Paid Placement: Advertising and Search on the Internet

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Introduction

• Paid-Placement
  – Online advertising in which links to advertisers’ products appear next to keyword search results.
  – Sellers (advertisers) bid payments to a search engine to be placed on its “recommended” list.
  – A predominant form of internet advertising, generating $8.2 billion ad revenues in 2005.
  – Enormous commercial successes for search engines.

• Interesting Issues
  – How do sellers form their bidding strategies?
  – How does paid-placement advertising affect consumer search and welfare?
  – What determines the revenues of a search engine?
This paper develops a market equilibrium model that sheds light on the economic forces behind the success of paid placement.

- Sellers are differentiated.
  - Different from existing models of internet competition assuming homogeneous sellers.

- The bidding process is embedded in a market game where consumers’ search and purchase decisions, as well as sellers’ pricing decisions, are all determined endogenously.
  - Relation to the literature on auctions with endogenous valuations.
  - Relation to Edelman et al. (2005), which studies pay-for-placement auctions.

- The search engine serves as an intermediary that provides information about the relevance of different sellers’ products.
  - Relation to the economics literature on advertising.
A Simple Model

- There are $m \geq 3$ differentiated sellers, selling to a unit mass of consumers at constant marginal cost $c$.

- With probability $\beta_i$ seller $i$’s product matches the preference of a consumer, in which case the consumer’s valuation for $S_i$’s product is $v$. cdf $F(v)$ and pdf $f(v)$.

- With probability $1 - \beta_i$ $S_i$’s product does not match the preference of the consumer, in which case the consumer’s valuation for $S_i$’s product is zero.

- We call $\beta_i$ the match (or relevance) probability of $S_i$, which is $S_i$’s private information; $\beta_i$ is independent of $F(v)$ and is i.i.d. for every consumer.

- Assume $\beta_1 \geq \beta_2 \geq \ldots \geq \beta_m$. Specifically, $\beta_i = \gamma^{i-1}\beta$ for $i = 1, 2, 3$ but $\beta_i = \gamma^3\beta$ for $i \geq 4$, where $\beta, \gamma \in (0, 1)$.

- Consumers can find out whether a seller’s product is desirable by visiting the seller’s website at some cost, and they can choose which sellers’ websites to visit by first searching through a search engine with a keyword for the product.
• The search engine, $E$, has 3 ad positions, $E_1, E_2, E_3$, that it can auction to the sellers in a second price auction, where the seller who bids the most gets listed the highest (at $E_1$) and pays the second highest bid, the seller who bids the second highest gets listed the second highest (at $E_2$) and pays the third highest bid, and so on.

• The timing of the game is as follows.
  – Sellers, having learned their private $\beta_i$, first bid to be listed on $E$.
  – The chosen sellers are listed on $E$.
  – Sellers then simultaneously and independently choose their prices, which are not observed by any consumer until the consumer searches the seller’s website.
  – Consumers decide whether and how to search the websites, possibly using information from $E'$s list.
We make the following technical assumptions:

**A 1.** There is a unique $p^o$ such that

$$p^o = \arg \max_{p \in [c, u]} (p - c)[1 - F(p)].$$

**A 2.** The cost for a consumer’s $j$'th search is $t_j$, $t_j = t$ for $j = 1, 2, 3, 4$ and $t_j = t^h$ for $j > 4$ where

$$t < \gamma^3 \beta \int_{p^o}^{\infty} (v - p^o) f(v) dv < t^h.$$  

- We define

$$\pi^o \equiv (p^o - c)[1 - F(p^o)].$$

$$\pi_3 = (1 - \gamma \beta)(1 - \beta) \gamma^2 \beta \pi^o$$
Equilibrium Analysis

**Theorem 1.** Assume $\beta \geq \max n \left( 2 - \frac{1}{\gamma}, \frac{1-\gamma}{2-\gamma} \right) \equiv \beta (\gamma)$. Then, there is an equilibrium in which seller $S_i$ bids to pay $E$

\[
b_1 = \gamma \beta^2 p^o + 1 - \frac{1-\gamma^2}{m-3} \pi_3,
\]

\[
b_2 = (1 - \beta) \gamma^3 \beta^2 p^o + 1 - \frac{1-\gamma^2}{m-3} \gamma \pi_3,
\]

\[
b_3 = 1 - \frac{1-\gamma^3}{m-3} \pi_3;
\]

\[
b_k = 1 - \frac{1-\gamma^2}{m-3} \gamma \pi_3, \quad k = 4, \ldots, m,
\]

$S_1, S_2, S_3$ are placed at $E_1, E_2, E_3$ and pay $b_2, b_3, b_4$, respectively. Each seller’s price is $p^o$, and each consumer searches sequentially $E_1, E_2, E_3$ and then one randomly selected seller not listed on $E$. The consumer stops searching either when she finds her desired product, in which case she purchases if and only if $v \geq p^o$, or when she has conducted these four searches without finding her desired product.
Observations About the Equilibrium:

- Sellers bid their endogenously determined values in equilibrium.
  - The value of a paid-placement position depends on consumers’ search and purchase behavior.
  - The value also depends on who is placed ahead on the list.

- The placement of sellers by the search engine is in the order of their relevance.
  - A more relevant seller bids more because it has a higher expected profit from a visiting consumer.
  - Paid-placement advertising conveys information about the relevance of different sellers.

- Paid-placement leads to efficient search by consumers.
  - Compared to the situation where consumers search the sellers randomly, paid placement reduces consumer search costs; it also results in higher expected output.
Corollary 1. The search engine’s profit, $\pi_E \equiv b_2 + b_3 + b_4$, is strictly increasing in the number of sellers, $m$. Furthermore, when $m$ is large and $\gamma \leq 0.82$, $\pi_E$ is increasing in the match probability $\beta$ for $\beta \in (\beta(\gamma), \hat{\beta}(\gamma)]$ but is decreasing in $\beta$ for $\beta \in (\hat{\beta}(\gamma), 1)$.

- With more sellers, a seller is less likely to be selected randomly by a buyer, and thus placement on the search engine’s recommended list is more valuable. This motivates the sellers to bid more for placement, increasing the search engine’s revenue.

- An increase in $\beta$ has a positive effect on the value of being placed at $E_1$, but has two opposite effects on the value of being placed at $E_2$ and $E_3$:
  - while it increases the probability of match when a consumer visits the seller’s website, it also reduces the probability that the consumer will visit $E_2$ or $E_3$, since the consumer is more likely to purchase at $E_1$.
  - The balance of these effects results in the search engine’s revenue being first increasing and then decreasing in $\beta$. 
Issue of Equilibrium Uniqueness

- The model is formally a dynamic game with asymmetric information—each seller’s type (relevance) is known to himself but unknown to consumers. There can generally be equilibrium other than the one characterized in Theorem.

- One possible (pooling) equilibrium is that consumers always search randomly without paying attention to the placement list by the search engine, and sellers bid zero amount to be placed on the list.

- Another possible (partially-pooling) equilibrium is that $S_1, S_2, S_3$ bid the same amount, but more than the rest of sellers, and consumers first search the listed sellers on $E$, in random order.

- The equilibrium in Theorem 1, the fully informative equilibrium, is the unique equilibrium of the game if we consumer search behavior is such that she will search in the order of the placement list when she does not have higher expected payoff from any other search order.
  - This will “break” the non-informative equilibrium, since sellers are willing to bid more to be placed on a higher position on the list.
Other Discussions

- Equilibrium Prices
  - Since consumers do not observe a seller’s price before visiting the seller, sellers charge a monopoly price in equilibrium—a familiar result due to Diamond (1971).
  - Still, it is possible to have price dispersion if sellers have different marginal costs, but such an extension does not seem to add significant value to the paper.

- It would be desirable for future work to explore other forms of price formation.

- Modelling “relevance”.
Conclusion

- Advertising through paid placement enables sellers to reveal information about their product relevance to consumers:
  - A seller with a more relevant product expects a higher expected profit from attracting a visiting consumer;
  - this motivates the seller to bid more and to receive a higher ad placement position.
- In equilibrium consumers have the same expected price from each seller, and thus optimally search sellers sequentially, according to their placement on the search engine’s list.
- Paid-placement advertising leads to efficient search by consumers and to higher total output.
- There is an inverted U-shaped relationship between the search engine’s profit and sellers’ product relevance.