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Working Paper #09-25

Revised Cr tkn2010

Switching Costs in Network Industries

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Switching Costs and Dynamic Price Competition in Network Industries

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April 5, 2010

Abstract

Motivated by policy makers' recent interest in reducing switching costs in various network industries to increase competition, this paper investigates how switching costs affect market outcome in such industries. The results show that the effects of switching costs on market concentration and prices critically depend on two factors: the strength of network effects and the quality of the outside good. For example, switching costs lower prices if network effects are modest and the outside good is attractive, but raise prices otherwise. Therefore, policy makers need to carefully evaluate those two factors in order to make informed decisions.

1 Introduction

An important feature of many network industries—industries in which the product's value increases in the number of consumers who use it—is the existence of switching costs: consumers can switch between networks but it is costly, in terms of money and/or effort, for them to do so. For example, switching from one PC operating system to another requires substantial effort in learning the new environment. Switching from one wireless phone network to another is also costly, as the consumer needs to inform her contacts of her new phone number (unless she can keep her old phone number) and may have to pay an early termination fee. Similarly, when a consumer switches from one bank to another, she needs to communicate the information about this change to all relevant parties (direct deposits, automatic payments, etc.), which is time-consuming. In fact, according to Shy (2001), switching costs are one of the main characteristics of network industries that distinguish them from other types of industries. While there have been many studies that analyze network effects and switching costs separately, few have looked at them jointly, and it is

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unclear whether the findings so far can be applied to markets in which those two factors coexist.

Research in this area is particularly pressing in light of policy makers' recent interest in reducing switching costs in various network industries to increase competition. For example, mobile number portability (MNP), which reduces mobile phone users' switching costs by enabling them to retain their phone numbers when changing from one network to another, was implemented in more than forty countries during the past few years. In the EU retail banking and payments systems markets, the European Competition Authorities Financial Services Subgroup recommends the implementation of switching facilities (objective and up-to-date comparison sites, switching services, etc.) and account number portability to lower switching costs (ECAFSS (2006)). In predicting the effects of policies like these, we can not just rely on past research that abstracts from network effects, because the interactions between switching costs and network effects can alter the picture and make previous findings inapplicable.

To address the above issue, this paper builds an oligopolistic model of price competition that incorporates both network effects and switching costs. Firms dynamically optimize. A Markov perfect equilibrium is numerically solved for, and I assess the effects of switching costs on the frequency with which market dominance occurs, as well as prices and welfare.

I find that results are quite different when we go from a setting with only switching costs to one with both network effects and switching costs, indicating that the interaction between network effects and switching costs does matter. In particular, the literature on switching costs without network effects finds that switching costs cause the larger firm to price less aggressively in order to exploit locked-in consumers. As a result, the larger firm loses consumers to the smaller firm. This is referred to as the "fat cat effect", with the larger firm being a nonaggressive "fat cat". This effect tends to prevent market dominance and hence markets with switching costs tend to be stable (Beggs and Klemperer (1992), Chen and Rosenthal (1996), Taylor (2003)). When we incorporate network effects into the analysis, switching costs have a new role. They reinforce network effects by heightening the exit barrier for locked-in consumers and making a network size advantage longer-lasting and more valuable. As a result, firms compete more fiercely when they have comparable size, and when an asymmetry arises, the larger firm prices more aggressively in order to obtain and keep a dominant position. This is referred to as the "top dog effect", with the larger firm being an aggressive "top dog". This effect facilitates tipping towards market dominance. The results show a non-monotonic relationship between switching costs and market dominance. At low switching costs, the top dog effect dominates and an increase in switching costs can change the market from fragmentation to being dominated by one firm. But at high switching costs, the fat cat effect takes over and an increase in switching costs can change the market from being dominated by one firm to fragmentation.

Regarding the effects of switching costs on prices, the traditional view in the theoretical literature is that firms' incentive to charge higher prices in order to exploit locked-in consumers (the "harvesting incentive") dominates their incentive to charge lower prices in order to build a larger customer base (the "investment incentive"), and hence switching costs increase equilibrium prices (Beggs and Klemperer (1992), To (1996)). However, several recent studies find that equilibrium prices can be decreasing in switching costs (Doganoglu and Grzybowski (2005), Doganoglu (2005), Cabral (2008)). Similar to the disagreement in the

theoretical literature, empirical studies on this subject also reach conflicting findings (Viard (2007), Dubé, Hitsch, and Rossi (2009), Park (2009)). Results from the current model offer a new perspective on this “price puzzle of switching costs”, by showing that both scenarios can arise as equilibrium outcomes and that network effects and competition from the outside good are important determinants. Specifically, I find that if the outside good is fairly inferior, the average price generally increases in switching costs, consistent with the findings in the traditional theoretical literature. But if there exists a reasonably attractive outside good, the average price tends to decrease in switching costs when the network effect is weak and increase in switching costs when the network effect is strong. The reason is that with an outside good, an increase in switching costs makes the firms’ products less attractive and shifts some of the demand from the firms’ products to the outside good. As a result, the size of the market is reduced and firms’ equilibrium prices tend to drop. Furthermore, a stronger network effect amplifies the “top dog effect” mentioned above, and hence an increase in switching costs becomes more effective in pulling the industry towards asymmetric states, in which the average price is generally higher than in symmetric states. Existing empirical studies provide anecdotal evidence that supports these predictions. For example, Dubé, Hitsch, and Rossi (2009) study the markets for refrigerated orange juice and tub margarine. Their simulations based on the calibrated model show that prices are lower with than without switching costs. On the other hand, Park (2009) studies the wireless phone industry and finds that the introduction of number portability caused prices to decrease, implying that switching costs lead to higher equilibrium prices. Since it is likely that the wireless phone industry has a stronger network effect and a weaker outside good than the markets for refrigerated orange juice and tub margarine, the findings in those studies support the notion that prices tend to rise in the switching cost when the network effect is strong and the outside good is weak.

For a recent survey of both the literature on switching costs and the literature on network effects, see Farrell and Klemperer (2007); see also Klemperer (1995) and Economides (1996). As discussed above, prior studies generally focus on one factor and abstract from the other. In particular, the literature on switching costs typically assumes zero network effects, and the literature on network effects typically fixes switching costs at either infinity or zero. There are two exceptions. Doganoglu and Grzybowski (2005) use a two-period differentiated-products duopoly model to study the effects of switching costs and network effects on demand elasticities and prices. Suleymanova and Wey (2008) use a Bertrand duopoly model with myopic agents to study market outcomes under network effects and switching costs, and find that market dynamics critically depend on the ratio of switching costs to network effects. My framework differs from theirs in that I work with an infinite-horizon model, which avoids the end-of-game effects in the last period and allows me to investigate both the short-run and the long-run industry dynamics. For example, in the presence of switching costs, in an infinite-horizon model firms have both the “investment incentive” and the “harvesting incentive” in every period, whereas in a two-period model they do not. Additionally, I endogenize the market size by modeling an outside good. Compared to competition for switching consumers within a mature and saturated market, competition that takes place when new technologies or platforms are being adopted is a lot more common and interesting. The modeling of an outside good is particularly needed in such cases, as the use of old technologies, or simply “doing without”, is the common

default choice around initial adoption. Furthermore, results from this model indicate that the quality of the outside good is also an important determinant of how switching costs affect the market outcome.

The model is described in Section 2. Markov perfect equilibria of the model are reviewed in Section 3. The effects of switching costs on firms' expected sales are explored in Section 4. Section 5 presents results on the tendency towards market dominance. Section 6 presents results on price and welfare. Section 7 concludes.

2 Model

This section describes a dynamic oligopolistic model of markets with network effects and switching costs, which essentially adds switching costs to the model in Chen, Doraszelski, and Harrington (2009). Since the objective is to provide some general insights about the effects of switching costs in network industries, the model is not tailored to a particular industry. Instead, a more generic model is developed to capture the key features of many markets characterized by network effects and switching costs.

2.1 State Space

The model is cast in discrete time with an infinite horizon. There are $N \geq 2$ single-product price-setting firms, who sell to a sequence of buyers with unit demands. Firms' products are durable subject to stochastic death. They are referred to as the inside goods. There is also an outside good ("no purchase"), indexed 0. At the beginning of a period, a firm is endowed with an installed base which represents users of its product. Let $b_i \in \{0, 1, \dots, M\}$ denote the installed base of firm i where M is the bound on the sum of the firms' installed bases. $b_0 = M - b_1 - \dots - b_N$ is taken as the outside good's "installed base", though it does not offer network benefits. The industry state is $b = (b_1, \dots, b_N)$, with state space $\Omega = \{(b_1, \dots, b_N) | 0 \leq b_i \leq M, i = 1, \dots, N; b_1 + \dots + b_N \leq M\}$.

2.2 Demand

Demand in each period comes from a random consumer who chooses one among the $N + 1$ goods. Let $r \in \{0, 1, \dots, N\}$ denote the good that has her loyalty.¹ Assume r is distributed according to $\Pr(r = j | b) = b_j / M$, $j = 0, 1, \dots, N$, so that a larger installed base implies a larger expected demand from loyal consumers. The utility that a consumer who is loyal to good r gets from buying good i is

$$v_i + \mathbf{1}(i \neq 0)\theta g(b_i) - p_i - \mathbf{1}(r \neq 0, i \neq 0, i \neq r)k + \epsilon_i.$$

Here v_i is the intrinsic product quality, which is fixed over time and is common across firms: $v_i = v$, $i = 1, \dots, N$. Since the intrinsic quality parameters affect demand only through

¹A consumer may be loyal to a firm's product because she previously used that product and now her product dies and she returns to the market. A consumer may also be loyal to a firm's product because of her relationship with current users. For example, if a consumer is familiar with a particular product because her relatives, friends, or colleagues are users of this product, then she may be loyal to this product even if she has never purchased from this market before.

the expression $v - v_0$, without loss of generality I set $v = 0$, but consider different values for v_0 .

The increasing function $\theta g(\cdot)$ captures network effects, where $\theta \geq 0$ is the parameter controlling the strength of network effect. There are no network effects associated with the outside good. The results reported below are based on linear network effects, that is, $g(b_i) = b_i/M$. I have also allowed g to be convex, concave, and S-shaped, and the main results are robust.

p_i denotes the price for good i . The price of the outside good, p_0 , is always zero.

The nonnegative constant k denotes switching cost, and is incurred if the consumer switches from one inside good to another. A consumer who switches from the outside good to an inside good incurs a start-up cost, which is normalized to 0. Increasing the start-up cost above 0 has the effect of lowering the inside goods' intrinsic quality relative to that of the outside good.

ϵ_i is the consumer's idiosyncratic preference shock. $(\epsilon_0, \epsilon_1, \dots, \epsilon_N)$ and r are unknown to the firms when they set prices.

The consumer buys the good that offers the highest current utility. I am then assuming that consumers make myopic decisions. Such a parsimonious specification of consumers' decision-making allows rich modeling of firms' prices and industry dynamics. Allowing consumers to be forward-looking with rational expectations in the presence of both network effects and switching costs is an important but challenging extension of the current work.

Assume ϵ_i , $i = 0, 1, \dots, N$ is distributed type I extreme value, independent across products, consumers, and time. The probability that a consumer who is loyal to good r buys good i is then

$$\phi_{ri}(b, p) \equiv \frac{\exp(v_i + \mathbf{1}(i \neq 0)\theta g(b_i) - p_i - \mathbf{1}(r \neq 0, i \neq 0, i \neq r)k)}{\sum_{j=0}^N \exp(v_j + \mathbf{1}(j \neq 0)\theta g(b_j) - p_j - \mathbf{1}(r \neq 0, j \neq 0, j \neq r)k)}, \quad (1)$$

where b is the vector of installed bases and p is the vector of prices.

2.3 Transition Probabilities

In each period, each unit of a firm's installed base independently depreciates with probability $\delta \in [0, 1]$, for example due to product death. Thus the expected size of the depreciation to a firm's installed base is proportional to the size of its installed base. Let $\Delta(x|b_i)$ denote the probability that firm i 's installed base depreciates by x units. We have

$$\Delta(x|b_i) = \binom{b_i}{x} \delta^x (1 - \delta)^{b_i - x}, \quad x = 0, \dots, b_i,$$

since x is distributed binomial with parameters (b_i, δ) .

Let $q_i \in \{0, 1\}$ indicate whether or not firm i makes the sale. Firm i 's installed base changes according to the transition function

$$\Pr(b'_i|b_i, q_i) = \Delta(b_i + q_i - b'_i|b_i), \quad b'_i = q_i, \dots, b_i + q_i.$$

If the joint outcome of the sale and the depreciation results in an industry state outside of the state space, the probability that would be assigned to that state is given to the nearest state(s) on the boundary of the state space.

2.4 Bellman Equation and Strategies

As mentioned above, the consumer's idiosyncratic preference shocks and the good that has her loyalty are stochastic and unknown to the firms when they set prices. Let $V_i(b)$ denote the expected net present value of future cash flows to firm i in state b . Firm i 's Bellman equation is²

$$V_i(b) = \max_{p_i} E_r \left[\phi_{ri}(b, p_i, p_{-i}(b)) p_i + \beta \sum_{j=0}^N \phi_{rj}(b, p_i, p_{-i}(b)) \bar{V}_{ij}(b) \right], \quad (2)$$

where $p_{-i}(b)$ are the prices charged by firm i 's rivals in equilibrium (given the installed bases), the (constant) marginal cost of production is normalized to zero, $\beta \in [0, 1)$ is the discount factor, and $\bar{V}_{ij}(b)$ is the expected continuation value to firm i given that firm j wins the current consumer:

$$\bar{V}_{ij}(b) = \sum_{b'} \Pr(b'|b, q_j = 1) V'_i(b').$$

Differentiating the right-hand side of equation (2) with respect to p_i and using the properties of logit demand yields the first-order condition

$$E_r \left[-\phi_{ri}(1 - \phi_{ri})(p_i + \beta \bar{V}_{ii}) + \phi_{ri} + \beta \phi_{ri} \sum_{j \neq i} \phi_{rj} \bar{V}_{ij} \right] = 0. \quad (3)$$

The pricing strategies $p(b)$ are the solution to the system of first-order conditions.

2.5 Equilibrium

I focus attention on symmetric Markov perfect equilibria (MPE), where symmetry means agents with identical states are required to behave identically. For example, if there are two firms, then symmetry means firm 2's price in state $(b_1, b_2) = (\hat{b}, \hat{b})$ is identical to firm 1's price in state $(b_1, b_2) = (\hat{b}, \hat{b})$, and similarly for the value function. I therefore define $p(b_1, b_2) \equiv p_1(b_1, b_2)$ and $V(b_1, b_2) \equiv V_1(b_1, b_2)$, and note that $p_2(b_1 = \hat{b}, b_2 = \hat{b}) = p(\hat{b}, \hat{b})$ and $V_2(b_1 = \hat{b}, b_2 = \hat{b}) = V(\hat{b}, \hat{b})$.

I restrict attention to pure strategies, which follows the majority of the literature on numerically solving dynamic stochastic games (Pakes and McGuire (1994), Pakes and McGuire (2001)). A symmetric MPE in pure strategies always exists (Doraszelski and Satterthwaite (2010)), but as is true with many other dynamic models, there may exist multiple MPE. I therefore take a widely used selection rule in the dynamic games literature by computing the limit of a finite-horizon game as the horizon grows to infinity (for details see Chen, Doraszelski, and Harrington (2009)). With this equilibrium selection rule in place, the iterative algorithm always converged and resulted in a unique MPE.

²Recall that firms' products are durable subject to stochastic death, and that demand in each period comes from a random consumer.

2.6 Parameterization

The key parameters of the model are the strength of network effect θ , the switching cost k , the rate of depreciation δ , and the quality of the outside good v_0 . I focus on two values for v_0 , $-\infty$ (fixed market size) and 0 (endogenous market size), but also consider some in-between values. The lower bound for δ is zero and corresponds to the unrealistic case in which installed bases never depreciate. On the other hand, if δ is sufficiently high then the industry never takes off. I consider many values for δ between 0 and 0.2. I investigate the following values for the strength of network effect and the switching cost: $\theta \in \{0, 0.2, \dots, 4\}$, and $k \in \{0, 0.2, \dots, 3\}$. While I extensively vary the key parameters, I hold the remaining parameters constant at $N = 2$, $M = 20$, and $\beta = \frac{1}{1.05}$, which corresponds to a yearly interest rate of 5%.

While the model is not intended to fit any specific product, the own-price elasticities for the parameterizations that I consider are reasonable compared with findings in empirical studies. As representative examples of the equilibria in the model, the own-price elasticities for the parameterizations in Figures 1-4 range from -0.75 to -0.62 . These numbers are in line with the own-price elasticities in Gandal, Kende, and Rob (2000) (-0.54 for CD players, computed according to results reported in the paper), Dranove and Gandal (2003) (-1.20 for DVD players), and Clements and Ohashi (2005) (ranging from -2.15 to -0.18 for video game consoles).

3 Sharing Equilibrium and Tipping Equilibrium

In this model two types of equilibria emerge, Sharing and Tipping. In the former, the market tends to be shared by firms that are of comparable size, whereas in the latter, the market tends to be dominated by a single firm. Real-world examples of market sharing in network industries include video game consoles, wireless phone networks, credit card payment systems, etc. Examples of market dominance include the QWERTY keyboard, the VHS format in the home VCR market, Windows PC operating system, etc.³ Below we examine these two types of equilibria in turn.

3.1 Sharing Equilibrium

The characteristic of a Sharing equilibrium is that the limiting distribution is unimodal with a lot of mass at reasonably symmetric states, indicating that market dominance is highly unlikely. Based on the shape of the equilibrium policy function, Sharing equilibria can be divided into two subtypes, Rising and Peaked.

3.1.1 Rising Equilibrium

A *Rising* equilibrium is characterized by a policy function in which the larger firm's price rises in its own base and falls in its rival's base, whereas the smaller firm's price is lower than the larger firm's and is fairly insensitive to the industry state (Panel 1 in Figure 1).

³See Farrell and Klemperer (2007) and the references given therein for a number of case studies.

A firm’s value (Panel 2) monotonically increases in its own base and decreases in its rival’s base.

To show the evolution of the industry structure over time, Panels 3 and 4 plot the 15-period transient distribution of installed bases (which gives the frequency with which the industry state takes a particular value after 15 periods, starting from state $(0, 0)$ in period 0) and the limiting distribution (which gives the frequency with which the state takes a particular value as the number of periods approaches infinity), respectively. The unimodal transient distribution and limiting distribution show that market dominance is unlikely, as the industry spends most of the time in fairly symmetric states.

Panel 5 plots the probability that a firm makes a sale, and Panel 6 plots the resultant forces, which report the expected movement of the state from one period to the next (for visibility of the arrows, the lengths of all arrows are normalized to 1, therefore only the direction, not the magnitude, of the expected movement is reported). We see that the larger firm’s installed base advantage, which results in a quality advantage and a larger group of locked-in consumers, allows the larger firm to charge a higher price and still win the consumer with a higher probability (Panel 5). However, the larger firm’s expected size of depreciation is also larger than that of the smaller firm. In a Rising equilibrium, the difference in expected depreciation more than offsets the difference in expected sales, and as a result, the difference in installed bases shrinks in expectation (Panel 6).

A Rising equilibrium occurs when both network effect and switching cost are weak. The next section gives more details on the parameterizations for which different types of equilibria occur.

3.1.2 Peaked Equilibrium

A *Peaked* equilibrium is characterized by a peak in the policy function when each firm has half of the consumer population in its installed base. Away from this peak, price drops rapidly for the smaller firm, and mildly for the larger firm. An example of a Peaked equilibrium is shown in Figure 2.

When each firm locks in half of the consumer population, price competition is weak as reflected in the peak in the policy function. Due to switching costs, both firms have strong incentives to charge high prices to “harvest” their large bases of locked-in consumers. Moreover, the absence of unattached consumers (that is, consumers who are not loyal to either of the firms) weakens firms’ “investment” incentive to lower prices. Off of the peak, the smaller firm drops its price substantially in order to increase expected sales and thereby reduce the installed base differential and move the industry back to the peak. The larger firm also drops its price, but that is a response to the smaller firm’s aggressive pricing rather than an effort to achieve market dominance. In fact, as the industry moves away from the peak, the smaller firm drops its price much more aggressively than the larger firm. For example, in the policy function in Figure 2, as the industry moves from the peak at $(10, 10)$ to $(11, 9)$ to $(12, 8)$, the smaller firm’s price drops from 3.99 to 2.73 to 1.68, whereas the larger firm’s price only drops from 3.99 to 3.76 to 3.31.

The pricing behavior of the firms results in the value function also having a peak when each firm locks in half of the consumer population. Off of the peak, the smaller firm’s value drops rapidly whereas the larger firm’s value drops mildly. Switching costs enable the firms

to segment the market and focus on their locked-in consumers rather than their rivals'. Locked-in consumers are heavily exploited, and firms enjoy high profits. When one firm gains a small installed base advantage, its value actually decreases because the balance is damaged, the smaller firm starts to price aggressively, and the larger firm has to respond by cutting its own price. Since firms have little incentive to induce tipping in their favor, market dominance is highly unlikely, as reflected in the unimodal transient distributions and limiting distribution presented in Figure 2.

While the resultant forces in Figure 2 resemble those in Figure 1 and show global convergence towards the symmetric modal state, the probability that a firm makes a sale exhibits a different pattern. In the Rising equilibrium in Figure 1, the larger firm always wins the consumer with a higher probability. In contrast, in the Peaked equilibrium in Figure 2, it is the smaller firm who has a higher probability of sale in states that are reasonably symmetric. This pattern results from the smaller firm's aggressive pricing aimed at bringing the industry back to the peak.

A Peaked equilibrium occurs when switching cost is strong and the outside good is inferior. A key function of switching costs is that they segment the market into submarkets, each containing consumers who have previously purchased from a particular firm. A Peaked equilibrium is based on such market segmentation, which allows firms to price in a fashion that resembles collusion even in a noncooperative equilibrium (see Klemperer (1987)). However, if the outside good is reasonably attractive, then it constitutes a non-strategic player and restrains firms' ability to exploit the locked-in consumers, weakening the basis for firms to engage in collusion-like pricing. In fact, with $v_0 = 0$ a Peaked equilibrium never occurs. We further investigate the role of the outside good in the next section.

3.2 Tipping Equilibrium

In a *Tipping* equilibrium, there is intense price competition when firms' installed bases are of comparable size, and the limiting distribution of installed bases is bimodal with a lot of mass at asymmetric states. An example of a Tipping equilibrium is shown in Figure 3. The policy function in a Tipping equilibrium features a deep trench along and around the diagonal. When the industry is sufficiently away from the diagonal, price is relatively high. A Tipping equilibrium occurs when network effect is strong and depreciation is modest. This type of equilibria is also found in prior dynamic models with increasing returns, such as Doraszelski and Markovich (2007), Besanko, Doraszelski, Kryukov, and Satterthwaite (2008), and Chen, Doraszelski, and Harrington (2009).

The value function of a firm presented in Figure 3 shows that the larger firm enjoys a much higher value than the smaller firm. For example, when the state moves from $(0, 0)$ to $(2, 0)$, the larger firm's value increases from 8.58 to 15.91, whereas the smaller firm's value decreases from 8.58 to 8.14. Note that in state $(2, 0)$, the larger firm's value is almost twice that of the smaller firm. It is this substantial difference between the market leader's value and that of the market follower that drives the intense price competition in symmetric states. Each firm prices aggressively in hope of getting an installed base advantage and forcing the rival to give up. Hence the deep trench along and around the diagonal.

When the industry is sufficiently away from the diagonal, the smaller firm gives up the fight by not pricing aggressively, and accepts having a low market share. If instead it were to

price aggressively and try to overtake the larger firm, it would have to price at a substantial discount for an extended period of time. Anticipating that such an aggressive strategy is not profitable, the smaller firm abandons the fight, thus ensuring that the larger firm enjoys a dominant position and high profits.

Both the transient distribution and the limiting distribution in Figure 3 are bimodal. They show that over time the industry state moves towards asymmetric outcomes, and that market dominance is likely. The probability that a firm makes a sale shows that the larger firm enjoys a significant advantage: the average probability that firm 1 makes a sale is 0.72 in states with $b_1 > b_2$, compared to an average probability of 0.28 in states with $b_1 < b_2$. Such a difference in the probabilities results from the smaller firm's willingness to surrender (by charging high prices), and gives rise to the forces that pull the industry away from the diagonal once an asymmetry arises, as shown in the resultant forces reported in Figure 3.

When there exists an outside good, there is a variant to Tipping equilibrium worth noting. In this variant, referred to as a *Mild Tipping* equilibrium, the modes of the limiting distribution are asymmetric but a fair amount of mass is spread over the reasonably symmetric states between the two modes, as exemplified by Figure 4. Here, the equilibrium is in the process of morphing from Rising to Tipping. When the outside good is reasonably attractive, the payoff to having an installed base advantage is reduced, as the outside good serves as a non-strategic player and restrains firms' ability to exploit the locked-in consumers. For example, in the Tipping equilibrium shown in Figure 3, the increase in firm 1's value from state $(0, 0)$ to state $(2, 0)$ is $15.91 - 8.58 = 7.33$, whereas in the Mild Tipping equilibrium shown in Figure 4, the increase is only $5.77 - 4.70 = 1.07$. As a result, when firms are of comparable size, they do not fight so fiercely in a Mild Tipping equilibrium as in a Tipping equilibrium. In Figure 3, the policy function features a deep trench along and around the diagonal, in which firms' prices are often negative (below cost). On the diagonal, prices range from $p_1(0, 0) = -2.78$ to $p_1(10, 10) = 0.29$. In contrast, in the policy function in Figure 4, although prices are still lowest when firms are of comparable size, there does not exist a visible trench. Moreover, prices are always positive. On the diagonal, prices range from $p_1(0, 0) = 0.74$ to $p_1(10, 10) = 0.92$. As a consequence of the relatively mild competition along the diagonal, the smaller firm finds it less painful to try to catch up: doing so does not involve going through a highly unprofitable trench. In response, the smaller firm does not give up the fight as completely as in a Tipping equilibrium. For example, in Figure 3, the smaller firm's price in states $(0, 0)$, $(0, 1)$, $(0, 2)$, $(0, 3)$ and $(0, 4)$ are -2.78 , 0.39 , 1.37 , 1.54 , and 1.55 , respectively, showing a substantial increase in price once the firm lags behind. In contrast, in Figure 4, the smaller firm's price in those states are 0.74 , 0.74 , 0.75 , 0.76 , and 0.77 , respectively, indicating that the smaller firm keeps its price relatively low in hope of reducing the base differential and catching up with the larger firm. Such pricing behavior explains why the industry spends a fair amount of time in the reasonably symmetric states between the two modes.

Turning to expected sales, we see that in a Mild Tipping equilibrium, the larger firms still has a higher probability of making a sale than the smaller firm, but the difference is less significant than in a Tipping equilibrium. Correspondingly, the resultant forces show that the tendency for the industry to move away from the diagonal is weaker than in a Tipping equilibrium.

4 Switching Costs and Firms' Expected Sales

In this section, we investigate how switching costs affect firm's expected sales, which forms the basis for our later analysis of how switching costs affect the tendency towards market dominance. In any given state, if the larger firm has a larger expected sale that more than compensates for its larger expected depreciation, then in expectation the asymmetry in the firms' installed bases will grow, and there is increasing dominance in that state; and vice versa.

Firms' expected sales depend on consumers' product choices, which in turn depend on both switching costs and firms' prices, as is evident from Eq. (1), which gives consumers' choice probabilities. An increase in switching costs affects both consumers' product choices and firms' prices, and therefore it affects firms' expected sales through two channels. In the direct channel, holding firms' prices fixed, an increase in switching costs makes a consumer more likely to buy the product that has her loyalty. In the indirect channel, an increase in switching costs leads to changes in firms' prices, which in turn affect consumers' choices.

In particular, prior studies have identified two opposite pricing incentives of firms due to switching costs, the *harvesting incentive* (firms' incentive to charge high prices to "harvest" the locked-in consumers for greater current profits) and the *investment incentive* (firms' incentive to charge low prices to "invest" in installed base and hence increase future profits). In the current dynamic model, firms have both the harvesting incentive and the investment incentive in every period. As we will see below, the larger firm and the smaller firm are asymmetric in their harvesting and investment incentives, and such asymmetry is key to understanding the indirect channel through which switching costs affect firms' expected sales.

4.1 Direct Channel and Indirect Channel

Consider a permanent increase in the switching cost k . Such an increase implies that both the current-period switching cost (denoted by k^c) and the switching cost in all future periods (denoted by k^f) are increased. At a given state, let $\phi = (\phi_1, \phi_2) = (E_r[\phi_{r1}], E_r[\phi_{r2}])$ denote the firms' expected sales. ϕ depends on firms' equilibrium prices $p = (p_1, p_2)$ and k^c .⁴ p , in turn, depends on k^c and firms' future value function, denoted by the $(M+1) \times (M+1)$ matrix V' (such dependence can be seen by examining the first order condition, Eq. (3), in which ϕ_r depends on k^c , and $\bar{V}_i$ depends on V').⁵ Furthermore, note that k^f affects V' , and therefore it affects p through its impact on V' .

We are interested in $d\phi/dk$, which tells how a permanent increase in the switching cost affects the firms' expected sales. For example, suppose firm 1 is the larger firm and firm 2 is the smaller firm. If $d\phi_1/dk > 0$ and $d\phi_2/dk < 0$, then we know an increase in k increases market concentration by giving a boost to the larger firms' expected sale relative to that of the smaller firm; and vice versa.

⁴Since consumers are myopic in this model, k^f affects ϕ only through p .

⁵Recall that $V(b_1, b_2) \equiv V_1(b_1, b_2)$, and note that $V_2(b_1 = \hat{b}, b_2 = \hat{b}) = V(\hat{b}, \hat{b})$ due to symmetry of the MPE. The (i, j) -th element of the matrix V' is firm 1's future value associated with the next-period state $(b'_1 = i-1, b'_2 = j-1)$. Furthermore, since any (b'_1, b'_2) such that $b'_1 + b'_2 > M$ is not part of the state space, any (i, j) -th element in V' such that $(i-1) + (j-1) > M$ does not enter into our analysis and can simply be fixed at 0 throughout.

Since firms' equilibrium prices p depends on the current-period switching cost k^c and the future value function V' , and since V' depends on the future switching cost k^f , we can rewrite firms' equilibrium prices as $p(k^c, V'(k^f))$, to highlight such dependence. Correspondingly, the expected sales ϕ can be written as $\phi(p(k^c, V'(k^f)), k^c)$. We now totally differentiate ϕ with respect to k , noting that $k^c(k) = k$ and $k^f(k) = k$:⁶

$$\begin{aligned}
\frac{d\phi}{dk} &= \left. \frac{d\phi}{dk^c} \right|_{dk^f=0} \frac{dk^c}{dk} + \left. \frac{d\phi}{dk^f} \right|_{dk^c=0} \frac{dk^f}{dk} \\
&= \left(\frac{\partial \phi}{\partial k^c} + \frac{\partial \phi}{\partial p} \frac{\partial p}{\partial k^c} \right) \times 1 + \left(\frac{\partial \phi}{\partial p} \frac{\partial p}{\partial V'} \frac{dV'}{dk^f} \right) \times 1 \\
&= \underbrace{\frac{\partial \phi}{\partial k^c}}_{2 \times 1} + \underbrace{\frac{\partial \phi}{\partial p} \left(\frac{\partial p}{\partial k^c} + \frac{\partial p}{\partial V'} \frac{dV'}{dk^f} \right)}_{2 \times 2 \quad 2 \times 1 \quad 2 \times (M+1)^2 (M+1)^2 \times 1}. \tag{4} \\
&\quad \text{direct channel} \qquad \qquad \qquad \text{indirect channel}
\end{aligned}$$

The above equation makes it clear that k^c affects ϕ through both the direct channel and the indirect channel (which goes through p), whereas k^f affects ϕ only through the indirect channel (which goes through V' and then p).

Before we look at the overall effects of switching costs on the equilibrium market outcome in Section 5, we first decompose these effects according to the direct channel and the indirect channel mentioned above, to clarify the intuition.

4.2 Direct Channel: Network Solidification Effect and Market Contraction Effect

In this subsection we examine the changes in firms' expected sales and industry dynamics brought about by switching costs through the direct channel, represented by $\partial \phi / \partial k^c$ in Eq. (4).

In the direct channel, an increase in switching costs heightens the “exit barrier” for locked-in consumers and solidifies existing networks. Everything else being equal, higher switching costs make consumers in the installed base less likely to switch to rival products. Consequently, an installed base advantage becomes better protected. We refer to this effect as the *network solidification effect* of switching costs. To the extent that there was reversion to the mean (the opposite of increasing dominance) and the larger firm was losing consumers to the smaller firm prior to the increase in k^c , the network solidification effect benefits the larger firm by slowing down the process of such reversion to the mean. To examine this effect, we consider the discrete analogue of $\partial \phi / \partial k^c$ by marginally increasing k^c from k^0 to $k^0 + 0.01$ while holding firms' policy function fixed at the one from $k = k^0$. The resulting changes in the firms' expected sales, $(\Delta \phi_1, \Delta \phi_2)$, is a vector that tells the change in the resultant forces.

We first look at the case in which there does not exist an outside good so that the market size is fixed. Panel 1 in Figure 5 reports the percentage changes in firm 1's expected

⁶In the terminology of Chiang and Wainwright (2005) (page 192), $\left. \frac{d\phi}{dk^c} \right|_{dk^f=0}$ and $\left. \frac{d\phi}{dk^f} \right|_{dk^c=0}$ in the equation below are the “partial total derivatives”.

sale for the parameterization $(v_0, \delta, \theta, k^0) = (-\infty, 0.06, 2.2, 0)$.⁷ The panel shows that as a result of the marginal increase in k^c while holding firms' prices fixed, the smaller firm's expected sale decreases, whereas the larger firm's expected sale increases. For the same parameterization, Panel 3 reports the changes in the resultant forces due to the changes in the expected sales. It shows that the marginal increase in k^c leads to *asymmetry movements* of the industry state, that is, movements parallel to the $(0, 20) - (20, 0)$ diagonal and in the direction of increasing asymmetry between the two firms. In addition, Panel 5 reports the corresponding changes in the limiting distribution, where a "+" symbol in a state indicates an increase in the probability of that state, and a "-" symbol indicates a decrease in the probability. This panel shows that as a result of the marginal increase in k^c while holding firms' prices fixed, the industry spends less time in relatively symmetric states and more time in asymmetric states. Thus both Panel 3 and Panel 5 show an increased tendency for the industry structure to become asymmetric. For this parameterization the marginal increase in k^c causes the expected long-run Herfindahl-Hirschman Index (HHI; based on installed bases and weighted by probabilities in the limiting distribution) to increase from 0.5611 to 0.5615.

We now turn to the case in which there exists an outside good so that the market size is endogenous. Panel 2 in Figure 5 reports the percentage changes in firm 1's expected sale for the parameterization $(v_0, \delta, \theta, k^0) = (0, 0.06, 2.2, 0)$. Similar to Panel 1, it shows that as a result of the marginal increase in k^c while holding firms' prices fixed, the smaller firm's expected sale decreases, whereas the larger firm's expected sale generally increases. For this parameterization, Panel 4 reports the changes in the resultant forces. Notice the difference between the arrows in Panel 3 and those in Panel 4. Whereas in Panel 3 all arrows are parallel to the $(0, 20) - (20, 0)$ diagonal, arrows in Panel 4 point towards larger asymmetry and smaller market size $(b_1 + b_2)$. Therefore, the changes in the resultant forces in Panel 4 are the combination of two types of movements of the industry state: *asymmetry movements* (as described above) and *contraction movements* (movements parallel to the $(0, 0) - (20, 20)$ diagonal and in the direction of decreasing market size). Thus Panel 4 shows that when there exists an outside good, an increase in switching costs also has a *market contraction effect*. Whereas the network solidification effect corresponds to a shift of the demand from one inside good to the other, the market contraction effect corresponds to a shift of the demand from the inside goods to the outside good. Consider a consumer loyal to firm 1's product. When the switching cost is increased while holding firms' prices fixed, the consumer's representative utility (her utility excluding the idiosyncratic preference shock) from firm 2's product is reduced, but her representative utilities from firm 1's product and from the outside good are unaffected. Consequently, the probability that she buys firm 2's product is reduced, but the balance is not given entirely to firm 1's product; the outside good also benefits. The case with a consumer loyal to firm 2's product is analogous. On the other hand, the choice probabilities of an unattached consumer are unaffected. Therefore, due to the marginal increase in k^c while holding firms' prices fixed, the demand for the outside good is increased, resulting in a smaller market size for the inside goods. The reduction in

⁷Due to symmetry, the change in firm 2's expected sale in state $(\hat{b}, \hat{b})$ is equal to the change in firm 1's expected sale in state $(\hat{b}, \hat{b})$. This also applies to the changes in equilibrium price, expected sale, and future value function presented in the subsequent figures.

market size is reflected in Panel 6, which reports the changes in the limiting distribution. In contrast to Panel 5, Panel 6 shows increased probabilities assigned to states that are close to the origin, in addition to states that are highly asymmetric. For this parameterization the combination of the network solidification effect and the market contraction effect of switching costs results in an increase of 0.0003 in the HHI, and a decrease of 0.0106 in the expected long-run market size (weighted by probabilities in the limiting distribution).

The parameterizations in Figure 5 give rise to Rising equilibria (in which there is global convergence towards symmetry). Figure 6 reports the changes in the expected sales, the resultant forces, and the limiting distribution for the other types of equilibria: Tipping (the left column of panels), Peaked (the center column), and Mild Tipping (the right column). Examination of such changes for all the parameterizations confirms that the network solidification effect generally causes asymmetry movements of the state, and that in the case with an outside good, the market contraction effect always causes contraction movements of the state. The only area in which the network solidification effect may cause *symmetry movements* of the state (that is, movements parallel to the $(0, 20) - (20, 0)$ diagonal and in the direction of increasing symmetry between the two firms) is between the two modal states in a Tipping or Mild Tipping equilibrium, if the smaller firm was rapidly losing consumers to the larger firm prior to the increase in k^c . But even in those parameterizations, the asymmetry movements still dominate and the overall impact of a marginal increase in k^c while holding firms' prices fixed is still an increased tendency towards asymmetry. For example, in the left column in Figure 6, the marginal increase in k^c results in an increase of the HHI from 0.6269 to 0.6273, despite causing symmetry movements between the two modal states. More generally, I verified that in all the parameterizations with $v_0 \in \{-\infty, 0\}$, $\delta \in \{0, 0.01, \dots, 0.2\}$, $\theta \in \{0, 0.2, \dots, 4\}$, $k \in \{0, 0.2, \dots, 3\}$, the changes in the HHI due to a marginal increase in k^c while holding firms' prices fixed are all positive. Moreover, in all the parameterizations with $v_0 = 0$ and $\delta > 0$, the changes in the market size due to a marginal increase in k^c while holding firms' prices fixed are all negative (when $\delta = 0$, the expected long-run market size is always 20 and not affected by the increase in k^c).

4.3 Indirect Channel: Fat Cat Effect and Top Dog Effect

As discussed above, a permanent increase in the switching cost k implies an increase in the current-period switching cost k^c and an increase in the future switching cost k^f , both of which have indirect impact on consumers' choice probabilities through their influence on firms' equilibrium prices.

The literature on switching costs without network effects finds that markets with switching costs tend to be stable: switching costs cause the larger firm to price less aggressively (in order to harvest locked-in consumers) and lose consumers to the smaller firm, and therefore asymmetries in market shares are dampened over time (Beggs and Klemperer (1992), Chen and Rosenthal (1996), Taylor (2003)). This is referred to as the *fat cat effect*, with the larger firm being a nonaggressive "fat cat".⁸

In markets with network effects, switching costs have another major effect on firms' prices and the tendency towards market dominance. First note that a basic property of

⁸The terms "fat cat effect" and "top dog strategy" are introduced by Fudenberg and Tirole (1984) in the context of strategic investment and entry deterrence/accommodation.

network effects is that they can tip the market to one firm as soon as it has an installed base advantage. However, for a firm to price aggressively and give up current profit, the prospect of future dominance by investing in its installed base must be sufficiently great, which requires that network effects are sufficiently strong and that the installed base does not depreciate too rapidly in the future. This is where switching costs come into play: just as an increase in the current-period switching cost k^c solidifies the current networks (the network solidification effect discussed above), an increase in the future switching cost k^f solidifies networks in the future, making an installed base advantage longer-lasting and more valuable. This effect intensifies price competition when firms' installed bases are of comparable size, and when an installed base differential emerges, this effect causes the larger firm to price more aggressively in order to build on its advantage (since the prospect of future dominance becomes better). We refer to this effect as the *top dog effect* of switching costs, with the larger firm being an aggressive "top dog". This top dog effect of switching costs reinforces network effects and makes market dominance more likely.

Below we examine the fat cat effect and the top dog effect of switching costs in turn.

4.3.1 Fat Cat Effect

To examine the fat cat effect, we consider the discrete analogue of $\partial p / \partial k^c$ by marginally increasing the current-period switching cost k^c from k^0 to $k^0 + 0.01$ while holding the future switching cost k^f fixed at k^0 . Consequently the firms' future value function $V'(b')$ is also fixed at the one from $k = k^0$, and changes in firms' prices are only due to the change in k^c .

When there is an increase in k^c , the harvesting incentive to raise price is stronger for the larger firm, because a majority of the loyal demand comes from consumers loyal to its product, and now these consumers find it more difficult to switch. Furthermore, the incentive to lower price is stronger for the smaller firm, because a majority of the loyal demand comes from consumers loyal to its rival's product, and it now takes a greater price discount to lure these consumers. Therefore the larger firm is more likely than the smaller firm to increase price. Such intuition is borne out by results from the computation. The three columns of panels in Figure 7 pertain to the parameterizations with $(\delta, \theta, k^0) = (0.06, 2.2, 0)$ and $v_0 = -\infty, -2$, and 0 , respectively.⁹ The top row of panels plot the change in firm 1's equilibrium price due to the marginal increase in k^c . They show that a firm's price increases in its own installed base and decreases in its rival's installed base, and the price change is generally positive for the larger firm and negative for the smaller firm. The bottom row of panels plot the percentage change in firm 1's expected sale. They show that as a result of the change in the equilibrium prices, the larger firm's expected sale drops whereas the smaller firm's expected sale increases.¹⁰ Corresponding to Panel 2 ($v_0 = -2$) in Figure 7, Figure 8 spells out the value of the price change in all the states. The states in which the price change is negative are framed. The figure shows that the price change is close to 0 in states near the diagonal (in which firms have comparable installed bases), but increases significantly as the firm gains an installed base advantage over its rival. For

⁹To aid comparison, the scale and location of the z -axis are the same for all the panels in the same row in Figure 7. This also applies to Figure 9 below.

¹⁰Here, when computing the change in the firms' expected sales, k^c is held fixed at k^0 , so that the direct channel and the indirect channel are not conflated, and the change in expected sales is only due to the change in prices.

example, the change in firm 1's price is 0 in state $(0, 0)$, but increases to 0.7×10^{-3} in state $(5, 0)$, 1.7×10^{-3} in state $(10, 0)$, 3.0×10^{-3} in state $(15, 0)$, and 4.3×10^{-3} in state $(20, 0)$. On the other hand, the change in the smaller firm's price is negative and has a relatively small absolute value (no larger than 1.5×10^{-3}). Thus the numbers reported in Figure 8 illustrate the asymmetric impact of an increase in the current-period switching cost on firms' prices and show that the larger firm tends to act as a nonaggressive fat cat by increasing its price. Such asymmetry in firms' price responses makes it likely that the larger firm will lose consumers to the smaller firm, and therefore there is an increased tendency for the industry to move towards symmetric states. Furthermore, results from the computation show that this pattern of asymmetric price responses is robust to different parameterizations and different types of equilibria.

Figure 7 also illustrates how the quality of the outside good affects the magnitude of the difference between the two firms' price responses. Specifically, the outside good serves as a non-strategic player and restrains firms' ability to harvest their locked-in consumers, because these consumers can resort to the outside good if prices charged by the firms are too high. Such a "threat" generated by the outside good can significantly dampen the fat cat effect, as illustrated by the three columns of panels in Figure 7, in which the value for v_0 is successively increased. For example, consider the change in firm 1's price in state $(20, 0)$. In Panel 1 ($v_0 = -\infty$), the price change is 6.5×10^{-3} , and it decreases to 4.3×10^{-3} in Panel 2 ($v_0 = -2$), and 1.4×10^{-3} in Panel 3 ($v_0 = 0$). Similarly, Panels 4-6 show that as v_0 is increased, the percentage increase in the smaller firm's expected sale and the percentage decrease in the larger firm's expected sale are both diminished. Together these panels show that the more attractive is the outside good, the weaker is the fat cat effect. Such diminishing of the fat cat effect due to the existence of an outside good has implications for the effects of switching costs on market dominance and prices, which we discuss later.

4.3.2 Top Dog Effect

Turning to the effects of the future switching cost k^f on firms' prices, we see a different picture. Whereas an increase in k^c makes the larger firm price less aggressively than the smaller firm, an increase in k^f generally causes the larger firm to act as a top dog by pricing more aggressively than the smaller firm.

Since k^f affects the firms' equilibrium prices through its impact on the firms' future value function, we first examine how the future value function is affected. As discussed above, to highlight the dependence of V' on k^f , we rewrite it as $V'(k^f)$. We then consider the discrete analogue of dV'/dk^f by marginally increasing k^f from k^0 to $k^0 + 0.01$.¹¹ The top row of panels in Figure 9 plot the resulting changes in firm 1's future value function, denoted by $\Delta V'_1$. In Panel 1, there does not exist an outside good ($v_0 = -\infty$). We see that the increase in k^f enlarges the reward to being a dominant firm: $\Delta V'_1$ is negative for the smaller firm, and increases rapidly for the larger firm as it widens its network size advantage over its rival. Also note that $\Delta V'_1$ is positive only if firm 1 has a dominant position; in fact, in Panel 1 $\Delta V'_1 > 0$ only if $b'_1 - b'_2 \geq 9$. Such patterns are consistent with our intuition that an increase in k^f makes a network size advantage longer-lasting and hence more valuable.

¹¹Since our solution concept is Markov perfect equilibrium in an infinite-horizon game, dV'/dk^f is identical to dV/dk , where k encompasses both k^c and k^f .

As a result, firms have greater incentive to fight for dominance, which lowers the firms' values except when a firm emerges as the dominant firm.

In Panels 1-3, we successively increase the value for v_0 , from $-\infty$ to -2 to 0 . When the quality of the outside good is increased, the dominant firm's ability to charge high prices to exploit its loyal consumers is impaired by the increased competition from the outside good. As a result, the extra reward to being a dominant firm created by the increase in k^f shrinks, as reflected by the flatter climb in $\Delta V'_1$ for the larger firm.

We next examine the change in firms' equilibrium prices due to the above change in the future value function. The middle row of panels in Figure 9 plot the change in firm 1's equilibrium price when k^f is marginally increased from k^0 to $k^0 + 0.01$ (which causes the $\Delta V'_1$ discussed above) while k^c is fixed at k^0 . We again first consider the case without an outside good, depicted in Panel 4. Here we see that firms drop their prices significantly when they are of comparable size, indicating that the fight for dominance is made fiercer by the increase in k^f and the resulting extra reward to being a dominant firm. Furthermore, as a network size differential emerges and widens, Δp_1 rises for both the smaller firm and the larger firm, but the smaller firm's price change rises more rapidly than the larger firm's. In fact, the larger firm's price change is always more negative than the smaller firm's, indicating that the increase in k^f causes the larger firm to act as a top dog, by pricing more aggressively than the smaller firm in order to build and keep its network size advantage. Such pricing behavior increases the tendency towards market dominance.

As we successively increase the quality of the outside good in Panels 4-6, the patterns regarding the change in equilibrium price mentioned above persist, but the magnitude is reduced. Firms do not increase their level of competition so much when they are of comparable size, as reflected by the modest drop in prices along the diagonal in Panels 5-6. It remains true that the larger firm's price change is always more negative than the smaller firm's, but the difference between the two firms' price changes shrinks noticeably as v_0 increases, indicating that the top dog effect is dampened by the increased competition from the outside good.

Finally, the bottom row of panels in Figure 9 plot the percentage change in firm 1's expected sale due to the marginal increase in k^f while k^c is fixed at k^0 . These panels show that as a result of the top dog effect, the smaller firm's expected sale decreases whereas the larger firm's generally increases, which implies an increased tendency towards market dominance. Furthermore, consistent with our discussion above, the percentage changes in both firms' expected sales are diminished as the outside good's quality is increased.

5 Switching Costs and Market Dominance

For the primary dynamic forces of our model to be at work, the relevant part of the parameter space is when the rate of depreciation is neither too low (so that there is customer turnover) nor too high (so that the investment incentive is not weak). In that part of the parameter space, the switching cost and its interaction with the network effect have significant impact on the likelihood of market dominance.

5.1 Fixed Market Size

First consider the case with fixed market size ($v_0 = -\infty$). Here, two major forces compete against each other in affecting the relationship between switching costs and the tendency towards market dominance: the fat cat effect, which works against dominance, and the combination of the network solidification effect and the top dog effect, which facilitates dominance. Which of these two forces dominates is then an empirical question. Results from this model suggest that the network solidification effect and the top dog effect dominate at low switching cost whereas the fat cat effect takes over at high switching cost. For example, the top row of panels in Figure 10 plot the expected long-run HHI for $v_0 = -\infty$ and $\delta = 0.05$, 0.06 , and 0.07 , respectively.¹² Each panel reports the HHI for the parameterizations with $\theta \in \{0, 0.2, \dots, 4\}$ and $k \in \{0, 0.2, \dots, 3\}$. The higher is the HHI, the more likely market dominance is to occur. The panels reveal a non-monotonic relationship between switching costs and market dominance. In all three panels and for all levels of network effect, the HHI initially increases and then decreases as the switching cost increases from 0 to 3, indicating a change in the dominant force.

When the network effect is low to modest ($\theta \in [0, 1.6]$ in Panel 1, $\theta \in [0, 2]$ in Panel 2, and $\theta \in [0, 2.2]$ in Panel 3), the HHI is low throughout, and the changes in the HHI caused by changes in the switching cost are small. For example, with $\theta = 0$ and $\delta = 0.06$ (Panel 2), the HHI is almost flat from 0.5323 at $k = 0$ to 0.5337 at $k = 1$, then slightly decreases to 0.5225 at $k = 2$ and 0.5159 at $k = 3$. Examination of the policy function and the limiting distribution in this part of the parameter space indicates that the equilibrium gradually morphs from a Rising equilibrium at low switching cost to a Peaked equilibrium at modest to high switching cost. Within the range in which a Peaked equilibrium occurs, the peak in the policy function becomes sharper and sharper as the switching cost increases.

When the network effect is modest to high ($\theta \in [2, 4]$ in Panel 1, $\theta \in [2.4, 4]$ in Panel 2, and $\theta \in [2.8, 4]$ in Panel 3), the HHI starts with a relatively high level (above 0.6) at $k = 0$. As the switching cost increases, the HHI initially increases but later drops significantly. For example, with $\theta = 3$ and $\delta = 0.06$ (Panel 2), the HHI starts with 0.645 at $k = 0$, increases to 0.664 at $k = 1$, then decreases to 0.647 at $k = 1.6$ before plummeting to 0.544 at $k = 1.8$. It then continues to decrease slightly, reaching 0.520 at $k = 3$. A Tipping equilibrium occurs for $k \leq 1.6$ and a Peaked equilibrium occurs for $k \geq 1.8$.

Finally, for some intermediate values of the network effect ($\theta = 1.8$ in Panel 1, $\theta = 2.2$ in Panel 2, and $\theta \in \{2.4, 2.6\}$ in Panel 3), the gradual increase in the switching cost causes the equilibrium to change from Rising to Tipping then to Peaked. For example, with $\theta = 2.2$ and $\delta = 0.06$ (Panel 2), there is a Rising equilibrium at $k = 0$ (HHI = 0.561), a Tipping equilibrium at $k = 1$ (HHI = 0.627), and a Peaked equilibrium at $k = 2$ (HHI = 0.527) (the equilibria for those three parameterizations are depicted in Figures 1-3).

5.2 Endogenous Market Size

Turning to the case with endogenous market size, both the fat cat effect and the top dog effect are dampened by competition from the outside good, yet the network solidification

¹²In Figure 10, in order to show the details in each panel, the scale of the z -axis is not fixed across different panels.

effect, which generally increases the asymmetry, is not much affected (for example, see Panels 1 and 2 in Figure 5). Consistent with these observations, results from the model show that when the outside good is reasonably attractive, an increase in switching costs generally increases the likelihood of market dominance. For example, the bottom row of panels in Figure 10 plot the expected long-run HHI for $v_0 = 0$ and $\delta = 0.05, 0.06$, and 0.07 , respectively. In contrast to the top row, in the bottom row the HHI almost always increases in the switching cost, suggesting that the quality of the outside good plays an important role in determining the effects of switching costs on the likelihood of market dominance. With $v_0 = 0$, there is a Rising equilibrium when the network effect is weak and a Tipping or Mild Tipping equilibrium when the network effect is strong. Examination of the policy functions and the limiting distributions show that an increase in the switching cost does not change the type of the equilibrium that occurs; instead, it results in the limiting distribution putting more mass in more asymmetric states and states closer to the origin.

The middle row of panels ($v_0 = -2$) complement the other panels in Figure 10 to illustrate the progressive changes that take place as the outside good becomes more and more attractive. Together the panels show that when v_0 increases, the ability of switching costs to reduce market asymmetry is weakened. In particular, in the top row ($v_0 = -\infty$), an increase in the switching cost from k^0 to $k^0 + 0.2$ results in a decrease of the HHI 73.4% of the time. That number drops to 20.0% for the middle row ($v_0 = -2$), and further drops to 0.8% for the bottom row ($v_0 = 0$).

In sum, the results presented in Figure 10 highlight the importance of the network effect and the outside good in the analysis of switching costs and market dominance. In contrast to the findings in studies of switching costs that abstract from network effects, we find that the existence of network effects enables switching costs to increase the likelihood of market dominance, sometimes dramatically so, via the combination of the network solidification effect and the top dog effect. Moreover, the existence of an outside good can significantly reduce the ability of switching costs to prevent market dominance by dampening the fat cat effect.

6 Price and Welfare

In this section we consider the effects of switching costs on price and welfare in network industries.

6.1 Price

Figure 11 plots the average price (weighted first by the expected sales then by the probabilities in the limiting distribution) against the switching cost, using $\delta = 0.06$. $v_0 = -\infty$ in the left column of panels and $v_0 = 0$ in the right column. From the top row to the bottom row, θ is 0, 1, 2, 3, 4, respectively. The figure reveals the following patterns. First, without an outside good ($v_0 = -\infty$), the average price generally increases in the switching cost, although there may be a noticeable drop when the equilibrium switches from Tipping to Peaked, such as when k increases from 1.6 to 1.8 in Panel 7 and from 2 to 2.2 in Panel 9. Second, with an outside good ($v_0 = 0$), the average price tends to decrease in the switching cost when the network effect is modest (Panels 2, 4, and 6) and increase in the switching

cost when the network effect is strong (Panel 10). Third, the changes in the average price caused by the changes in the switching cost are much bigger when $v_0 = -\infty$ than when $v_0 = 0$.

Table 1 provides confirming results from a broad set of parameterizations. It reports statistics for the percentage change in the equilibrium average price when the switching cost increases from k^0 to $k^0 + 0.2$, for $v_0 \in \{-\infty, -2, 0\}$, $\delta \in \{0.04, 0.05, \dots, 0.1\}$, $\theta \in \{0, 0.2, \dots, 4\}$, and $k^0 \in \{0, 0.2, \dots, 2.8\}$. In the table, the parameterizations are first grouped according to the value of v_0 , and then within each v_0 group, they are further divided into subgroups according to the value of θ . The statistics reported are the 5th, 25th, 50th, 75th, and 95th percentiles, as well as the percentage of negative changes.

The first three rows in Table 1 show that the quality of the outside good plays an important role in determining the effects of switching costs on price. An increase in the switching cost tends to increase price if there does not exist an outside good, but this is gradually reversed as the quality of the outside good increases. When $v_0 = -\infty$, an increase in the switching cost reduces price only 10.1% of the time. That number increases to 20.7% for $v_0 = -2$, and further jumps to 62% for $v_0 = 0$. Furthermore, as the outside good becomes more attractive, the impact of switching costs on price is diminished. For example, as v_0 increases from $-\infty$ to 0, the 25th-75th percentile interval moves down and narrows from $[2.83\%, 6.98\%]$ to $[-0.24\%, 0.09\%]$. The outside good is a non-strategic player whose competition weakens firms' ability to change their prices in response to the switching costs.

The remaining rows in Table 1 show that a stronger network effect makes it more likely that the average price will increase in the switching cost. This pattern is particularly salient when there exists a reasonably attractive outside good. For example, with $v_0 = -2$, the percentage of negative price changes is 33.3% when $\theta \in [0, 1)$, but drops to 12.4% when $\theta \in [3, 4]$. With $v_0 = 0$, the percentage of negative price changes is 93.7% when $\theta \in [0, 1)$, but drops dramatically to 27.0% when $\theta \in [3, 4]$. In the case with $v_0 = 0$, the increase of θ from $[0, 1)$ to $[3, 4]$ results in the median changing from negative (-0.08%) to positive (0.18%).

6.1.1 The Four Effects

To understand the above patterns, we start by examining how the average price is affected by the network solidification effect, the market contraction effect, the fat cat effect, and the top dog effect of switching costs. Note that the network solidification effect and the market contraction effect represent movements *on* the price surface, since the price policy function is fixed but the position of the industry state is changed. On the other hand, the fat cat effect and the top dog effect represent both movements *on* the price surface and movements *of* the price surface: firms change their price policy function in response to a change in the switching cost, and consequently, the position of the industry state is also changed.

We first consider the network solidification effect. Computation using the policy functions in all the parameterizations with $\theta > 0$ shows that an asymmetry movement of the state (that is, a movement from state (b_1, b_2) to state $(b_1 + 1, b_2 - 1)$ if $b_1 \geq b_2$, or to state $(b_1 - 1, b_2 + 1)$ if $b_1 \leq b_2$) causes the average price to increase except when there is a Peaked equilibrium (for an example of the latter see Panel 1 in Figure 2). An asymmetry

movement represents a widening of the installed base differential between the two firms, so it allows the larger firm to charge a higher price and drive up the average price—except when high switching costs result in a Peaked equilibrium, in which case both firms drop their prices when the state moves away from the (symmetric) peak. Therefore, the network solidification effect, which in general causes asymmetry movements of the state, increases the average price except when there is a Peaked equilibrium.

Next consider the market contraction effect. Computation using the policy functions in all the parameterizations with $v_0 \in \{-2, 0\}$ shows that a contraction movement of the state (that is, a movement from state (b_1, b_2) to state $(b_1 - 1, b_2 - 1)$) causes the average price to decrease 96% of the time. A contraction movement represents a reduction in the size of the market, and firms find it optimal to lower their prices in response. Therefore, the market contraction effect, which shrinks the market size by making the inside goods less attractive relative to the outside good, reduces the average price.

Next consider the fat cat effect. The main feature of this effect is a price increase by the larger firm in order to harvest its loyal consumers. Given that the larger firm has a bigger expected sale than the smaller firm, such movement of the price surface increase the average price. On the other hand, the fat cat effect also drives the industry towards more symmetric states, and such movement on the price surface decreases the average price except when there is a peaked equilibrium (as discussed above). Examination of the computation results indicates that the net impact of the fat cat effect on the average price is generally positive.

Finally, consider the top dog effect. The main feature of this effect is a price reduction by the larger firm in order to build and keep its network size advantage. The smaller firm also generally decreases its price, though by a smaller amount. The result of such movement of the price surface is a reduction in the average price. On the other hand, the top dog effect also pulls the industry towards more asymmetric states, and such movement on the price surface generally increases the average price. Examination of the computation results indicates that the net impact of the top dog effect on the average price is generally positive.

6.1.2 The Outside Good

The above analysis sheds light on the patterns we observe in Figure 11 and Table 1. Among the four effects, the main force that reduces the average price is the market contraction effect. Without an outside good, the market contraction effect does not operate, and hence the average price generally increases in the switching cost. As the quality of the outside good increases, the market contraction effect is gradually strengthened, making it more likely that an increase in the switching cost will cause the average price to drop.

6.1.3 Strength of the Network Effect

The relationship between switching costs and price also critically depends on the strength of the network effect, particularly when the outside good is reasonably attractive. As v_0 increases, the Peaked equilibrium gradually becomes extinct, and therefore any asymmetry movement of the industry state tends to raise the average price. Under this circumstance, a stronger network effect amplifies the network solidification effect to make it more likely that the average price will increase in the switching cost. The amplification happens in two ways.

First, the basic function of the network solidification effect is that it gives the larger firm an extra advantage by making the installed base differential longer-lasting. At the same time, the fundamental property of network effects is that they create a “bandwagon” or “snowball” effect that allows a small base differential to quickly widen. Hence, a stronger network effect expands the extra advantage for the larger firm that is created by the network solidification effect and causes the larger firm to price even more aggressively as a “top dog” (recall that the top dog effect is based on the network solidification effect in future periods), thus giving rise to a larger increase in the asymmetry in the industry.

Second, a stronger network effect translates an installed base differential into a larger quality differential and allows the larger firm to raise its price more substantially. To illustrate, Figure 12 plots the policy functions for two parameterizations that differ only in θ : $\theta = 1$ in Panel 1 and $\theta = 3$ in Panel 2. The biggest difference between the two policy functions is that as a firm gains an installed base advantage over its rival, its price increases only mildly in Panel 1 but significantly in Panel 2. For instance, when the state moves parallel to the $(0, 20) - (20, 0)$ diagonal from $(8, 8)$ to $(14, 2)$, in Panel 1 firm 1’s price increases only 7.1% from 1.19 to 1.27, whereas in Panel 2 its price increases a sizeable 59.4% from 0.86 to 1.37. Therefore, when the network solidification effect and the top dog effect of switching costs cause the industry to move towards more asymmetric states, a stronger network effect translates such asymmetry movements into larger price increases, making it more likely that the overall impact of the switching cost on the average price is positive.

To summarize, switching costs tend to increase prices when there does not exist an outside good so that the market size is fixed. When there exists a reasonably attractive outside good, switching costs tend to increase prices when the network effect is strong and tend to decrease prices when the network effect is weak. The key to understanding these patterns lies in understanding the ability of the outside good to induce the market contraction effect, as well as the ability of the network effect to amplify the network solidification effect and the top dog effect. Therefore, results from the current model offer a new perspective on the price puzzle of switching costs discussed in the Introduction, by showing that both scenarios (“prices increase in switching costs” and “prices decrease in switching costs”) can arise as equilibrium outcomes and that network effects and competition from the outside good are important determinants.

6.2 Welfare

Here we examine expected producer surplus, consumer surplus, and total welfare (the sum of the previous two measures), based on the limiting distribution.¹³ We consider the changes in the equilibrium levels of these measures when the switching cost increases from k^0 to $k^0 + 0.2$, again for $v_0 \in \{-\infty, -2, 0\}$, $\delta \in \{0.04, 0.05, \dots, 0.1\}$, $\theta \in \{0, 0.2, \dots, 4\}$, and $k^0 \in \{0, 0.2, \dots, 2.8\}$.

Table 2 reports statistics for the percentage change in producer surplus. It shows that switching costs help the firms when there does not exist an outside good, but become more

¹³The producer surplus is equal to the combined profits of the firms, and the consumer surplus is the log-sum term (Train (2003)). In the case without an outside good, the producer surplus is equal to the average price, since the sum of the firms’ expected sales is always 1 and the marginal cost is normalized to 0.

and more harmful to the firms as v_0 increases. Producer surplus increases in the switching cost 89.9% of the time when $v_0 = -\infty$, but decreases in the switching cost 92.5% of the time when $v_0 = 0$. For a same v_0 , a stronger network effect makes it more likely that the firms will benefit from the switching cost. For example, with $v_0 = -2$, the firms suffer a loss due to an increase in the switching cost 86.1% of the time if $\theta \in [0, 1)$, but only 14.0% of the time if $\theta \in [3, 4]$.

As we have previously discussed, the existence of an outside good gives consumers greater flexibility in their product choices and restrains firms' ability to raise prices. Therefore a stronger outside good makes switching costs more harmful to firms and less harmful to consumers. On the other hand, when the network effect is strong, switching costs are less capable of reducing the size of the market and more likely to result in price increases, and hence firms are more likely to benefit from switching costs.

Prior studies on switching costs, which often assume a fixed market size and do not consider network effects, tend to find that switching costs raise oligopoly profits (see Section 2.4.4. in Farrell and Klemperer (2007) and the references cited therein). By incorporating network effects and allowing the market size to be endogenously determined, this model allows us to look at a broader picture, in which the relationship between switching costs and firm profits depends on the strength of the network effect and the quality of the outside good.

Turning to consumer surplus (Table 3) and total welfare (Table 4), we find that both of them consistently decrease in the switching cost.¹⁴ Furthermore, the magnitude of the decrease tends to be smaller as the outside good becomes stronger. For example, The median change in consumer surplus diminishes (in absolute value) from -0.123 when $v_0 = -\infty$ to -0.005 when $v_0 = 0$, and the median change in total welfare diminishes from -0.038 when $v_0 = -\infty$ to -0.008 when $v_0 = 0$.

These results show that consumers are harmed by switching costs. And even when firms benefit from switching costs, consumers' losses more than offset firms' gains and result in a reduction in total welfare. Therefore, these results give support to public policies that aim at reducing switching costs. In the case with a reasonably attractive outside good, both firms and consumers will benefit from a reduction in switching costs. If the outside good is fairly inferior, industry lobbyists and consumer advocates are likely to be on opposing sides of a policy debate.

7 Conclusion

Switching costs are an important characteristic of network industries. In this paper I use a dynamic oligopolistic model of price competition to investigate the effects of switching costs in network industries. I find that switching costs have two opposite influences on the tendency towards market dominance. First, the fat cat effect causes the larger firm to price less aggressively in order to harvest the locked-in consumers. As a result, the larger firm loses consumers to the smaller firm. This effect works against market dominance. Second, the network solidification effect reinforces network effects by heightening the exit

¹⁴Note that Tables 3 and 4 report on the actual changes in consumer surplus and total welfare rather than the percentage changes, as the latter are not invariant to normalization of the intrinsic qualities of the goods.

barrier for locked-in consumers and making a network size advantage longer-lasting and more valuable. As a result, the larger firm acts as a top dog by pricing more aggressively in order to obtain and keep a dominant position. This effect facilitates tipping towards market dominance. The results show a non-monotonic relationship between switching costs and market dominance. At low switching costs, the top dog effect dominates and an increase in switching costs can change the market from fragmentation to being dominated by one firm. But at high switching costs, the fat cat effect takes over and an increase in switching costs can change the market from being dominated by one firm to fragmentation.

Furthermore, I find that network effects and competition from the outside good are important determinants of the relationship between switching costs and prices. If the outside good is fairly inferior, the average price generally increases in switching costs. But if there exists a reasonably attractive outside good, the average price tends to decrease in switching costs when the network effect is modest and increase in switching costs when the network effect is strong. The reason is that with an outside good, an increase in switching costs makes the firms' products less attractive and shifts some of the demand from the firms' products to the outside good. As a result, the size of the market is reduced and firms' equilibrium prices tend to drop. Furthermore, a stronger network effect amplifies the top dog effect mentioned above, and hence an increase in switching costs becomes more effective in pulling the industry towards asymmetric states, in which the average price is generally higher than in symmetric states. These results shed new light on the price puzzle of switching costs, and are supported by anecdotal evidence from existing empirical studies.

Thus the findings in this paper highlight the interactions between switching costs and network effects, and call for more research that explores their impact on industry dynamics and market outcome. A good understanding of such issues will allow regulators to make informed decisions about switching costs policies in network industries.

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(1) Firm 1's policy function

(2) Firm 1's value function

(3) Transient distribution after 15 periods

(4) Limiting distribution

(5) Probability that firm 1 makes a sale

(6) Resultant forces

Figure 1. Rising equilibrium: $v_0 = -\infty$, $\delta = 0.06$, $\theta = 2.2$, $k = 0$

(1) Firm 1's policy function

(2) Firm 1's value function

(3) Transient distribution after 15 periods

(4) Limiting distribution

(5) Probability that firm 1 makes a sale

(6) Resultant forces

Figure 2. Peaked equilibrium: $v_0 = -\infty$, $\delta = 0.06$, $\theta = 2.2$, $k = 2$

(1) Firm 1's policy function

(2) Firm 1's value function

(3) Transient distribution after 15 periods

(4) Limiting distribution

(5) Probability that firm 1 makes a sale

(6) Resultant forces

Figure 3. Tipping equilibrium: $v_0 = -\infty$, $\delta = 0.06$, $\theta = 2.2$, $k = 1$

(1) Firm 1's policy function

(2) Firm 1's value function

(3) Transient distribution after 15 periods

(4) Limiting distribution

(5) Probability that firm 1 makes a sale

(6) Resultant forces

Figure 4. Mild Tipping equilibrium: $v_0 = 0$, $\delta = 0.04$, $\theta = 2.6$, $k = 1$

Figure 5. Network solidification effect and market contraction effect: $\delta = 0.06$, $\theta = 2.2$, $k^0 = 0$. Presents changes in expected sale, resultant forces, and limiting distribution, when k^c is marginally increased from k^0 to $k^0 + 0.01$ while firms' policy function is fixed at the one from $k = k^0$.

Figure 6. Network solidification effect and market contraction effect.

Presents changes in expected sale, resultant forces, and limiting distribution, when k^c is marginally increased from k^0 to $k^0 + 0.01$ while firms' policy function is fixed at the one from $k = k^0$.

Figure 7. Fat cat effect of switching costs: $\delta = 0.06$, $\theta = 2.2$, $k^0 = 0$. Presents the change in equilibrium price when the current-period switching cost k^c is marginally increased from k^0 to $k^0 + 0.01$ while the future switching cost k^f is fixed at k^0 . Also presents the % change in expected sale due to the change in equilibrium price while holding k^c fixed.

Figure 8. Fat cat effect of switching costs: $v_0 = -2$, $\delta = 0.06$, $\theta = 2.2$, $k^0 = 0$. Presents the change in firm 1's equilibrium price when the current-period switching cost k^c is marginally increased from k^0 to $k^0 + 0.01$ while the future switching cost k^f is fixed at k^0 . Below the 45 degree line, firm 1 is the larger firm. Negative entries are framed.

Figure 9. Top dog effect of switching costs: $\delta = 0.06$, $\theta = 2.2$, $k^0 = 0$.

Presents changes in future value function, equilibrium price, and expected sale, when the future switching cost k^f is marginally increased from k^0 to $k^0 + 0.01$ while the current-period switching cost k^c is fixed at k^0 .

(1) $v_0 = -\infty, \delta = 0.05$

(2) $v_0 = -\infty, \delta = 0.06$

(3) $v_0 = -\infty, \delta = 0.07$

(4) $v_0 = -2, \delta = 0.05$

(5) $v_0 = -2, \delta = 0.06$

(6) $v_0 = -2, \delta = 0.07$

(7) $v_0 = 0, \delta = 0.05$

(8) $v_0 = 0, \delta = 0.06$

(9) $v_0 = 0, \delta = 0.07$

Figure 10. Expected long-run HHI

Figure 11. Average price: $\delta = 0.06$

Figure 12. Firm 1's policy function: $v_0 = 0$, $\delta = 0.05$, $k = 0$

Table 1. Percentage change in average price from $k = k^0$ to $k = k^0 + 0.2$
 $\delta \in [0.04, 0.1]$, $k^0 \in [0, 3]$

	Percentiles					% negative
	5th	25th	50th	75th	95th	
$v_0 = -\infty, \theta \in [0, 4]$	-1.93%	2.83%	4.77%	6.98%	12.95%	10.1%
$v_0 = -2, \theta \in [0, 4]$	-1.75%	0.18%	0.86%	2.00%	3.43%	20.7%
$v_0 = 0, \theta \in [0, 4]$	-0.92%	-0.24%	-0.02%	0.09%	0.95%	62.0%
$v_0 = -\infty, \theta \in [0, 1]$	-1.94%	2.43%	4.72%	6.73%	11.96%	13.3%
$v_0 = -\infty, \theta \in [1, 2]$	-2.29%	2.71%	5.24%	7.62%	14.70%	12.8%
$v_0 = -\infty, \theta \in [2, 3]$	-1.72%	3.19%	5.27%	7.50%	14.71%	8.2%
$v_0 = -\infty, \theta \in [3, 4]$	-1.15%	2.83%	4.06%	6.13%	10.47%	6.7%
$v_0 = -2, \theta \in [0, 1]$	-1.82%	-0.36%	0.20%	0.46%	1.05%	33.3%
$v_0 = -2, \theta \in [1, 2]$	-2.03%	0.04%	0.57%	1.25%	2.93%	24.4%
$v_0 = -2, \theta \in [2, 3]$	-1.75%	0.69%	1.43%	2.57%	3.87%	14.5%
$v_0 = -2, \theta \in [3, 4]$	-1.47%	1.15%	1.93%	2.63%	3.67%	12.4%
$v_0 = 0, \theta \in [0, 1]$	-0.70%	-0.27%	-0.08%	-0.02%	0.00%	93.7%
$v_0 = 0, \theta \in [1, 2]$	-0.91%	-0.31%	-0.06%	-0.01%	0.10%	78.7%
$v_0 = 0, \theta \in [2, 3]$	-1.07%	-0.26%	-0.01%	0.12%	0.92%	55.4%
$v_0 = 0, \theta \in [3, 4]$	-0.99%	-0.02%	0.18%	0.56%	2.08%	27.0%

Table 2. Percentage change in producer surplus from $k = k^0$ to $k = k^0 + 0.2$
 $\delta \in [0.04, 0.1]$, $k^0 \in [0, 3]$

	Percentiles					% negative
	5th	25th	50th	75th	95th	
$v_0 = -\infty, \theta \in [0, 4]$	-1.93%	2.83%	4.77%	6.98%	12.95%	10.1%
$v_0 = -2, \theta \in [0, 4]$	-2.26%	-0.36%	0.31%	1.34%	2.59%	38.2%
$v_0 = 0, \theta \in [0, 4]$	-2.09%	-1.01%	-0.43%	-0.18%	0.19%	92.5%
$v_0 = -\infty, \theta \in [0, 1]$	-1.94%	2.43%	4.72%	6.73%	11.96%	13.3%
$v_0 = -\infty, \theta \in [1, 2]$	-2.29%	2.71%	5.24%	7.62%	14.70%	12.8%
$v_0 = -\infty, \theta \in [2, 3]$	-1.72%	3.19%	5.27%	7.50%	14.71%	8.2%
$v_0 = -\infty, \theta \in [3, 4]$	-1.15%	2.83%	4.06%	6.13%	10.47%	6.7%
$v_0 = -2, \theta \in [0, 1]$	-2.52%	-1.10%	-0.29%	-0.06%	0.12%	86.1%
$v_0 = -2, \theta \in [1, 2]$	-2.43%	-0.61%	0.12%	0.47%	1.66%	39.2%
$v_0 = -2, \theta \in [2, 3]$	-2.09%	0.31%	0.87%	1.74%	2.78%	18.5%
$v_0 = -2, \theta \in [3, 4]$	-1.60%	0.73%	1.37%	2.03%	3.04%	14.0%
$v_0 = 0, \theta \in [0, 1]$	-2.07%	-1.07%	-0.54%	-0.26%	-0.13%	100.0%
$v_0 = 0, \theta \in [1, 2]$	-2.45%	-1.21%	-0.59%	-0.28%	-0.15%	100.0%
$v_0 = 0, \theta \in [2, 3]$	-2.29%	-1.11%	-0.48%	-0.20%	0.10%	93.5%
$v_0 = 0, \theta \in [3, 4]$	-1.72%	-0.62%	-0.18%	-0.04%	0.99%	79.2%

Table 3. Change in consumer surplus from $k = k^0$ to $k = k^0 + 0.2$
 $\delta \in [0.04, 0.1]$, $k^0 \in [0, 3)$

	Percentiles					% negative
	5th	25th	50th	75th	95th	
$v_0 = -\infty, \theta \in [0, 4]$	-0.378	-0.222	-0.123	-0.082	-0.026	99.0%
$v_0 = -2, \theta \in [0, 4]$	-0.076	-0.047	-0.027	-0.016	-0.007	99.9%
$v_0 = 0, \theta \in [0, 4]$	-0.024	-0.009	-0.005	-0.002	-0.001	100.0%
$v_0 = -\infty, \theta \in [0, 1)$	-0.388	-0.246	-0.129	-0.081	-0.031	100.0%
$v_0 = -\infty, \theta \in [1, 2)$	-0.412	-0.249	-0.129	-0.081	-0.025	100.0%
$v_0 = -\infty, \theta \in [2, 3)$	-0.425	-0.235	-0.124	-0.086	-0.022	99.2%
$v_0 = -\infty, \theta \in [3, 4]$	-0.288	-0.170	-0.114	-0.080	-0.029	97.3%
$v_0 = -2, \theta \in [0, 1)$	-0.036	-0.022	-0.014	-0.010	-0.005	100.0%
$v_0 = -2, \theta \in [1, 2)$	-0.060	-0.034	-0.021	-0.014	-0.008	100.0%
$v_0 = -2, \theta \in [2, 3)$	-0.079	-0.057	-0.033	-0.021	-0.011	100.0%
$v_0 = -2, \theta \in [3, 4]$	-0.084	-0.063	-0.047	-0.033	-0.016	99.7%
$v_0 = 0, \theta \in [0, 1)$	-0.009	-0.004	-0.002	-0.001	-0.001	100.0%
$v_0 = 0, \theta \in [1, 2)$	-0.014	-0.006	-0.003	-0.002	-0.001	100.0%
$v_0 = 0, \theta \in [2, 3)$	-0.025	-0.010	-0.005	-0.003	-0.001	100.0%
$v_0 = 0, \theta \in [3, 4]$	-0.035	-0.016	-0.009	-0.005	-0.002	100.0%

Table 4. Change in total welfare from $k = k^0$ to $k = k^0 + 0.2$
 $\delta \in [0.04, 0.1]$, $k^0 \in [0, 3)$

	Percentiles					% negative
	5th	25th	50th	75th	95th	
$v_0 = -\infty, \theta \in [0, 4]$	-0.089	-0.054	-0.038	-0.024	-0.010	99.8%
$v_0 = -2, \theta \in [0, 4]$	-0.056	-0.037	-0.027	-0.015	-0.007	100.0%
$v_0 = 0, \theta \in [0, 4]$	-0.029	-0.014	-0.008	-0.004	-0.002	100.0%
$v_0 = -\infty, \theta \in [0, 1)$	-0.077	-0.055	-0.040	-0.023	-0.010	100.0%
$v_0 = -\infty, \theta \in [1, 2)$	-0.075	-0.057	-0.041	-0.023	-0.009	100.0%
$v_0 = -\infty, \theta \in [2, 3)$	-0.092	-0.053	-0.040	-0.025	-0.009	99.8%
$v_0 = -\infty, \theta \in [3, 4]$	-0.107	-0.048	-0.032	-0.024	-0.011	99.5%
$v_0 = -2, \theta \in [0, 1)$	-0.058	-0.033	-0.020	-0.011	-0.006	100.0%
$v_0 = -2, \theta \in [1, 2)$	-0.060	-0.038	-0.024	-0.013	-0.007	100.0%
$v_0 = -2, \theta \in [2, 3)$	-0.054	-0.041	-0.028	-0.015	-0.008	100.0%
$v_0 = -2, \theta \in [3, 4]$	-0.051	-0.036	-0.030	-0.023	-0.011	100.0%
$v_0 = 0, \theta \in [0, 1)$	-0.018	-0.009	-0.005	-0.002	-0.001	100.0%
$v_0 = 0, \theta \in [1, 2)$	-0.027	-0.012	-0.006	-0.003	-0.001	100.0%
$v_0 = 0, \theta \in [2, 3)$	-0.034	-0.016	-0.008	-0.004	-0.002	100.0%
$v_0 = 0, \theta \in [3, 4]$	-0.030	-0.019	-0.011	-0.006	-0.003	100.0%