equiv \frac{1+\gamma_1}{1+\gamma_0} \cdot \frac{[q_M-1+\gamma_0 \cdot (q_M- q_R)]^2}{[q_M-1+\gamma_1 \cdot (q_M- q_R)]^2}$, then

$$f(q_R) = \varphi(q_R) - \mu(q_R).$$

It is easy to see that $\frac{\partial \varphi}{\partial A} > 0$, $A'(q_R) < 0$, $\frac{\partial \varphi}{\partial B} < 0$, and $B'(q_R) > 0$.

Thus, we have $\varphi'(q_R) < 0$. Note that $\mu'(q_R) = (\gamma_1 - \gamma_0) \cdot \frac{1+\gamma_1}{1+\gamma_0} \cdot \frac{2(qM-1)qM^{-1}+\gamma_0 \cdot (qM- q_R)}{[qM-1+\gamma_1 \cdot (qM- q_R)]^3}$

$$0 \text{ if and only if } \gamma_0 \leq \gamma_1.$$
So when $\gamma_0 \leq \gamma_1$, we must have $f'(q_R) < 0$. In this case, the equilibrium $\hat{q}_R^*$ is either at the endpoint 1 or $q_M$ or satisfies $f(q_R) = 0$ (FOC). When $\gamma_0 > \gamma_1$, we may not always have $f'(q_R) < 0$, but we are still able to characterize the equilibrium with the help of a property of $\hat{r}_R$. 

**Step 3: Property of $\hat{r}_R(q_R)$.**

We show how $\hat{r}_R$ and thus $\hat{r}_R^*$ change when $\gamma_0$ and $\gamma_1$ vary. That is, 

$$\frac{\partial \hat{r}_R^*}{\partial \gamma_0} \leq 0, \text{ and } \frac{\partial \hat{r}_R^*}{\partial \gamma_1} \geq 0.$$ 

From (A.3), 

$$\frac{\partial \hat{r}_R}{\partial \gamma_0} = \frac{\partial r_R}{\partial A} \cdot A'(\gamma_0) + \frac{\partial r_R}{\partial B} \cdot B'(\gamma_0).$$ 

Note that $A'(\gamma_0) = 0, \frac{\partial r_R}{\partial B} = \frac{A^2(4A+5B)}{(4A+3B)^3} > 0,$ and $B'(\gamma_0) = -\frac{(q_R-1)(q_M-1)(q_M-q_R)}{[q_M-1+y_0(q_M-q_R)]^2} < 0$ for $q_R \in (1, q_M)$. Thus, we have $\frac{\partial \hat{r}_R}{\partial \gamma_0} < 0$ for 

$q_R \in (1, q_M)$, which implies $\frac{\partial \hat{r}_R^*}{\partial \gamma_0} \leq 0$. Parallel, we can show $\frac{\partial \hat{r}_R}{\partial \gamma_1} > 0$ for $q_R \in (1, q_M)$, which implies $\frac{\partial \hat{r}_R^*}{\partial \gamma_1} \geq 0$. Therefore, the curve $\hat{r}_R$ between 1 and $q_M$ falls as $\gamma_0$ increases.

Note that $\hat{r}_R(1)$ and $\hat{r}_R(q_M)$ are independent of $\gamma_0$ or $\gamma_1$, because the transparent products do not involve any regret.

**Step 4: Characterize the thresholds on $\gamma_0$**

Define $\Gamma_1(\gamma_1) \equiv \begin{cases} L(\gamma_1), & \text{if } q_H \leq \frac{7}{4}, \\ M(\gamma_1), & \text{if } q_H \geq \frac{7}{4} q_M. \end{cases}$

(1) If $q_H \leq \frac{7}{4}$, 

From Step 1, we know that $L(\gamma_1) \leq \gamma_1 \leq M(\gamma_1)$. Thus, for $\gamma_0 < \Gamma_1(\gamma_1) = L(\gamma_1)$, we have $f(1) > 0$ and $f(q_M) < 0$, and hence we must have a unique $\hat{q}_R^* \in (1, q_M)$ such that $f(\hat{q}_R^*) = 0$. When $\gamma_0 = L(\gamma_1) \leq \gamma_1$, we have $f(1) = 0$, and $f'(q_R) < 0$ for any $q_R \in$
[1, q_M], which implies \( q_R^* = 1 \) and Firm R sells transparent product L. When \( \gamma_0 > L(\gamma_1) \), profit \( \hat{\pi}_R(q_R) \) on \((1, q_M)\) decreases and Firm R still sells transparent product L.

(2) If \( q_H \geq \frac{7}{4} q_M \),

It follows parallel, where \( \Gamma_1(\gamma_1) = M(\gamma_1) \).

(3) If \( \frac{7}{4} < q_H < \frac{7}{4} q_M \),

We will show by contradiction that when \( \gamma_0 \) goes to infinity, \( q_R^* \) converges to the endpoint 1 or \( q_M \).

Suppose as \( \gamma_0 \to \infty \), we have \( \hat{q}_R^* \to q \in (1, q_M) \). That is, for \( \epsilon \equiv \min\{q_M - q, q - 1\} > 0 \), there exists a constant \( N \), as long as \( \gamma_0 > N \), \( 1 \leq q - \epsilon < \hat{q}_R^* < q + \epsilon \leq q_M \). That means \( \hat{q}_R^* \) satisfies the FOC \( f(q_R) = \varphi(q_R) - \mu(q_R) = 0 \). However, as \( \gamma_0 \to \infty \), \( \varphi(q_R) \leq 2A^2 \leq 2(q_H - 1)^2 \) is bounded, whereas \( \mu(q_R) \to \infty \) (note that \( q_R < q_M \)). That means, when \( \gamma_0 \) is large enough, \( \mu(q_R) > \varphi(q_R) \). This is a contradiction. Therefore, \( \hat{q}_R^*(\gamma_0) \) cannot converge to a constant with in \((1, q_M)\). Since \( \hat{q}_R^* \) is in a bounded interval, as \( \gamma_0 \) increases the sequence \( \hat{q}_R^*(\gamma_0) \) must have a convergent subsequence. Use the same contradiction argument, the convergent subsequence cannot converges to any point within \((1, q_M)\). It must converges to either 1 or \( q_M \). Since the profit at the two endpoints is not affected by \( \gamma_0 \).

If \( q_M \leq \frac{16(q_H-1)q_H}{16q_H-7} \), we have \( \hat{\pi}_R(q_M) \geq \hat{\pi}_R(1) \); otherwise we have \( \hat{\pi}_R(q_M) < \hat{\pi}_R(1) \). We assume that Firm R always chooses \( q_M \) when \( q_M = \frac{16(q_H-1)q_H}{16q_H-7} \). Then we cannot have two convergent subsequences that one converges to 1 and the other converges to \( q_M \). Therefore, the sequence \( \hat{q}_R^* \) is convergent, either to 1 or \( q_M \). Two cases may follow: 1) \( \hat{q}_R^* \) never reaches the endpoint, then \( \Gamma_2(\gamma_1) = \infty \); 2) \( \hat{q}_R^* \) reaches the endpoint, then \( \Gamma_2(\gamma_1) = \min_{\hat{q}_R^*(\gamma_0)=q_M} \gamma_0 < \)
We have shown that $\hat{\pi}_R^*$ is non-increasing in $\gamma_0$. For any $\gamma_0 \geq \Gamma_2(\gamma_1)$, we must have $\hat{\pi}_R^*$ remains the same and $\hat{q}_R^*(\gamma_0)$ remains at the endpoint. From Step 1, we know that if $\gamma_0 < \min\{L(\gamma_1), M(\gamma_1)\}$, then $f(1) > 0$ and $f(q_M) < 0$. Thus, Firm R would sell a random product, implying a lower bound of $\Gamma_2(\gamma_1)$ is $\min\{L(\gamma_1), M(\gamma_1)\}$.

Next we show the monotonicity of $\Gamma_2(\gamma_1)$. Given $\gamma_1$, if $\Gamma_2(\gamma_1) < \infty$, when $\gamma_0 = \Gamma_2(\gamma_1)$ there are two possible cases: 1) the derivative $\frac{d\hat{\pi}_R}{dq_R}$ at one endpoint is 0; 2) the FOC point in the middle gives the same profit as the higher end point. Then for any $\bar{\gamma}_1 > \gamma_1$, in case 1), if $f(1) = \frac{1}{4}$,

$$13 - \frac{4(1+\gamma_0)}{1+\gamma_1} - 18q_H + 8q_M^2 - \frac{9}{1+2q_H} = 0, \text{ at } \bar{\gamma}_1, f(1) > 0; \text{ if } f(q_M) = -\frac{1+\gamma_1}{1+\gamma_0} + \frac{(q_H-q_M)(4q_H+q_M)}{q_M^2(2q_H+q_M)} = 0, \text{ at } \bar{\gamma}_1, f(q_M) < 0.\text{ Therefore the endpoints 1 or } q_M \text{ is no longer the maximum point, and Firm R chooses to offer random product, implying } \Gamma_2(\bar{\gamma}_1) > \Gamma_2(\gamma_1).$$

In case 2), since $\frac{d\hat{\pi}_R}{d\gamma_1} > 0$ for $q_R \in (1, q_M)$, we have the optimum profit is larger than the endpoints, and Firm R chooses to offer random product, implying $\Gamma_2(\bar{\gamma}_1) > \Gamma_2(\gamma_1)$.

(Profit Comparison) First, we show that $\hat{\pi}_R^* = \pi_R^*$ when $\gamma_0 = \gamma_1$. When $\gamma_0 = \gamma_1 = \gamma$,

$f(q_R) \propto 4q_H \cdot (q_M - 1) + \gamma \cdot q_M \cdot (4q_H - 7q_M) - [7(q_M - 1) + \gamma \cdot (4q_H - 7)] \cdot q_R$.

When $q_H \leq \frac{7}{4} L(\gamma_1) \leq \gamma_1 = \gamma_0 \leq M(\gamma_1)$. We have $f(1) \leq 0$, and we must have $f(q_R) < 0$ for any $q_R \in [1, q_M]$, which implies $q_R^* = 1$. This yields a profit the same as that in Proposition 1, as shown in Table A.1. The case for $q_H \geq \frac{7}{4} q_M$ follows parallel.

When $\frac{7}{4} < q_H < \frac{7}{4} q_M$, $\gamma_1 = \gamma_0 < \min\{L(\gamma_1), M(\gamma_1)\}$, we have $f(1) > 0$ and $f(q_M) < 0$. Thus, we must have a unique $q_R^* \in (1, q_M)$ such that $f'(q_R^*) = 0$. That is, $q_R^*$ =
\[
\frac{4q_H(q_M-1)+\gamma q_M(4q_H-7q_M)}{7(q_M-1)+\gamma(4q_H-7)}
\]
which yields \( \hat{\pi}^*_R = \frac{q_H}{48} \) the same as that in Proposition 1, as shown in Table A.1.

### Table A.1. The Profit Comparison

<table>
<thead>
<tr>
<th>Cases</th>
<th>( \gamma_1 = \gamma_0 ) (no regret)</th>
<th>( \gamma_1 &gt; \gamma_0 )</th>
<th>( \gamma_1 &lt; \gamma_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_H \leq \frac{7}{4} )</td>
<td>( q_H(q_H-1)/(4q_H-1)^2 )</td>
<td>( q_H(q_H-1)/(4q_H-1)^2, ) if ( \gamma_0 &lt; \Gamma(\gamma_1) )</td>
<td>( q_H(q_H-1)/(4q_H-1)^2, ) otherwise</td>
</tr>
<tr>
<td>( \frac{7}{4} &lt; q_H &lt; \frac{7}{4}q_M )</td>
<td>( q_H/48 )</td>
<td>( q_H/48 )</td>
<td>( q_H/48 )</td>
</tr>
<tr>
<td>( q_H \geq \frac{7}{4}q_M )</td>
<td>( q_H(q_H-q_M)/q_M(4q_H-q_M)^2 )</td>
<td>( q_H(q_H-q_M)/q_M(4q_H-q_M)^2, ) if ( \gamma_0 &lt; \Gamma(\gamma_1) )</td>
<td>( q_H(q_H-q_M)/q_M(4q_H-q_M)^2, ) otherwise</td>
</tr>
</tbody>
</table>

Consequently, we have \( \hat{\pi}^*_R = \pi^*_R \) when \( \gamma_0 = \gamma_1 \). Then by applying \( \frac{\partial \hat{\pi}^*_R}{\partial \gamma_0} \leq 0 \) and \( \frac{\partial \hat{\pi}^*_R}{\partial \gamma_1} \geq 0 \), this part follows.

### Proof of Corollary 2.

The limiting property on \( \gamma_0 \) is proven in Proof of Proposition 2. Here we prove the limiting property on \( \gamma_1 \).

Given \( \gamma_0 \), as \( \gamma_1 \) increases to infinity, both \( L(\gamma_1) \) and \( M(\gamma_1) \) increase to infinity, therefore, both \( \Gamma(\gamma_1) \) increases to infinity and larger than \( \gamma_0 \), and Firm R sells random product. When \( \gamma_1 \geq \gamma_0, f'(q_R) < 0 \). Therefore, \( \hat{\pi}^*_R \) satisfies the FOC \( f(q_R) = 0 \). That is

\[
\frac{A^2 \cdot (4A + 5B)}{B^2 \cdot (2A + 3B)} = \frac{1 + \gamma_1}{1 + \gamma_0} \cdot \frac{[q_M - 1 + \gamma_0 \cdot (q_M - q_R)]^2}{[q_M - 1 + \gamma_1 \cdot (q_M - q_R)]^2}.
\]
As $\gamma_1 \to \infty$, $A \to q_H - 1$. $B$ is a positive constant. The left-hand-side converges to a constant. If $\hat{q}_R^*(\gamma_1)$ has a subsequence converging to $q \in [1, q_M)$, then the right-hand-side goes to 0. To equalize the two sides, we must $\hat{q}_R^* \to q_M$. More specifically, we need $q_M - \hat{q}_R^* = O\left(\frac{1}{\gamma_1^{1/2}}\right)$. ■

**Proof of Lemma 3.**

From Proof of Proposition 2, in equilibrium, we have

$$p_R = \frac{A \cdot B}{4A + 3B}, p_H = \frac{2A \cdot (A + B)}{4A + 3B}. $$

$$D_R = \frac{A + B}{4A + 3B}, D_H = \frac{2(A + B)}{4A + 3B}. $$

$$\hat{\pi}_R = \frac{A \cdot B \cdot (A + B)}{(4A + 3B)^2}, \hat{\pi}_H = \frac{4A \cdot (A + B)^2}{(4A + 3B)^2}. $$

We can verify that

$$\frac{dp_R}{d\gamma_0} < 0, \frac{dp_H}{d\gamma_0} < 0, \frac{dD_R}{d\gamma_0} < 0, \frac{dD_H}{d\gamma_0} < 0, \frac{d\hat{\pi}_R}{d\gamma_0} < 0, \frac{d\hat{\pi}_H}{d\gamma_0} < 0. $$

■


2 The “no return” policy is common in probabilistic selling, see more details at [https://www.hotwire.com/helpcenter/hotels/after-booking/reservation-changes-cancellations/can-i-cancel-or-make-changes.jsp](https://www.hotwire.com/helpcenter/hotels/after-booking/reservation-changes-cancellations/can-i-cancel-or-make-changes.jsp). This policy can prevent consumers from arbitrage (e.g. repeatedly returning undesirable purchase until obtaining the favorable one), especially when quality differences exist in probabilistic selling.

3 According to a consumer relationship manager at Hotwire, many consumer complaints are about feeling regret buying products sold through probabilistic selling when consumers get an inferior product. Also, see more complaints on TripAdvisor: [http://www.tripadvisor.com/ShowUserReviews-g154998-d281720-r138268528-Advantage_Inn-Niagara_Falls_Ontario.html](http://www.tripadvisor.com/ShowUserReviews-g154998-d281720-r138268528-Advantage_Inn-Niagara_Falls_Ontario.html)

4 This result also provides a potential interpretation for the strict “no return” policy. Specifically, our findings suggest that, if this “no return” policy aggravates consumer regret, this actually benefits the probabilistic selling and encourages the use of this selling strategy.

The infrastructure/fixed investments costs across vertical products/services (e.g. a suite room versus an ordinary room) are significant in the travel industry but they are often sunk. The maintenance costs are relatively less salient compared to the infrastructure/fixed investments costs.

Such a sweet spot $\frac{4}{7}q_H$ is found in Choi and Shin (1992), where two firms freely choose any quality levels. In ours, Firm $R$ can only choose quality between 1 and $q_M$, implying possible corner solutions.