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Group Coupons: Interpersonal Bundling on the Internet

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Abstract. Sellers sometimes offer goods for sale under both a regular price and a discount for group purchase if the consumer group reaches some minimum size. This selling practice, which we term *interpersonal bundling*, has been popularized on the Internet by companies such as Groupon. We explain why interpersonal bundling is a profitable strategy in the presence of demand uncertainty, and how it may further boost profits by stimulating product information dissemination. Other reasons for its profitability are also discussed. We provide sufficient conditions for interpersonal bundling to dominate separate selling, and identify factors that determine the size of its profit advantage.

Keywords: Interpersonal bundling, group coupon, group discount, demand uncertainty

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1. INTRODUCTION

This paper studies a form of product bundling where a good is offered for sale under both a regular price and a discount for group purchase if the consumer group reaches some minimum size—the bundle size. The defining characteristic of this selling format is that the purchase of the bundle is made by different consumers—and hence we term it *interpersonal bundling*—rather than by a single consumer as under traditional mixed bundling.¹

While interpersonal bundling has long existed (multiple consumers may form a purchase group to qualify for a group discount, as, for example, when buying tickets for a concert or purchasing a tour),² the Internet dramatically reduced the transaction cost for different consumers, even without knowing each other, to form a purchase group in order to qualify for the sale price. In recent years, many Internet sites have emerged that allow sellers to offer interpersonal bundling, where consumers purchasing with group coupons receive substantial discounts when the minimum group size is reached. Launched in November 2008, Groupon was a pioneer in this selling format on the Internet, and in less than four years it has grown into a major public company with a market capitalization of US\$7.72 billion (based on data on May 15, 2012).³ What are the economic forces behind the popularity of this new Internet institution? When will interpersonal bundling be more profitable than separate selling?⁴

¹Mixed bundling refers to offering goods for sale both as a package and as individual components.

²Miller Farms, a local family farm in Colorado, runs the Fall Harvest Festival each year. In 2012, a customer is charged \$15 to participate in the Festival and pick up vegetables to take home. For a group of 10 or more, the price per person is lowered to \$13.

³There are now many other group buying websites offering variants of interpersonal bundling with group coupons, including Livingsocial, where a consumer receives a free deal if she gets three people buy the product. There are many interpersonal bundling sites around the global, including popular cites such as uBuyiBuy, Gaopeng, and Lashou in Asia, MyCityDeal in Europe, Downtown Colombia in South America, and Spreets in Australia.

⁴Here, separate selling means offering a good for sale under a single unit price to all consumers, whereas a pure bundle would consist of multiple units of the same good under a unit price for group purchase. The recent economics literature has investigated product bundling that is different from traditional mixed bundling. See, for example, the study of bundle size pricing by Chu, Leslie, and Sorensen (2011), and of inter-firm bundling by Armstrong (2012).

What factors determine the size of the profit advantage of interpersonal bundling? How does this selling format affect consumers and welfare? We provide some answers to these questions in this study.

The economics literature on product bundling has found that mixed bundling often is more profitable than separate selling through two main mechanisms: segmenting consumer population to facilitate price discrimination and reducing the dispersion of consumer values to extract consumer surplus (e.g., Adams and Yellen 1976; Schmalensee, 1984; Long, 1984; McAfee, McMillan, and Whinston 1989; and Chen and Riordan, 2011).⁵ This paper will suggest two new motives for bundling that have not been explored in existing studies: as a profitable strategy in response to demand uncertainty and as a mechanism to stimulate the dissemination of product information. While these motives can also arise when each bundle is purchased by an individual consumer,⁶ they are especially relevant and important for interpersonal bundling.

We start with a stylized model where a monopolist sells to a consumer population with an uncertain number of low-value consumers and possibly also an uncertain number of high-value consumers.⁷ The seller may optimally pursue either a high-price strategy targeting only the high-value consumers, or a low-price strategy that will also attract low-value consumers. The low price will be profitable only if it results in a sufficiently high sales volume—if the number of low-value consumers is sufficiently high. However, because price is set before the uncertainty is resolved, setting a single price is generally not optimal. By

⁵In a standard model of two goods, some consumers may value one good highly but another very little, while others may value two goods together relatively highly, and values for the bundle may be less dispersed than values for individual goods. By charging the former (who purchase only a single unit) a higher price and the latter a bundle discount, mixed bundling generally leads to higher profit than separate selling.

⁶Under standard mixed bundling with two goods, there can be uncertainties on each individual consumer's valuation for the two goods, and mixed bundling can thus be viewed as a form of option pricing, where a consumer will obtain the bundle discount only if she has sufficiently high demand for both goods.

⁷A seller typically offers interpersonal bundling on the Internet through an intermediary such as Groupon. To focus on the properties of interpersonal bundling as a selling format, we assume that the seller and the intermediary act as a single party to maximize their joint profits. We shall later discuss potential conflicts in the seller-intermediary relationship to further shed light on this Internet institution.

offering the good for sale under interpersonal bundling, the low price will become effective only if it will indeed lead to a sufficiently high increase in sales, while the high price will prevail when the number of low-value consumers turns out to be relatively small. Thus, interpersonal bundling potentially enables the seller to use optimal option pricing under uncertain demand, leading to higher profit than separate selling.

The benefits of interpersonal bundling to the seller can be enhanced if some consumers are initially uninformed about the existence of the seller's product. Then, in order to qualify for the discount available only when the minimum bundle size is reached, informed consumers may take (costly) actions to transmit product information to the uninformed, and the seller will consider this incentive when strategically setting the bundle size. Interpersonal bundling will thus facilitate the dissemination of product information and further increase the seller's profit.

Interpersonal bundling can also be more profitable due to traditional benefits of bundling. In particular, if (some) high-value consumers need to incur (higher) transaction costs to participate in group purchase, interpersonal bundling allows the seller to (partially) segment the consumer population and charge a higher price to high-value consumers who may choose to purchase at the regular price. Furthermore, if some consumers may demand multiple units of the product, with decreasing values for additional units, interpersonal bundling can reduce the dispersion of product values for such consumers, thereby expanding sales to them.

In addition to contributing to the economics of product bundling, this paper is also closely related to the literature on pricing under demand uncertainty. Dana (2001), for example, considers a model in which demand can be either high or low. He finds that a monopolist optimally offers two prices, with only a limited quantity offered under a low price, which is set for the low demand state. A high price then allows the firm to extract additional consumer surplus when demand turns out to be high, in which case the limited quantity available at the low price will sell out so that some high-priced units will be purchased. By contrast, in our model there are both high- and low-value consumers, the uncertainty is over their numbers, and under interpersonal bundling the discounted price is available only

when sufficiently many consumers will purchase at that price. Also related are Gale and Holmes (1992, 1993), who study how a monopolist may use advance purchase discounts to allocate capacity more efficiently in the presence of demand uncertainty. They consider a setting where the monopolist does not know which of two demand periods will be associated with the peak or the off-peak demand. Advance purchase discounts allow the firm to screen consumers so that those with weaker time preferences will purchase earlier and are more likely to consume at the off-peak period (See also Dana, 1998, for a related analysis in competitive markets). Our finding, that interpersonal bundling leads to higher profit than separate selling under demand uncertainty, offers a new insight to this literature.

Interpersonal bundling through group coupons adds to an expanding list of new Internet institutions, including targeted advertising, paid placement, and online auctions. As such, our paper joins many recent contributions to the economics of Internet markets (as, for example, surveyed in Levin, 2012). Our analysis will shed light on the economic forces behind the rapid growth of companies like Groupon by explaining why interpersonal bundling can be more profitable than separate selling.⁸ Our study will further reveal what determine the potential profitability of interpersonal bundling, relative to separate selling, thereby offering insights for sellers on when to adopt this selling format and how to avoid its inappropriate uses.

The rest of the paper is organized as follows. Section 2 contains our core analysis, where we formulate a basic model, characterize optimal interpersonal bundling, and develop a sufficient condition for interpersonal bundling to dominate separate selling. Remarkably, this condition is invariant to the functional forms of the distributions of consumer numbers. We also show how the profit difference between interpersonal bundling and separate selling may vary with parameter values of the market environment, and provide simple conditions for interpersonal bundling to have higher or lower welfare than separate selling. Furthermore, in the simple setting of the basic model, we argue that interpersonal bundling is in fact an optimal selling scheme among all selling mechanisms that may depend on realized aggregate demand. This section ends with the analysis of a variant of the basic model under alter-

⁸Groupon exceeded a billion dollars in revenue in just its third year of operation (Levin, 2012).

native assumptions on the distributions of consumer values. Section 3 considers another variant of the basic model where some consumers are initially uninformed of the product. This allows us to explore the beneficial role of interpersonal bundling in promoting the dissemination of product information. Section 4 discusses two additional variants of the basic model to show that, in the presence of demand uncertainty, interpersonal bundling can also increase the seller's profit through either price discrimination or reducing the dispersion of consumer values, similarly as mixed bundling. Section 5 concludes.

2. DEMAND UNCERTAINTY AND INTERPERSONAL BUNDLING

2.1 Basic Model

A monopolist offers a product for sale. There are two types of consumers, high-value and low-value, whose product valuations are respectively H and L , with $H > L > 0$. A consumer's type is her private information, and each consumer desires to purchase at most one unit. The numbers of low- and high-value consumers are respectively x and y , which are realizations of random variables X and Y that have joint distribution function $G(x, y)$ on support $[a_x, b_x] \times [a_y, b_y]$, where $0 \leq a_x < b_x$ and $0 \leq a_y \leq b_y$. The marginal distribution function of X is $F(x)$, with density function $f(x) > 0$ on $[a_x, b_x]$. Production cost is normalized to zero, and the firm maximizes expected profit.

Let $\bar{x}$ and $\bar{y}$ be the expected number of low- and high-value consumers, respectively. Then

$$\bar{x} = \int_{a_x}^{b_x} x dF(x); \quad \bar{y} = \int_{a_x}^{b_x} \int_{a_y}^{b_y} y dG(x, y). \quad (1)$$

We allow the possibility that $y = \bar{y}$ is a constant, in which case $G(x, y)$ degenerates to $F(x)$.⁹

As a benchmark, consider the case of separate selling where the firm posts a single unit price to all consumers. Then, profit is higher under $p = H$ if $H\bar{y} > L(\bar{x} + \bar{y})$ and it is higher under $p = L$ if $H\bar{y} < L(\bar{x} + \bar{y})$. It follows that the optimal price and the corresponding

⁹The uncertainty in x is essential for our analysis and is maintained throughout the paper.

profit are, respectively:¹⁰

$$p^s = \begin{cases} H & \text{if } \bar{y} \geq \frac{L}{H-L}\bar{x} \\ L & \text{if } \bar{y} < \frac{L}{H-L}\bar{x} \end{cases}, \quad \pi^s = \begin{cases} H\bar{y} & \text{if } \bar{y} \geq \frac{L}{H-L}\bar{x} \\ L(\bar{x} + \bar{y}) & \text{if } \bar{y} < \frac{L}{H-L}\bar{x} \end{cases}. \quad (2)$$

Therefore, the firm will only sell to the high-value consumers at $p^s = H$ if the expected number of low-value consumers is relatively low, and it will sell to all consumers at $p^s = L$ if $\bar{x}$ is high enough.

Under interpersonal bundling, the firm sets a stand-alone unit price p , a discounted unit price under group purchase $q \leq p$, and a minimum group size m for the discounted price to take effect (i.e., for the deal to be on). Each consumer can separately purchase the good at price p , but consumers who use the group coupon can purchase at the discounted price q if and only if $M (\geq m)$ consumers participate in the group purchase. Notice that if $q = p$, then bundling is equivalent to separate selling under price p . In our basic model, we assume that there is no transaction cost for a consumer to use the group coupon, which implies that if $q < p$, all consumers will attempt to purchase at the discounted bundle price. Thus, bundled selling with (p, q, m) is equivalent to separate selling with price q if $m \leq a_x + a_y$, and to separate selling with price p if $m \geq b_x + b_y$.

2.2 Profitability of Interpersonal Bundling

Under bundling, with (p, q, m) , all consumers will purchase at price q if $x + y \geq m$ and $q \leq L$, whereas when $x + y < m$ only high-value consumers will purchase at price p if $L < p \leq H$. The firm's problem is to maximize (expected) profit:

$$\max_{q \leq L < p \leq H, m} \pi(p, q, m) = q \int \int_{x+y \geq m} (x+y) dG(x, y) + p \int \int_{x+y < m} y dG(x, y). \quad (3)$$

Since $\pi(p, q, m)$ weakly increases in p and q for any m , the optimal p and q that maximize $\pi(p, q, m)$ are $p^* = H$ and $q^* = L$. Hence the firm's maximum profit under bundling and the optimal (minimum) bundle size are

$$\pi^* \equiv \max_m \pi(H, L, m); \quad m^* = \arg \max_m \pi(H, L, m). \quad (4)$$

¹⁰For ease of exposition, when profit is the same under $p = H$ and $p = L$, we assume $p^s = H$.

Notice that m can be set low enough so that it is not a constraint, or high enough so that the minimum group size can never be reached. In particular, $\pi(H, L, a_x + a_y) = L(\bar{x} + \bar{y})$ and $\pi(H, L, b_x + b_y) = H\bar{y}$, which implies $\pi^* \geq \pi^s$. Thus, same as mixed bundling, interpersonal bundling will always be at least as profitable as separate selling. We are, however, interested in when interpersonal bundling is more profitable than separate selling, and how large its profit advantage is. Condition (A1) below provides a sufficient condition for $\pi^* > \pi^s$:

$$\left(1 + \frac{a_x}{a_y}\right) < \frac{H}{L} < \left(1 + \frac{b_x}{b_y}\right). \quad (\text{A1})$$

Proposition 1 *Interpersonal bundling is always at least as profitable as separate pricing, and it is more profitable than separate selling if condition A1 holds.*

Proof. We show that under (A1) interpersonal bundling is more profitable than separate selling whether $L(\bar{x} + \bar{y}) \leq H\bar{y}$ or $L(\bar{x} + \bar{y}) > H\bar{y}$.

(i) If $L(\bar{x} + \bar{y}) \leq H\bar{y}$, $\pi^s = H\bar{y}$ under separate selling; and, if in addition $Hb_y < L(b_x + b_y)$, then for $\varepsilon \equiv \frac{1}{2}(b_x + b_y - \frac{H}{L}b_y) > 0$,

$$\begin{aligned} \pi^* &\geq \pi(H, L, b_x + b_y - \varepsilon) = \int \int_{x+y \geq b_x + b_y - \varepsilon} [L(x+y) - Hy] dG(x, y) + H\bar{y} \\ &\geq \int \int_{x+y \geq b_x + b_y - \varepsilon} [L(b_x + b_y - \varepsilon) - Hy] dG(x, y) + H\bar{y} > H\bar{y} = \pi^s. \end{aligned}$$

(The first inequality above is due to revealed preference, the second to $x+y \geq b_x + b_y - \varepsilon$, and the last to $L(b_x + b_y - \varepsilon) - Hy = \frac{1}{2}(L(b_x + b_y + Hb_y)) - Hy > \frac{1}{2}L(b_x + b_y - Hb_y) > 0$.)

(ii) If $L(\bar{x} + \bar{y}) > H\bar{y}$, $\pi^s = L(\bar{x} + \bar{y})$ under separate selling; and, if in addition $Ha_y > L(a_x + a_y)$, then for $\varepsilon \equiv \frac{1}{2}[\frac{H}{L}a_y - (a_x + a_y)] > 0$,

$$\begin{aligned} \pi^* &\geq \pi(H, L, a_x + a_y + \varepsilon) = L(\bar{x} + \bar{y}) + \int \int_{x+y < a_x + a_y + \varepsilon} [Hy - L(x+y)] dG(x, y) \\ &> L(\bar{x} + \bar{y}) + \int \int_{x+y < a_x + a_y + \varepsilon} [Hy - L(a_x + a_y + \varepsilon)] dG(x, y) > L(\bar{x} + \bar{y}) = \pi^s. \end{aligned}$$

■

The proof of Proposition 1 uses simple arguments that start from the optimal prices under separate selling: if $p^s = H$, profit can be increased by keeping the regular price but adding

a group bundle with unit price L and a minimum size that is slightly lower than $b_x + b_y$ (the maximum possible total number of consumers); if $p^s = L$, profit can be increased by raising the regular price to H and adding a bundle with unit price L and a minimum size that is slightly higher than $a_x + a_y$ (the minimum possible total number of consumers). Condition (A1), which requires $b_y H < (b_x + b_y) L$ and $(a_x + a_y) L < a_y H$, ensures that these changes starting from separate selling will indeed strictly increase profit. This condition is thus invariant to the functional form of the joint distribution of X and Y , depending only on the upper and lower limits of the support for the distribution. It holds if the H/L ratio is relatively large compared to a_x/a_y but small compared to b_x/b_y . Intuitively, when (A1) holds, profit can be higher either with high price (H) or with low price (L), depending on the demand realization. Interpersonal bundling allows the firm to sell at the low price only if profit is higher under the low price—otherwise the high price will prevail, thereby assuring a higher profit than separate selling.¹¹

In many situations where group coupons are issued by sellers such as restaurants and hair salons, H could be considered as the regular price at which the seller has less uncertainty about the number of consumers. Thus the difference between a_y and b_y tends to be relatively small. On the other hand, there might be more uncertainty about the number of consumers who will purchase at the sale price L , so the difference between a_x and b_x tends to be relatively large. In such situations, condition (A1) is likely satisfied.¹²

To illustrate our result and to make explicit profit comparisons, consider the example below:

Example 1 *Suppose that X and Y are independently and uniformly distributed on $[0, 3]$ and $[1, 2]$, respectively. Then, $\bar{x} = \bar{y} = \frac{3}{2}$, $p^s = H$ if $H \geq 2L$, $p^s = L$ if $H < 2L$, and (A1)*

¹¹If H/L is too small, it may be optimal always to sell at $p^s = L$, so the option to sell at alternative prices under interpersonal bundling has no value. Likewise, if H/L is too large, it could be optimal always to sell at $p^s = H$, which would then achieve the same profit as interpersonal bundling. Notice that if $a_x = 0$, then (A1) becomes $H < L(1 + b_x/b_y)$ and bundling is always more profitable than charging $p^s = L$.

¹²We may view interpersonal bundling as allowing the seller to experiment with a lower price that will prevail only when the number of purchasing consumers reaches a minimum size, or only when it is more profitable than the regular price.

holds if $H < \frac{5}{2}L$. Under interpersonal bundling,

$$\pi(H, L, m) \equiv L \int_{\max\{1, m-3\}}^2 \int_{\max\{m-y, 0\}}^3 (x+y) \frac{1}{3} dx dy + H \int_1^{\min\{m, 2\}} \int_0^{\min\{m-y, 3\}} y \frac{1}{3} dx dy.$$

Setting $\partial\pi(H, L, m)/\partial m = 0$, we find the optimal (minimum) bundle size as

$$m^* = \begin{cases} \frac{H}{2L-H}, & \text{with } \pi^* > \pi^s \quad \text{if} \quad H \leq \frac{4}{3}L \\ \frac{3}{2} \frac{H}{L}, & \text{with } \pi^* > \pi^s \quad \text{if} \quad \frac{4}{3}L < H < 2.6L \\ \geq 5, & \text{with } \pi^* = \pi^s \quad \text{if} \quad 2.6L \leq H \end{cases}.$$

For instance, if $L = 1$ and $H = 2$, then $m^* = 3$ and $\pi^* = 3.3333 > \pi^s = 3$, so interpersonal bundling increases (expected) profit by about 11%.

Several observations can be made in Example 1. First, condition (A1) is sufficient, but not necessary, for the profitability of interpersonal bundling. In Example 1, while (A1) holds for $H < 2.5L$, bundling is also profitable when $H \in [2.5L, 2.6L)$.

Second, (A1) is fairly tight as a sufficient condition. When $H \geq 2.6L$, interpersonal bundling is no longer profitable. In this case, $\frac{3}{2} \frac{H}{L} \geq \frac{3}{2} (2.6) = 3.9$. However, for any $m \in [3.9, 5)$, the expected profit under $x+y \geq m$, in which case all sales will occur at the discounted price L , is lower than the expected profit under separate selling. Therefore, it is optimal for the seller not to offer the bundle, which is equivalent to setting a sufficiently large bundle size ($m^* \geq 5$).

Third, when interpersonal bundling is profitable, m^* increases in H but decreases in L . A marginal increase in m reduces the probability that the sale will occur at the low price (with a large volume) and raises the probability that the sale will occur at the high price. Thus, m^* , which balances these two effects, increases with the high price and decreases with the low price. Put differently, a higher H (or a lower L) makes sales under the stand-alone price H relatively more profitable, reducing the benefit of selling at the bundle discount. Consequently the optimal (minimum) bundle size with which the discount price will become effective is larger.

We now turn to the question of how the advantage of interpersonal bundling, relative to separate selling, may vary with the market environment. We first consider how the ratio

H/L , or the difference between the reservation prices of the high- and low-value consumers, affects the relative profitability of bundling.

Corollary 1 *Suppose that (A1) holds and L is fixed. Then, $\pi^* - \pi^s$ exhibits an inverted-U shape with respect to changes in H , first increasing and then decreasing, reaching maximum at $H = \left(1 + \frac{\bar{x}}{\bar{y}}\right) L$.*

Proof. When $H < \left(1 + \frac{\bar{x}}{\bar{y}}\right) L$, $\pi^* - \pi^s = \max_m \pi(H, L, m) - L(\bar{x} + \bar{y})$. From (3), $\pi(H, L, m)$ increases in H for all interior m . Thus, if (A1) holds so that $\pi^* > \pi^s$, $\max_m \pi(H, L, m)$ is also increasing in H , and so is $\pi^* - \pi^s$. Similarly, when $H \geq \left(1 + \frac{\bar{x}}{\bar{y}}\right) L$, $\pi^* - \pi^s = \max_m \int_{x+y \geq m} [L(x+y) - Hy] dG(x, y)$, which decreases in H . ■

When H/L is low (or high), the profit advantage of bundling is low relative to separate selling, because selling at price L (or H) is often more profitable than at price H (or L), which implies that the option to sell at one of the two prices contingent on the realizations of $X + Y$ under bundling has very limited value. This option becomes more valuable when H/L is at some intermediate level, implying more profound profit advantage of bundling.

We next consider how the dispersion of X affects the profits under interpersonal bundling. Intuitively, when X is more dispersed, demand is more uncertain and the advantage of interpersonal bundling is larger. The result below shows that this is indeed the case under some conditions, assuming that X and Y are independent with the (marginal) distribution of Y being $J(y)$, and comparing profits under two different distributions of X .

Following Johnson and Myatt (2006), we say that distribution $\hat{F}(x)$ is more dispersed than $F(x)$ if $\hat{F}(x)$ is a rotation of $F(x)$ such that $x \geq \hat{x} \iff \hat{F}(x) \leq F(x)$ for some rotation point $\hat{x}$. Under $\hat{F}(x)$ and $F(x)$, respectively, let $\bar{x}_{\hat{F}}$ and $\bar{x}_F$ be the expected values of X , $\hat{b}_x$ and b_x the upper limits of $\hat{F}$ and F , and $\hat{m}^*$ and m^* the optimal bundle sizes, where $\hat{b}_x \geq b_x$ and $\bar{x}_{\hat{F}} \geq \bar{x}_F$. Let the corresponding profits be $\hat{\pi}^*$ and π^* under bundling, and $\hat{\pi}^s$ and π^s under separate selling.

Corollary 2 *Suppose (A1) holds and $\hat{F}$ is a rotation of F such that: (i) $H\bar{y} \geq L(\bar{y} + \bar{x}_{\hat{F}})$, (ii) $\hat{x} \leq m^* - b_y$, and (iii) $\int_{a_y}^{b_y} [Lm^* - Hy] [\hat{F}(m^* - y) - F(m^* - y)] dJ(y) \leq 0$. Then,*

$\hat{\pi}^* - \hat{\pi}^s > \pi^* - \pi^s$; that is, the profit advantage of bundling relative to separate selling is larger if X is more dispersed.

Proof. See the appendix. ■

Although the result seems intuitive, the comparison of profits under $\hat{F}(x)$ and $F(x)$ turns out to be subtle. Condition (i) ensures that $p^s = H$ under separate selling for both $\hat{F}(x)$ and $F(x)$. Under condition (ii), $\hat{F}(x) < F(x)$ for $x \geq m^* - y$, so that more dispersion under $\hat{F}$ leads to higher probabilities for higher realizations of x ; and under (iii) this similarly holds on average weighted by the density of Y . Together, conditions (i) and (iii) ensure that unambiguous comparisons can be made. All three conditions can be easy to verify. For instance, in Example 1, where $F(x) = \frac{x}{3}$, these conditions are satisfied for any rotation $\hat{F}(x) = \frac{x}{\alpha}$ with $\alpha > 3$ and $H/L \in [(3 + \alpha)/3, 2.6)$, where $m^* = \frac{3}{2} \frac{H}{L} > 3 > b_y = 2$, and $\hat{x} = 0$.

Finally, comparing consumer and social welfare under interpersonal bundling and separate selling are straightforward in our simple setting. When $p^s = L$, interpersonal bundling raises expected price and lowers expected output, whereas the opposite is true when $p^s = H$. From Proposition 1, we can therefore state the following sufficient conditions for the welfare effects of interpersonal bundling:

Corollary 3 *Suppose that (A1) holds. Interpersonal bundling increases consumer and social welfare if $H \geq L \left(1 + \frac{\bar{x}}{y}\right)$, but it reduces consumer and social welfare if $H < L \left(1 + \frac{\bar{x}}{y}\right)$.*

Hence, interpersonal bundling generally has ambiguous effects on consumer and social welfare. It increases consumer and social welfare when H is relatively high so that under separate selling only the high value consumers will be served, whereas the opposite is true when H and L are relatively close.

2.3 Interpersonal Bundling as an Optimal Selling Scheme

We now further argue that, in our simple setting, interpersonal bundling is an optimal selling scheme. Since all consumers are *ex ante* the same, we can consider mechanisms for

a representative consumer. From the revelation principle, we can limit our search for an optimal selling scheme to direct mechanisms where the consumer is asked to report her type $\theta \in \{H, L\}$, who will receive a unit of the good with probability $\lambda(\cdot)$ by paying $p(\cdot)$,¹³ and truth reporting is optimal for the consumer. Given that there is a continuum of consumers, $\lambda(\cdot)$ and $p(\cdot)$ will depend on θ and on some aggregate measure(s) of consumers. We assume that a mechanism may depend on the realized aggregate demand, $x + y$, but not on individual values of x and y . One possible motivation for this assumption is that x and y are not separately verifiable while $x + y$ potentially is. Under this assumption, which we shall call the verifiability restriction, a mechanism specifies $\{\lambda(\theta, x + y), p(\theta, x + y)\}$.

The seller chooses $\{\lambda(\theta, x + y), p(\theta, x + y)\}$ to maximize

$$\pi = \int \int [xp(L, x + y)\lambda(L, x + y) + yp(H, x + y)\lambda(H, x + y)] dG(x, y), \quad (5)$$

subject to individual rationality constraints

$$(L - p(L, x + y))\lambda(L, x + y) \geq 0, \quad (6)$$

$$(H - p(H, x + y))\lambda(H, x + y) \geq 0; \quad (7)$$

and incentive compatibility constraints

$$(L - p(L, x + y))\lambda(L, x + y) \geq (L - p(H, x + y))\lambda(H, x + y), \quad (8)$$

$$(H - p(H, x + y))\lambda(H, x + y) \geq (H - p(L, x + y))\lambda(L, x + y). \quad (9)$$

From standard arguments, $p(L, x + y) = L$ so that the low-value type receives no information rents, and (8) holds with $p(H, x + y) \geq L$. From (9), which holds in equality at the optimum, and with $p(L, x + y) = L$, we have

$$p(H, x + y)\lambda(H, x + y) = H\lambda(H, x + y) - (H - L)\lambda(L, x + y). \quad (10)$$

Thus (7) and (10) are the two remaining constraints. Substituting (10) into (5), with $p(L, x + y) = L$, we obtain

$$\pi = \int \int \{[xL - y(H - L)]\lambda(L, x + y) + yH\lambda(H, x + y)\} dG(x, y),$$

¹³We can also allow a transfer payment when the consumer does not receive the good, but it would be optimal for the seller to set this payment to zero.

which increases in $\lambda(H, x + y)$. Since constraint (7) is not less likely satisfied with an increase in λ , it follows that $\lambda(H, x + y) = 1$ at the optimum. Then, subject to (7), the seller chooses $\lambda(L, x + y)$ to maximize

$$\pi = \int \int \{(x + y) L \lambda(L, x + y) + y H [1 - \lambda(L, x + y)]\} dG(x, y).$$

Hence, the optimal solution must involve a cut-off value for $x + y$, m^* , such that $\lambda(L, x + y) = 1$ when $x + y \geq m^*$ and $\lambda(L, x + y) = 0$ when $x + y < m^*$,¹⁴ where

$$m^* = \arg \max_m \int \int_{x+y \geq m} L(x + y) dG(x, y) + \int \int_{x+y < m} H y dG(x, y),$$

with $p(H, x + y) = L$ if $\lambda(L, x + y) = 1$ and $p(H, x + y) = H$ if $\lambda(L, x + y) = 0$. But this is exactly optimal interpersonal bundling under (4). We have thus shown:

Proposition 2 *Interpersonal bundling is an optimal selling scheme among all mechanisms satisfying the verifiability restriction.*

Note that if Y is a constant and takes the value y , then $m^* = \frac{H}{L}y$ and interpersonal bundling is an optimal selling scheme among all selling mechanisms, with no need for the verifiability restriction.

2.4 Continuous Distributions of Consumer Values

Our basic model assumes that the high- and low-value consumers have constant reservation prices H and L , respectively. This allows us to illustrate our ideas in a most transparent setting. Our analysis can be extended to situations where the product values of these two types of consumers are v_H and v_L , which are realizations of continuous random variables. To illustrate this, we assume that the number of low-value consumers, x , again follows distribution $F(x)$, and the number of high-value consumers is a given parameter $a > 1$.

¹⁴If mechanisms could depend on the realizations of x and y separately, then the optimal mechanism would set $\lambda(L, x + y) = 1$ when $(x + y)L > yH$ and $\lambda(L, x + y) = 0$ when $(x + y)L \leq yH$. Notice that with a continuum of consumers, a single consumer cannot change the realizations of x or y by reporting or not reporting her type.

Furthermore, v_L , v_H , and x are independently and uniformly distributed on $[0, 1]$, $[0, a]$, and $[0, 2]$, respectively.¹⁵ Thus a higher a indicates a higher product valuation or higher demand from the high-value consumers.

First, under separate selling, the firm's expected profit is:

$$\pi(p) = \begin{cases} p(a-p) & \text{if } 1 < p \leq a \\ \int_0^2 p(x(1-p) + a-p) \frac{1}{2} dx & \text{if } 0 \leq p \leq 1 \end{cases},$$

which is maximized if either $p = \frac{a}{2}$ or $p = \frac{a+1}{4}$. Since $\pi(\frac{a}{2}) - \pi(\frac{a+1}{4}) \geq 0$ if $a \geq \sqrt{2} + 1$, we have $p^s = \frac{a}{2}$ if $a > \sqrt{2} + 1$ and $p^s = \frac{a+1}{4}$ if $1 < a \leq \sqrt{2} + 1$. It follows that

$$\pi^s = \begin{cases} \frac{a^2}{4} & \text{if } a > \sqrt{2} + 1 \\ \frac{(a+1)^2}{8} & \text{if } 1 < a \leq \sqrt{2} + 1 \end{cases}.$$

Next, under interpersonal bundling, given (p, q, m) with $q < p$ and $m \geq 0$, consumers whose value is at least q will purchase at price q if $x(1-q) + a - q \geq m$, or

$$x \geq \frac{m - a + q}{1 - q}. \quad (11)$$

If inequality (11) does not hold, then no group purchase will occur and consumers can only purchase at the “regular” price p . Thus, the seller chooses (p, q, m) to maximize

$$\pi(p, q, m) = q \int_{\min\{\frac{m-a+q}{1-q}, 2\}}^2 \frac{x(1-q) + a - q}{2} dx + p \int_0^{\min\{\frac{m-a+q}{1-q}, 2\}} \frac{\max\{x(1-p), 0\} + a - p}{2} dx.$$

We can now establish:

Proposition 3 *For the variant of the basic model with continuous distributions of v_H and v_L , interpersonal bundling dominates separate selling (i.e., $\pi^* > \pi^s$) if and only if $1 < a < \sqrt{3} + 1$.*

¹⁵Equivalently, we can relax the unit demand assumption and allow each of these two types of consumers to have a downward-sloping demand curve. In particular, each low-value consumer has demand $q_L = 1 - p$, and each high-value consumer has demand $q_H = a - p$. Note that the number of each type of consumers is a continuum.

The proof of Proposition 3, formally presented in the appendix, starts with two observations linking profits under interpersonal bundling and under separate selling: (1) $\pi(p, p^s, 0) = \pi^s$ for $p > p^s$ and (2) $\pi(p^s, q, m) = \pi^s$ for $q < 1$ and $m = a + 2 - 3q$, where, under interpersonal bundling, all consumers purchase with bundle discount in case (1) and no consumer qualifies for the bundle discount in case (2). Differentiating $\pi(\cdot, \cdot, m)$ with respect to m , we can then show that, if $1 < a < \sqrt{3} + 1$, π is increasing in m at $m = 0$ in case (1) and decreasing in m at $m = a + 2 - 3q$ in case (2), so that interpersonal bundling achieves higher profit than separate selling. Furthermore, $\pi(p, q, m)$ is concave in m when $a > \sqrt{2} + 1$, which, together with π increasing at $m = a + 2 - 3q$ if $a \geq \sqrt{3} + 1$, leads to the conclusion that $\pi^* = \pi^s$ if $a \geq \sqrt{3} + 1$.

Table 1 below contains some comparisons for three values of a :

Table 1

	p^*	q^*	m^*	π^*	π^s	$\frac{\pi^* - \pi^s}{\pi^s}$
$a = 2$	1	0.728 71	1.372 3	1.130 9	1.125	0.5%
$a = 2.4$	1.2	0.778 30	1.850 2	1.480 5	1.44	2.8%
$a = 2.7$	1.35	0.788 60	2.311 1	1.823	1.822 5	0.03%

Apparently, in this simple variant of the basic model, interpersonal bundling achieves higher profit than separate selling if the value of a is in an intermediate range, and the profit advantage, $\pi^* - \pi^s$, exhibits an inverted-U shape with respect to changes in a . These findings are analogous to those in Proposition 1 and Corollary 1 of the basic model.

3. DISSEMINATION OF PRODUCT INFORMATION

The existence of a seller's product may be known to some consumers but unknown to others. In order to achieve the group size to qualify for the low (bundle) price, an informed potential buyer may have the incentive to transmit the information about the sale to other consumers. Interpersonal bundling can thus stimulate the dissemination of product

information, thereby expanding market demand.¹⁶

To formalize this idea in a simple setting, we consider a variant of the basic model by assuming that the number of high-value consumers is initially a given number $n \geq 1$, and each of them ($i = 1, \dots, n$) can make an effort in order to inform a set of $k > 0$ high-value consumers who are initially unaware of the seller's product and prices.¹⁷ Define set $N \equiv \{i : i = 1, \dots, n\}$. Each $i \in N$ succeeds in transmitting the information to the k uninformed consumers with probability β_i at a personal cost $C(\beta_i)$, where $C'(\cdot) > 0$ with $C'(0) \rightarrow 0$, $C''(\cdot) \geq 0$, and the k uninformed consumers become informed if at least one $i \in N$ succeeds. Thus, the number of high-value consumers is potentially

$$y = \begin{cases} n + k & \text{with probability } 1 - \prod_{i=1}^n (1 - \beta_i) \\ n & \text{with probability } \prod_{i=1}^n (1 - \beta_i) \end{cases}.$$

Other aspects of the model are the same as the basic model in Section 2. In particular, all low-value consumers are informed about the seller's product and price(s), and their number, x , is the realization of random variable X that has distribution $F(x)$. The reservation prices of the high- and low-value consumers are again H and L , respectively. Under separate selling, informed consumers have no incentive to incur the cost to transmit product information to uninformed consumers. Hence $p^s = L$ and $\pi^s = L(n + \bar{x})$ if $L(n + \bar{x}) > Hn$, whereas $p^s = H$ and $\pi^s = Hn$ if $L(n + \bar{x}) \leq Hn$.

Under interpersonal bundling, the seller first posts (p, q, m) , after which all $i \in N$ simultaneously choose β_i . Both x and y are then realized, and possible purchases are made. For convenience, we again treat m as a continuous number, and without loss of generality, we

¹⁶As a form of advertising, interpersonal bundling through group coupons can inform consumers about the product, and may also serve as a promotional device that encourages consumers to try the product and become repeat customers. We do not model these traditional roles of advertising under interpersonal bundling, although they can also be important. Some new group buying sites such as Google Offers do not impose a bundle size restriction on the deal being offered, which appears to act more as a form of advertising/promotion.

¹⁷Unlike in Section 2, the number of initial high-value consumers is now an integer. This avoids the problem that no consumer is willing to incur the information transmission cost when the number is a continuum. For convenience, we assume that the initially uninformed consumers also are all of the high-value type.

can confine our search for the optimal (p, q, m) to $q \leq L < p \leq H$.

We consider a symmetric equilibrium where each $i \in N$ chooses the same β . Given (p, q, m) , and all other high-value consumers' choice $\tilde{\beta}$, consumer i maximizes her expected surplus:

$$U(\beta | m, \tilde{\beta}) = (H - q) \Pr(X + Y \geq m) + (H - p) \Pr(X + Y < m) - C(\beta),$$

where $\Pr(X + Y \geq m) =$

$$[1 - F(m - n - k)] \left[1 - (1 - \tilde{\beta})^{n-1} (1 - \beta) \right] + (1 - F(m - n)) (1 - \tilde{\beta})^{n-1} (1 - \beta).$$

Denote the equilibrium bundle by (p^*, q^*, m^*) .

Notice that interpersonal bundling now can increase profit for two distinct reasons. First, as a profitable pricing strategy under uncertainty, it increases profit even if $\beta = 0$ (in which case uninformed consumers do not learn about the product information). From Proposition 1 and (A1), this is ensured if

$$\left(1 + \frac{a_x}{n}\right) < \frac{H}{L} < \left(1 + \frac{b_x}{n}\right). \quad (\text{A1}')$$

Second, interpersonal bundling can motivate consumers to transmit product information to the uninformed, or to choose $\beta > 0$ at a personal cost, in hope of reaching the minimum bundle size so that the discount will be effective. Our next result, which provides a sufficient condition for higher profits under interpersonal bundling with the additional channel of encouraging information transmission to expand demand (i.e., in equilibrium $\beta = \beta^* > 0$), refers to the following condition

$$\left(1 + \frac{a_x}{n}\right) < \frac{H}{L} \leq \left(1 + \frac{b_x}{n + k}\right). \quad (\text{A2})$$

Note that (A2), which implies the weaker condition (A1'), similarly holds if H/L is in an intermediate range.

Proposition 4 *Suppose that (A2) holds. Then, interpersonal bundling has higher profit than separate selling with $\beta^* > 0$, $p^* = H$, and $m^* \in (n + a_x, n + k + b_x)$.*

Proof. See the appendix. ■

Since the discount price can be valid only if the minimum bundle size is purchased, the informed consumers have the incentive to transmit costly product information to the uninformed, hoping that more consumers will join the group purchase. The optimal bundle size, m^* , is now chosen also to provide this incentive, in addition to responding optimally to demand uncertainty. Therefore, interpersonal bundling also provides a mechanism to expand market demand.

To illustrate, consider the next example:

Example 2 Suppose that $n = 2$, $k = 1$, $C(\beta) = \frac{1}{2}\beta^2$, $F(x) = \frac{x}{3}$ for $x \in [0, 3]$, and $L < H < \frac{5}{2}L$. Then, condition (A1') is satisfied, which is sufficient for interpersonal bundling to increase profit. With $L = 1$, Table 2 below lists the equilibrium interpersonal bundle and the profit comparisons with separate selling.

Table 2

	p^*	q^*	m^*	β^*	π^*	π^s	$\frac{\pi^* - \pi^s}{\pi^s}$
$H = 2$	2	1	4.875	0.25	4.805	4	10%
$H = 1.8$	1.8	1	4.278	0.21	4.377	3.6	22%
$H = 1.5$	1.5	1	3.398	0.143	3.914	3.5	12%

As in Example 1, given L , m^* is higher for higher H . Furthermore, β^* is also higher for higher H , directly because of the larger bundle discount ($H - L$), and indirectly because of the higher bundle size (m^*).

4. PRICE DISCRIMINATION AND DISPERSION OF VALUES

As is well known in the existing literature, monopoly product bundling can achieve higher profit than separate selling both through segmenting the consumer population to facilitate price discrimination and by reducing the dispersion of values for the product offering. In this section, we show that interpersonal bundling may also serve these two purposes in the presence of demand uncertainty.

4.1 Price Discrimination

Couponing is a textbook example of price discrimination: if high-value consumers have higher time costs, they are less likely to use coupons. Interpersonal bundling can thus also be a device for price discrimination. To obtain the bundle discount, a consumer needs to sign up for the group coupon, which may involve additional time costs. With interpersonal bundling there is an additional instrument to screen the buyers: Through the choice of the (minimum) bundle size that may not be reached due to uncertainty, the seller can further discourage high-value consumers from attempting to receive the bundle discount.

To illustrate this, consider another variant of the basic model, where the low-value consumers have no cost to participate in group purchase, but the high-value consumers incur a transaction cost t to do so. Assume that t is distributed on $[\underline{t}, \bar{t}]$ with p.d.f. $\phi(t) > 0$, c.d.f. $\Phi(t)$, and $0 \leq \underline{t} < \bar{t}$. The number of low-value consumers is again x with cumulative distribution function $F(x)$, while the mass of high-value consumers is normalized to 1. As in the basic model, these two types of consumers value the product respectively at L and H . Thus, under separate selling, $p^s = H = \pi^s$ if $H \geq L(\bar{x} + 1)$, whereas $p^s = L$ and $\pi^s = L(\bar{x} + 1)$ if $H < L(\bar{x} + 1)$.

Under bundling, the seller again offers (p, q, m) . Assume that t needs to be incurred by a high-value consumer before it becomes known whether the bundle discount is available (whether m is reached), if she wishes to participate in the group purchase. If there is some $t^* \in [0, \bar{t}]$ that solves

$$H - p = \int_{x+\Phi(t^*) \geq m} (H - q) f(x) dx + \int_{x+\Phi(t^*) < m} (H - p) f(x) dx - t^*, \quad (12)$$

then there is an equilibrium where a high-value consumer will sign up for group purchase if and only if $t \leq t^*$, and we focus on this equilibrium.¹⁸ Rearranging (12), we obtain

$$t^* = (p - q) [1 - F(m - \Phi(t^*))]. \quad (13)$$

¹⁸Potentially, there could also be an equilibrium where no high-value consumers sign up for the group coupon, due to there being a continuum of them.

The seller's problem is, with $t^* = t^*(p, q, m)$:

$$\max \pi(p, q, m) = \int_{m-\Phi(t^*)}^{b_x} [q(x + \Phi(t^*)) + p(1 - \Phi(t^*))] f(x) dx + p \int_{a_x}^{m-\Phi(t^*)} f(x) dx \quad (14)$$

subject to $q \leq L$, $L \leq p \leq H$, $a_x \leq m - \Phi(t^*) \leq b_x$.

The seller can increase its profit by charging a lower price to the low-value consumers (a price no higher than L) and a higher price to the high-value consumers (as high as H). With regular price p and discounted bundle price q , a high-value consumer may nevertheless prefer to purchase at p , because she incurs sign-up cost t for the group coupon and she may lose t without receiving the bundle discount if the minimum bundle size is not reached. Hence, a higher m will reduce the incentive of a high-value consumer to engage in group purchase. Interpersonal bundling may thus price discriminate more effectively both than traditional coupons and than usual mixed bundling.

A higher m , however, may also cost the seller if the sales to the low-value consumers are not realized, which should also be taken into account when the seller chooses its optimal m . Notice that any q below L will lower the seller's profit when the good is sold at a discount and will also make participating in group purchase more attractive to the high-value consumers. Thus it is optimal for the seller to set $q^* = L$. On the other hand, a higher p may have the opposing effects of increasing the profit from the high-value consumers purchasing at the regular price but also making purchasing at the bundle discount more attractive. Consequently, the optimal value of p is determined jointly with m . Because $\pi(H, L, a_x) \geq \pi(L, L, m)$ for any m , to search for the optimal (p, m) we can limit our attention to situations where $p > L$.

Again denote the seller's equilibrium profit under interpersonal bundling by π^* . To derive a sufficient condition under which $\pi^* > \pi^s$, we utilize the condition below

$$(i) \bar{t} > H - L; \quad (ii) L\bar{x} > (H - L)\Phi(H - L). \quad (A3)$$

Since $p^* \leq H$, part (i) in (A3) ensures that some high-value consumers will not incur t for the bundle discount, and, from (14),

$$\pi(H, L, a_x) = L[\bar{x} + \Phi(H - L)] + H[1 - \Phi(H - L)] > L(\bar{x} + 1) = \pi^s|_{p^s=L},$$

so that bundling with $(p, q, m) = (H, L, a_x)$ is always more profitable than separate selling with $p^s = L$. Moreover, condition (ii) in (A3) ensures that

$$\pi(H, L, a_x) = L[\bar{x} + \Phi(H - L)] - H\Phi(H - L) + H > H = \pi^s|_{p^s=H},$$

so that bundling with $(p, q, m) = (H, L, a_x)$ is also always more profitable than separate selling with $p^s = H$. Therefore, since $p^s = L$ or H , under condition (A3) it must be true that $\pi^* \geq \pi(H, L, a_x) > \pi^s$ and $p^* > L = q^*$. We have therefore established:

Proposition 5 *Suppose that condition (A3) is satisfied. Then, the seller's profit is higher under interpersonal bundling than under separate selling with $p^* > L = q^*$.*

We illustrate the result with the following example:

Example 3 *Assume that $\phi(t) = 1$ on $[0, 1]$, $f(x) = \frac{1}{2}$ on $[0, 2]$, and $L = 1$. For different values of H , Table 3 below lists the equilibrium interpersonal bundle and the profit comparisons with separate selling.*

Table 3

	p^*	q^*	m^*	t^*	π^*	π^s	$\pi^* - \pi^s$	$\frac{\pi^* - \pi^s}{\pi^s}$
$H = 2.5$	2.5	1	1.846	0.462	2.808	2.5	0.308	12%
$H = 2$	2	1	1.5	0.5	2.5	2	0.5	25%
$H = 1.5$	1.5	1	0.8	0.167	2.263	2	0.263	13%

Several observations can be made from Table 3. First, $\pi^* > \pi^s$ for all $H \in \{1.5, 2, 2.5\}$, and for profitable interpersonal bundling, (A3) is sufficient but not necessary. For instance, when $H = 2.5$, (A3) is not satisfied but $m^* > a_x$, $t^* < \bar{t}$, and $\pi^* > \pi^s$. Second, m^* increases in p^* , so as to discourage the high-value consumers from using the group coupon. Third, similarly as in the basic model, $\pi^* - \pi^s$ varies non-monotonically as H changes, reaching maximum when H is some intermediate value.

4.2 Dispersion of Values

A standard insight for why product bundling can increase profit is that it reduces the dispersion of consumer values to the seller—or it pools the incentives of a buyer to purchase different products through the bundle. Interpersonal bundling can increase profit also through this mechanism. To illustrate this, suppose that a subset of high-value consumers, $N_1 \subset N$, also have demand for a second unit of the product. The number of consumers in N_1 is n_1 . Each $i \in N_1$ values the first unit at the amount of H but a second unit at the amount of $L - \delta$ with $\delta > 0$, whereas the rest of the high-value consumers, whose number is $n_2 \equiv n - n_1$, still have unit demand with valuation H . The low-value consumers, whose number x still follows distribution $F(x)$, again have unit demand with valuation L .

One setting in which the possibility of multi-units demand may arise is that a high-value consumer in N_1 has a family member or friend who has a lower value for the product, and utilities are (partially) transferable within the family or among the friends (possibly through repeated interactions) so that we can consider a consumer in N_1 as willing to purchase up to two units of the product, with values H and $L - \delta$ for the first and the second unit, respectively.

For this analysis, we will refer to the following condition:

$$\frac{n_1}{(n + n_1 + \bar{x})} < \delta < (H - L) < \frac{b_x + n_1}{n} L. \quad (\text{A4})$$

The first inequality in (A4) is from $L(n + \bar{x}) > (L - \delta)(n + n_1 + \bar{x})$, which ensures that $p^s \neq L - \delta$ under separate selling. Then, $p^s = L$ and $\pi^s = L(n + \bar{x})$ if $L(n + \bar{x}) > Hn$, whereas $p^s = H$ and $\pi^s = Hn$ if $L(n + \bar{x}) \leq Hn$. The second inequality in (A4) ensures that any consumer in N_1 has positive surplus by purchasing two units at price L . The third inequality ensures that $Hn < L(b_x + n + n_1)$, so that selling at price L can be profitable.

Under interpersonal bundling with (p, q, m) , provided that $q > L - \delta$ and $2H - \delta \geq H - p$, in equilibrium all n high-value consumers will purchase only one unit if $n + x \geq m$, and all $i \in N_1$ will purchase two units if $m = x + n + n_1$. When $x + n < m \leq x + 2n_1 + n_2$, in equilibrium a subset of $\tilde{N}_1 \subset N_1$, with the number of consumer in $\tilde{N}_1$ being $\tilde{n}_1 \leq n_1$ so that

$m = x + n + \tilde{n}_1$, will purchase two units.¹⁹ The seller's problem is then

$$\max_{q \leq L < p \leq H, m} \pi(p, q, m) = q \int_{x+n+n_1 \geq m} \max\{x+n, m\} dF(x) + p \int_{x+n+n_1 < m} ndF(x). \quad (15)$$

We can rewrite $\pi(p, q, m)$ as

$$qm[F(m-n) - F(m-n-n_1)] + q \int_{m-n}^{b_x} (x+n) f(x) dx + pnF(m-n-n_1). \quad (16)$$

For any m , $\pi(p, q, m)$ increases in p and q . We thus have $p^* = H$ and $q^* = L$. Furthermore, m^* satisfies $\frac{\partial \pi(H, L, m)}{\partial m} = 0$, or

$$L[F(m-n) - F(m-n-n_1)] + (Hn - Lm)f(m-n-n_1) = 0. \quad (17)$$

Notice that $\pi(H, L, a_x + n) = \pi^s|_{p^s=L}$ and $\pi(H, L, b_x + n + n_1) = \pi^s|_{p^s=H}$. Thus interpersonal bundling is always weakly profitable. From (17), $m^* > a_x + n$, and $m^* < b_x + n + n_1$ if $Hn < L(b_x + n + n_1)$. Therefore, (A4) is a sufficient condition for interpersonal bundling to be more profitable than separate selling. We thus have:

Proposition 6 *Suppose that condition (A4) holds. Then, the seller's profit is higher under interpersonal bundling than under separate selling, with $p^* = H$, $q^* = L$, and $a_x + n < m^* < b_x + n + n_1$.*

We further illustrate with the following example:

Example 4 *Suppose that $f(x) = \frac{1}{4}$ for $x \in [0, 4]$, $n = 4$, $n_1 = 2$, $L = 1$, $\frac{9}{8} < H < \frac{5}{2}$, and $\frac{1}{4} < \delta < H - 1 \leq \frac{3}{2}L$. Then, (A4) holds and $\pi^s = \max\{4H, 6\}$. From (17), $m^* = 4H + 2$. From (16), $\pi^* = \frac{1}{2}(4H^2 - 8H + 17)$. Therefore,*

$$\pi^* - \pi^s = \begin{cases} \frac{1}{2}(4H^2 - 8H + 5) \geq \frac{1}{2} & \text{if } 4H < 6 \\ \frac{1}{2}(4H^2 - 16H + 17) \geq \frac{1}{2} & \text{if } 4H \geq 6 \end{cases}.$$

¹⁹The identities of consumers in $\tilde{N}_1$ are not uniquely determined, and we may assume that they are randomly chosen from N_1 .

In this case, when the “deal is on” (m is reached), all consumers pay the discounted price $q^* = L = 1$, so there is no price discrimination. But interpersonal bundling increases profit potentially through two channels: it optimally responds to demand uncertainty (as in other cases of interpersonal bundling), and it reduces the dispersion of product values for consumers who purchase multiple units (as under standard product bundling).

5. CONCLUDING REMARKS

This paper has conducted an economic analysis of interpersonal bundling. We have suggested two motives that are especially relevant for this form of product bundling: as optimal option pricing under demand uncertainty, and as a mechanism to disseminate product information. The profit advantage of interpersonal bundling (relative to separate selling) tends to be the highest under an intermediate H/L ratio, and it is more profound when the number of low-value consumers has a more dispersed distribution or when product information is transmitted to more initially uninformed consumers. Interpersonal bundling can also increase profit if high-value consumers are less likely to sign up for group purchases, or if some high-value consumers have demand for a lower-valued second unit.²⁰

Our analysis also reveals that interpersonal bundling is not always more profitable than separate selling, even if no additional selling cost is required. Moreover, like other selling formats, interpersonal bundling can achieve its potential benefits for the seller only if it is properly implemented. In particular, losses may occur if the bundle discount under group purchase is too big. For example, when a restaurant offers a group coupon for 60% off its regular price, it could be unwisely pricing below marginal cost.²¹ While many businesses have profited from selling with interpersonal bundling on the Internet, there have also been reports in the press about how a merchant is hurt by its deep group discount through

²⁰Interpersonal bundling can also serve as a product promotion device, possibly to attract new customers (as mentioned earlier), or to achieve scale economies.

²¹The restaurant may want to attract repeat customers by taking a one-time loss, but is the loss necessary? Our analysis suggests that interpersonal bundling can be profitable without the repeat-business effect, and a seller need not incur losses in order to generate repeat businesses.

Groupon and other “social buying” intermediaries.²² It would not be in the best interests of the sellers and their Internet intermediaries (such as Groupon) to offer below-cost group sale prices.²³

We have studied monopoly interpersonal bundling in this paper. It would be desirable for future research to analyze interpersonal bundling by competing firms. The profitability of interpersonal bundling, and its potential adoption by a firm, may then depend on competitive conditions, possibly also including considerations such as product differentiation. It could also be interesting to extend our analysis to markets with more complex uncertain demand, where firms may use more general nonlinear pricing schemes.

As a new online selling format, it is not entirely surprising that interpersonal bundling on the Internet does not always produce the desired outcomes. Nevertheless, there are profound economic forces behind the popularity of this Internet institution, and understanding the economics of interpersonal bundling will help its profitable use both on the Internet and in other market settings.

APPENDIX

The appendix contains proofs for Corollary 2, Proposition 3, and Proposition 4.

Proof of Corollary 2. From (i), $H\bar{y} \geq L(\bar{y} + \bar{x}_{\hat{F}})$. Hence under separate selling the

²²See, for example, “Groupon demand almost finishes cupcake-maker” (November 22, 2011, *The Telegraph*), which tells the story of a British cakemaker who offered her product at 75% off its regular price through Groupon and had to produce at costs substantially above price in order to meet a huge demand increase.

²³According to a survey reported in “Groupon hurt by lack of repeat biz” (January 4, 2012, *The New York Post*), although 8 out of 10 merchants who ran a daily group coupon deal were satisfied with the results, 52 percent of those surveyed were not planning to run a daily deal in the next six months. The article states that “[Groupon] has been accused of coercing businesses to basically give away goods and services while it takes its up to 50 percent cut.”

optimal price is H for either $\hat{F}$ or F . It follows that

$$\begin{aligned}\hat{\pi}^* - \hat{\pi}^s &= \int \int_{x+y \geq \hat{m}^*} [L(x+y) - Hy] d\hat{F}(x) dJ(y) + H\bar{y} - H\bar{y} \\ &\geq \int_{a_y}^{b_y} \left\{ \int_{m^*-y}^{\hat{b}_x} [Lx - (H-L)y] d\hat{F}(x) \right\} dJ(y),\end{aligned}$$

where the inequality is due to revealed preference. Since $\hat{F}(x) < F(x)$ for $x \geq m^* - y$ from (ii), we have

$$\begin{aligned}&\int_{m^*-y}^{\hat{b}_x} [Lx - (H-L)y] d\hat{F}(x) \\ &= [L\hat{b}_x - (H-L)y] - [Lm^* - Hy] \hat{F}(m^* - y) - \int_{m^*-y}^{b_x} L\hat{F}(x) dx - \int_{b_x}^{\hat{b}_x} L\hat{F}(x) dx \\ &> [Lb_x - (H-L)y] - [Lm^* - Hy] \hat{F}(m^* - y) - \int_{m^*-y}^{b_x} LF(x) dx.\end{aligned}$$

Thus

$$\begin{aligned}\hat{\pi}^* - \hat{\pi}^s &> \int_{a_y}^{b_y} [Lb_x - (H-L)y] dJ(y) - \int_{a_y}^{b_y} [Lm^* - Hy] \tilde{F}(m^* - y) dJ(y) \\ &\quad - \int_{a_y}^{b_y} \int_{m^*-y}^{b_x} LF(x) dx dJ(y). \\ (\text{from (iii)}) &\geq \int_{a_y}^{b_y} [Lb_x - (H-L)y] dJ(y) - \int_{a_y}^{b_y} [Lm^* - Hy] F(m^* - y) dJ(y) \\ &\quad - \int_{a_y}^{b_y} \int_{m^*-y}^{b_x} LF(x) dx dJ(y) \\ &= \int_{m^*-y}^{b_x} [Lx - (H-L)y] dF(x) = \pi^* - \pi^s.\end{aligned}$$

■

Proof of Proposition 3. We consider in turn two cases:

Case 1: $1 < a \leq \sqrt{2} + 1$. Then $\pi^s = \frac{(a+1)^2}{8} = \pi(p, q, 0)$ with $q = \frac{a+1}{4} < 1 < p < a$.

Notice that

$$\left. \frac{\partial \pi(p, q, m)}{\partial m} \right|_{m=0, q=\frac{a+1}{4}, 1 < p < a} = \frac{1}{2(1-q)} (a-p)p > 0.$$

Therefore starting from separate selling at $p^s = \frac{a+1}{4}$, introducing interpersonal bundling with $q = \frac{a+1}{4} < 1 < p < a$ and $m > 0$ leads to a higher profit than π^s .

Case 2: $a > \sqrt{2} + 1$. We argue that in this case $\pi^* > \pi^s$ if $\sqrt{2} + 1 < a < \sqrt{3} + 1$, and $\pi^* = \pi^s$ if $a \geq \sqrt{3} + 1$.

First, notice that $\pi^s = \frac{a^2}{4} = \pi(p, q, m)$ with $p = \frac{a}{2}$, $q < 1$ and $m = a + 2 - 3q$; and, for interpersonal bundling to have a higher profit than separate selling, it is necessary that $q < 1 < p$ and $a - q < m < a + 2 - 3q$. Next, since

$$\frac{\partial \pi(p, q, m)}{\partial p} = -\frac{(2p - a)(m - a + q)}{2(1 - q)},$$

the optimal p satisfies $p^* = a/2$. Furthermore, since $\frac{\partial \pi(p, q, m)}{\partial m} = -\frac{mq - ap + p^2}{2(1 - q)}$ and $\frac{\partial^2 \pi(p, q, m)}{\partial m^2} < 0$, $\pi^* > \pi^s$ if and only if $\pi(a/2, q, m)$ is decreasing in m at $m = a + 2 - 3q$ for some $q \in (0, 1)$.

Finally,

$$-\frac{(mq - ap + p^2)}{2(1 - q)} \Big|_{m=a+2-3q, p=\frac{a}{2}} = \frac{a^2 - 4aq - 8q + 12q^2}{8(1 - q)} < 0$$

if and only if both $a < \sqrt{3} + 1$ and

$$q \in \left(\frac{1}{3} + \frac{a}{6} - \frac{\sqrt{2}}{6} \sqrt{2a - a^2 + 2}, \frac{1}{3} + \frac{a}{6} + \frac{\sqrt{2}}{6} \sqrt{2a - a^2 + 2} \right) \equiv \Omega(a),$$

where $\Omega(a)$ is an interval on $[0.5, 1)$ when $a < \sqrt{3} + 1$. We conclude that $\pi^* > \pi^s$ if and only if $1 < a < \sqrt{3} + 1$. ■

Proof of Proposition 4. First, in equilibrium, $\tilde{\beta} \equiv \tilde{\beta}(p, q, m)$ satisfies $\partial U(\beta | m, \tilde{\beta}) / \partial \beta \Big|_{\beta=\tilde{\beta}} = 0$, or

$$(p - q)[F(m - n) - F(m - n - k)](1 - \tilde{\beta})^{n-1} - C'(\tilde{\beta}) = 0. \quad (18)$$

The firm's problem is:

$$\begin{aligned} & \max_{q \leq L < p \leq H, m} \pi(p, q, m) \\ &= q \left[\left[1 - (1 - \tilde{\beta})^n \right] \int_{x \geq m-n-k} (x + n + k) dF(x) + (1 - \tilde{\beta})^n \int_{x \geq m-n} (x + n) dF(x) \right] \\ & \quad + p \left[\left[1 - (1 - \tilde{\beta})^n \right] (n + k) F(m - n - k) + (1 - \tilde{\beta})^n n F(m - n) \right]. \end{aligned} \quad (19)$$

Next, from (18) and with $C'' \geq 0$, we have $\tilde{\beta} \equiv \tilde{\beta}(p, q, m)$ increasing in p and decreasing in q ; and furthermore

$$\frac{\partial \tilde{\beta}(p, q, m)}{\partial m} = \frac{(p - q)[f(m - n) - f(m - n - k)](1 - \tilde{\beta})^{n-1}}{(n - 1)(p - q)[F(m - n) - F(m - n - k)](1 - \tilde{\beta})^{n-2} + C''}.$$

Thus $\tilde{\beta}(p, q, m)$ is increasing in m at $m = n + a_x$ but decreasing in m at $m = n + k + b_x$. At the optimum, $\pi(p, q, m)$ must increase in $\tilde{\beta}$. Thus, since $\pi(p, q, m)$ and $\tilde{\beta}(p, q, m)$ both increase in p , the solution to problem (19) must have $p = H$, so that problem (19) becomes $\max_{q \leq L, m} \pi(H, q, m)$.

Next,

$$\begin{aligned} \frac{\partial \pi(H, q, m)}{\partial \tilde{\beta}} &= qn \left(1 - \tilde{\beta}\right)^{n-1} \left[\int_{x \geq m-n-k} (x + n + k) dF(x) - \int_{x \geq m-n} (x + n) dF(x) \right] \\ &\quad + Hn \left(1 - \tilde{\beta}\right)^{n-1} [(n + k) F(m - n - k) - nF(m - n)], \end{aligned}$$

with

$$\begin{aligned} \frac{\partial \pi(p, q, m)}{\partial \tilde{\beta}} \Big|_{m=n+a_x} &= qn \left(1 - \tilde{\beta}\right)^{n-1} k > 0, \\ \frac{\partial \pi(p, q, m)}{\partial \tilde{\beta}} \Big|_{m=n+k+b_x} &= Hn \left(1 - \tilde{\beta}\right)^{n-1} k > 0. \end{aligned}$$

Next, since $Hn \geq L(n + a_x)$ by assumption (A2),

$$\begin{aligned} &\frac{\partial \pi(H, q, m)}{\partial m} \Big|_{m=n+a_x} \\ &= \left[1 - \left(1 - \tilde{\beta}\right)^n \right] [H(n + k) - qm] f(m - n - k) \Big|_{m=n+a_x} \\ &\quad + \left(1 - \tilde{\beta}\right)^n (Hn - qm) f(m - n) \Big|_{m=n+a_x} + \frac{\partial \pi(p, q, m)}{\partial \tilde{\beta}} \frac{\partial \tilde{\beta}(p, q, m)}{\partial m} \Big|_{m=n+a_x} \\ &\geq \frac{\partial \pi(p, q, m)}{\partial \tilde{\beta}} \Big|_{m=n+a_x} \frac{\partial \pi(p, q, m)}{\partial \tilde{\beta}} \Big|_{m=n+k+b_x} > 0. \end{aligned}$$

On the other hand, at $m = n + k + b_x$, $\frac{\partial \pi(p, q, m)}{\partial \tilde{\beta}} \frac{\partial \tilde{\beta}(p, q, m)}{\partial m} < 0$, $f(m - n) = 0$, $f(m - n - k) > 0$, $\tilde{\beta}$ is not affected by q from (18), but $\pi(H, q, m)$ increases in q , which implies that $q^* = L$ at $m = n + k + b_x$. And since $H(n + k) \leq L(n + k + b_x)$ by assumption (A2), we have

$$\frac{\partial \pi(H, q, m)}{\partial m} \Big|_{m=n+k+b_x} < 0.$$

Therefore, the equilibrium m is interior: $m^* \in (n + a_x, n + k + b_x)$. It follows from (18) that $\beta^* > 0$. ■

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