Optimal Crowdfunding Design

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Abstract

This paper investigates the optimal design of crowdfunding where crowdfunders are potential consumers with standard motivations and entrepreneurs are profit-maximizing agents. We characterize the typical crowdfunding mechanism where the entrepreneur commits to produce only if aggregate funding exceeds a defined threshold. We study how the entrepreneur uses this threshold, in conjunction with a minimal price, for rent extraction. Compared to a standard posted-price mechanism, total welfare may rise because the entrepreneur can adapt the production decision to demand conditions, but may fall because rent-seeking can worsen. Crowdfunding platforms can raise threshold credibility. So we also compare outcomes when the entrepreneur commits to a threshold against those where the entrepreneur simply decides on production after observing crowdfunder bids. Finally, we contrast crowdfunding with the optimal mechanism where production is contingent on a general function of all bids, rather than the simple sum of bids obliged by the aggregate threshold rule. Crowdfunding is very different, for instance, never committing to produce the good when aggregate bids fall short of the fixed cost (even absent credit constraints).

Keywords: Crowdfunding, mechanism design.

JEL: C72, D42, L12

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1 Introduction

Crowdfunding is a recent but rapidly growing phenomenon in which entrepreneurs can pitch their projects for producing goods to a set of funders who are typically interested in purchasing the good for their own consumption.¹ So far, the literature on crowdfunding has focused on behavioral explanations of crowdfunding. (See, for instance, Belleflamme et al. (2014).) Supporting innovation is an important motivation for some funders, especially in the creative sector, but entrepreneurs often have strong profit motives and the most successful crowdfunding platforms are highly profit-oriented. In addition, most funders have a material interest in consuming the product at a moderate price, even in the creative sector, and their possible non-standard motives, such as a value from participation, certainly do not preclude strategic play by entrepreneurs. So to analyze crowdfunding’s performance and potential for growth, it is critical to understand the strategic choice of entrepreneurs.

In this paper, we investigate the optimal design of crowdfunding where crowdfunders are potential consumers with standard motivations and entrepreneurs and crowdfunding platforms are simply profit-maximizing agents. The most salient feature of typical crowdfunding mechanisms is the commitment to implement the project when aggregate funding reaches a threshold. There is also a commitment to not implement if the threshold is not reached. We analyze the profit and welfare impact of this facet of the aggregate rule separately, because it is harder to enforce and may not be credible: e.g., if wishing to implement ex post (that is, after observing funder contributions), the entrepreneur might get a contact to contribute funds, paid by the entrepreneur, to top up the aggregate fund and reach the threshold.

Our analysis revolves around the fact that an aggregate threshold rule can facilitate adaptation to market demand and price discrimination. The market testing property can save paying fixed production costs when demand is low. Similarly, the enhanced price discrimination option of crowdfunding may enhance welfare, by making valuable projects viable, but may also lower welfare, since aggressive extraction of consumer rents reduces the chances of executing valuable projects. An important question is how the design of crowdfunding and the ability to commit to a threshold that cannot be manipulated downwards affect welfare and entrepreneur profits.

We provide a characterization of the optimal design of a simple crowdfunding mechanism. We focus on reward-based crowdfunding where funders are rewarded for their contributions by receiving the product if it gets produced. We assume that funders come in two types: those who value the product highly and those who value the product less. In this way, the funders are also consumers who buy the entrepreneur’s product if sufficient funds are raised.

¹See Agrawal et al. (2014).
for the project to go ahead. Indeed, to draw attention most starkly to the possibility of strategic play by the entrepreneur, we abstract from the entrepreneur’s true credit needs.\(^2\)

In brief, we study an entrepreneur who sets a minimum total amount, \(K\), that needs to be collected or pledged in order for production to take place. If the minimum amount is not reached (by some defined deadline) no production takes place and fixed costs, \(C\), are avoided. We call this a “funding threshold” in line with the literature and practical usage.\(^3\)

Our main results are as follows. The profit maximizing crowdfunding mechanism excludes low valuation buyers by setting a high minimum bid if and only if low valuation buyers are excluded in the posted price mechanism even when there is no fixed cost. That is, the decision to exclude low valuation buyers is independent of the true fixed cost. When the optimal mechanism does not exclude low valuation buyers, the funding threshold is often used to induce price discrimination where high types bid strictly more than the minimal bid in order to increase the probability of project implementation. The funding threshold also serves as an instrument to commit to implement the project if and only if the number of high types exceeds a certain level \(\hat{n}\), which is (weakly) increasing in fixed cost \((C)\) and the probability of a buyer being of high type \((p)\). In many cases the funding threshold strictly exceeds \(C\). We show that if the entrepreneur cannot commit to not produce when aggregate funds exceed \(C\) but fall short of the funding threshold \(K\) (perhaps because the entrepreneur is nor credit constrained and can manipulate by using fake bids), then the entrepreneur can still implement viable projects by avoiding fixed costs in low demand states, but will not be able to extract as much rents from high type buyers. We also consider that the entrepreneur can restrict feasible bids (which seems a typical crowdfunding feature in practice). This option is irrelevant when she has commitment power but is crucial when she has not: the entrepreneur can extract more rents with restricted bids and sometimes as much as with commitment power.

Crowdfunding mechanisms are simpler than the optimal selling mechanism in general, which is carefully analyzed in an important paper by Cornelli (1996). We highlight three simplifications which in some respects complicate the optimization challenge, but which we believe are important to the success of crowdfunding in making funders willing to participate and therefore important to model. First, and most important, is the aggregate rule: implementation depends only on the sum of funds relative to the aggregate threshold. Second,

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\(^2\)Later we will return to this issue, since financial constraints are an important motive for seeking crowdfunding. We also suppose that the cost of implementing the project is observed by all parties, abstracting from the role that entrepreneurs and crowdfunding platforms may play in credibly transmitting information about the project, including viability and concerns about fraud (the risk that the entrepreneur takes the funds and runs away without providing the promised goods).

\(^3\)For now, we do not impose credit constraints on the entrepreneur. For example, we could impose \(K \geq C\), but in fact we derive this as an optimal property under certain conditions.
bidders know what they pay: conditional on project implementation, each funder’s contribution is independent of other funders’ choices, so funders know exactly what they will pay if they get the good. To know what they would pay is trivial; certainly, it does not require making any calculation as to what other funders will do. Also, funders can rest assured that their price is immune to manipulation by the entrepreneur (or crowdfunding platform) of what other funders actually did.\footnote{Project implementation is not immune to manipulation. Indeed, as just noted, the aggregate threshold as commitment to only implement when the threshold is reached requires a commitment that the entrepreneur not use a fake funder identity to raise the funds to the threshold after seeing that the true aggregate lies short (and yet exceeds the fixed cost).} Third, “no good, no pay”; no funder pays anything if the project is not implemented.\footnote{This last is not a universal property of crowdfunding platforms. Indeed, the major platform, “Kickstarter,” takes money from funders in the form of holding fund pledges and earning interest on these funds during the period of fund-seeking. (In our stylized model, fund-seeking is instantaneous.)} Cornelli (1996) does not contemplate the first and most important feature of crowdfunding, though she does consider the other simplifications as extensions of her general model.

Barbieri and Malueg (2010) investigate whether Cornelli’s (1996) optimal design ever takes the form of a crowdfunding mechanism. They find a set of highly restrictive conditions (only two buyers and their valuations come from a distribution with linear inverse hazard rate) under which this is true. Our project is complementary and essentially asks the opposite question: given that typically, crowdfunding involves many buyers and deviates heavily from the optimal unconstrained design, what is the optimal design under the simplicity constraints that characterize crowdfunding?

We make a number of simplifying assumptions in our analysis. We assume that funders perfectly observe their own private valuations of the good that would be produced.\footnote{This captures the major role of internet and web technologies in enabling dispersed potential consumers to learn about entrepreneurs’ projects from a distance.} While we allow for the possibility that the entrepreneur’s actual credit constraint is non-binding, and could implement production even when aggregate funding lies below the fixed cost $C$, under restriction to the aggregate rule which we treat as defining feature of crowdfunding, the entrepreneur always collects enough funding to cover the fixed cost. This result does not hold under the general optimal mechanism. And of course it does not hold when an important segment of the product’s potential buyers are unable to participate as funders during the crowdfunding stage.

**Related Literature.** Notice that the bids are not standard voluntary contributions as in a pure public goods game, because individuals do not get the good unless they pay the minimum bid (our setup is equivalent to a public good problem with exclusion; however, the club goods literature that studies this type of environment does not analyze the simple...
aggregate rule that is pertinent to crowdfunding; also that literature generally looks at socially optimal rules rather than profit-maximization). However, any bid in excess of the minimal bid, is a voluntary contribution in the standard sense.

Notice that crowdfunding is equivalent to interpersonal bundling (“IPB”) in this scenario. (See for example Chen and Zhang (2012).) But the relatively more advanced literature on IPB only treats the case where the good is produced in advance and has no fixed cost of production.

2 Illustration and simplified model

We begin with a simple illustration of the problem. There are \( N = 2 \) potential consumers. Each consumer has either a low valuation \( v_L \) or a high valuation \( v_H \), with the probability of high valuation equal to \( 0 < p < 1 \), independently drawn for each person. Under a standard, posted-price selling procedure, the entrepreneur must first produce the good and then offer it for sale, at which point the entrepreneur would either set a price of \( v_L \) (yielding a revenue of \( 2v_L \)) or a price of \( v_H \) (yielding an expected revenue of \( 2pv_H \)). When there are neither fixed nor marginal costs, the former is optimal when \( p \leq \hat{p} = v_L/v_H \). In the presence of a fixed cost \( C \), production would take place as long as \( C < \max\{2v_L, 2pv_H\} \).

When there is no fixed cost, the posted-price mechanism is profit-maximizing. However, for positive fixed costs, assuming that potential buyers can evaluate their value for the good prior to its production, some alternative mechanism may perform better, because pre-production inspection may allow the entrepreneur to avoid incurring fixed costs when the implied sales revenues would fail to cover the costs.

Suppose, for instance, that the entrepreneur commits to produce only if both buyers commit to pay \( v_H \). She would then be able to obtain an expected profit of \( p^2(2v_H - C) \) which strictly exceeds \( \max\{2pv_H - C, 2v_L - C, 0\} \) when either \( p > \hat{p} \) and \( 2v_H > C > 2pv_H/(1 + p) \) or \( p < \hat{p} \) and \( 2v_H > C > 2v_L/v_H - p^2v_H \). Given any \( C \in (2v_L, 2pv_H) \), each these cases, \( p \) above or below \( \hat{p} \), arises for a non-trivial range of the parameters \( p, v_L, v_H \). The upper cost limit just reflects how commitments are useless if production should never occur. The lower cost limit is more interesting. The intuition is that high enough fixed costs make it strictly advantageous to save on fixed costs of production in low demand states and the threshold commitment is a way to achieve this.\(^7\)

We will argue that a key contribution of crowdfunding is to enable this kind of mechanism.\(^8\)

\(^7\)The cases can also be written as \( p^2 \in \left(2v_L - C, \frac{2v_L}{v_L + v_H} \right) \) and \( p \in \left[\hat{p}, \frac{C}{2v_H - C}\right] \), respectively.

\(^8\)For completeness, notice that a distinct commitment might be even better. When \( C < v_H \), it is strictly better to sell even when there is only one high type buyer.
Before defining crowdfunding and solving for the optimal mechanism, notice that the vital feature for the mechanism in the example is the ability to exhibit the proposed good, make price offers to buyers, and receive buyer responses, all before producing the good. The entrepreneur commits to a minimal price $t$ as with a standard posted-price, but any sales agreement remains contingent on choosing to produce. So we say that a buyer bids $t$ and only actually buys (at price $t$) if production occurs, and the entrepreneur choose production *ex post* (that is, after observing seller responses).

This key feature may be sufficient. We just described the example of a commitment to produce contingent on receiving two bids of $v_H$. But it may suffice for the entrepreneur to simply retain an option to not produce after observing buyer responses. In fact, so long as $v_H < C < 2v_H$, setting a price of $t = v_H$ prior to production, leads the entrepreneur with this ex post option to produce only after two $v_H$ bids. So no additional commitment technology is needed there. For a lower cost, $C < v_H$, the entrepreneur would require an additional commitment power to commit to produce contingent on two high bids, because with an unconstrained option to produce, the entrepreneur would produce so long as there is one high bid. Of course, with $C < v_H$, the entrepreneur would anyway prefer to produce after just one bid of $v_H$. So additional commitment power is not strictly useful there, but more generally, we find that commitment can be necessary for the optimal mechanism.

Commitment helps the entrepreneur if it can be used to force high types to pay more than $v_L$, despite a plan to sell to low types, which obliges setting the minimal price no higher than $v_L$. Even without commitment, high types may pay more than $v_L$ if the endogenous production decision makes them pivotal with high probability.

**Crowdfunding.** Crowdfunding offers a simple and robust way of implementing these benefits to the entrepreneur. The crowdfunding mechanism allows for precisely the type of contingent commitment just described: through it, the entrepreneur can commit to respond to bids from the buyers by producing when aggregate revenues exceed a threshold. Concretely, the entrepreneur sets up a mechanism of the following form:

The entrepreneur chooses $(b, K)$ where $b$ denotes the minimum bid or contribution and $K$ is the minimum amount to be raised, which we call the funding threshold. Potential consumers, called buyers, $i$ decide on a bid $b_i$. If the sum of their bids is at least $K$, the entrepreneur implements the project (at cost $C$) and produces the goods. Each consumer who bids a positive amount, always weakly above the minimum $b$, then pays his bid and receives one unit of the good. If the sum of bids is insufficient, nothing is invested, produced or paid. In particular, consumers do not pay their bids.\(^9\)

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\(^9\)In practice, bids are often paid as contributions to the crowdfund, with the condition that consumers get their money back if the threshold is not met and the project is accordingly not implemented. This is
**Three properties.** Notice the three attractive properties of this mechanism. First, implementation depends on a simple *aggregate rule:* the sum of bids is compared with a deterministic threshold value that the entrepreneur announces to all bidders. Second, every bidder knows exactly what he will pay if the project is implemented. Third, bidders do not pay anything if they do not get the good. The first and second property make the mechanism easy to understand. The second and third property provide reassurance to buyers that they exactly know what to expect, only facing uncertainty over which scenario, getting the good or not, will materialize; thanks to the first property, they can immediately see how this probability depends on the bids. In addition, it is transparent that bidding more weakly raises the implementation probability.

We now ask, what is the equilibrium of the resulting game and how should the entrepreneur choose the parameters of the mechanism in order to maximize profits? For many parameters, there will be multiple equilibria because of coordination problems. For example, if \( K > v_H \) then it is always an equilibrium for both consumers to bid zero (or not participate). We focus on the best pure strategy equilibrium from the entrepreneur’s perspective.\(^\text{10}\)

The entrepreneur has four different potentially optimal strategies. These can be divided into strategies with a minimal price of \( v_H \) (these strategies exclude low types), and those with minimal price equal to \( v_L \) (both types buy in these strategies, potentially at different prices).

1. The entrepreneur could set a minimum price \( b = v_H \) and sell whenever there is at least one high type buyer. That is funding threshold is set at \( K = v_H \). This yields profit \( \tilde{\pi}_1(p) = 2p(1-p)(v_H - C) + p^2(2v_H - C) \).

2. The entrepreneur could set a minimum price of \( v_H \) but sell only if both types are high by setting \( K = 2v_H \). This yields profit \( \tilde{\pi}_2(p) = p^2(2v_H - C) \).

3. The entrepreneur could set a minimum price of \( b = v_L \) and sell even if there are no buyers who bid strictly more than the minimum bid by setting any \( K \leq 2v_L \). This yields \( \pi_0(p) = 2v_L - C \).

4. The entrepreneur could set a minimum price of \( b = v_L \) but produce only when at least one type is high and sell to the low type at \( v_L \) and the high type at price \( b_H = v_H - p(v_H - v_L) = (1-p)v_H + pv_L > v_L \) by setting funding threshold \( K = v_L + b_H \). This yields \( \pi_1(p) = 2p(1-p)(v_L + b_H - C) + p^2(2b_H - C) \).

\(^{10}\)We believe the restriction to pure strategy equilibria is without loss of generality.
The high type will reveal his type (weakly) because he is indifferent between lying and truth-telling: If he lies and bids \( v_L \) he receives expected utility \( p(v_H - v_L) \). If he reports the truth he gets \( v_H - b_H \) for sure, which is the same (by construction of \( b_H \)). Bidding anything strictly between \( v_L \) and \( b_H \) (or above \( b_H \)) is of course worse than bidding \( v_L \).

Notice that our notation uses a tilde to indicate when the minimum price is \( v_H \) and low types are excluded from consumption. The subscript indicates the minimum number of high type buyers needed for production to take place. Note that in principle the entrepreneur can set the minimum price at \( v_L \) but set the threshold at \( K = 2v_H \) so that two high type buyers are required. However, this yields \( \pi_2 = \pi_1 \).

For \( C = 0 \), the last option is never strictly beneficial and only (weakly) optimal at the exact value \( p = \hat{p} \). However, for some \( C > 0 \) there exists a range of values of \( p \) for which the fourth option is optimal (taking the entrepreneur’s perspective). Note that for any \( p \in (0, 1) \), \( \pi_2(p) > \pi_1(p) \) if and only if \( C > v_H \) and that \( \pi_1(p) > \tilde{\pi}_2(p) \) if and only if \( p \in (0, \hat{p}) \).

To illustrate graphically, we now fix the specific values: \( v_L = 1 \) and \( v_H = 3 \) so that \( \hat{p} = 1/3 \).

So, using the above, if \( C < 3 \), \( \pi_1(p) > \pi_2(p) \) for all \( p \); \( \pi_1(p) > \tilde{\pi}_1(p) \) if and only if \( p < \hat{p} \). In addition, comparing cases 4 and 3 reveals: \( \pi_1(p) > \pi_0(p) \) if and only if \( p > \frac{2 - C}{6 - C} \). So, provided \( 2 < C < 3 \) and \( 0 < p < \hat{p} \), option 4 is optimal, with the funding threshold set at \( K = 4 - 2p > C \).

If \( C > 3 \), \( \pi_1(p) > \tilde{\pi}_1(p) \) for all \( p \); \( \pi_1(p) > \pi_0(p) \) for all \( p \in (0, 1) \); \( \pi_1(p) > \pi_2(p) \) if and only if \( p < \frac{4 - C}{6 - C} \). So if \( 4 < C < 6 \), the second option is optimal and the threshold is set at \( K = 6 \). The fourth option is optimal when \( C < 4 \) and \( p < \frac{4 - C}{6 - C} \) and the funding threshold equals \( K = 4 - 2p > C \).\(^{11}\) Figure 1 shows which option is optimal depending on cost (\( C \) on the horizontal axis) and the \( H \)-type probability (\( p \) on the vertical axis), for the values \( v_L = 1 \) and \( v_H = 3 \).

\[ \text{[Figure 1 about here.]} \]

It may help to see how this figure arises by looking at Figure 2 which plots profits from each option as a function of the probability \( p \) in the specific example where \( C = 1.5 \) (for values \( v_L = 1 \) and \( v_H = 3 \)).\(^ {12}\)

The flat line is the fixed profit from always selling at price \( v_L = 1 \) to both consumers, earning profit \( 2 - 1.5 = 0.5 \). This is clearly optimal when \( p \) is very low, but as \( p \) grows (above

\(^{11}\)To see this, note that \( K - C > 4 - 2 \left( \frac{4 - C}{6 - C} \right) = \frac{(4 - C)^2}{6 - C} > 0 \).

\(^{12}\)We do not show the full range of probabilities, \( p \in (0, 1) \), because \( \pi_1 \) remains the highest on the remainder of the range. To see this note that any pair of quadratics intersect at most twice and the curves \( \pi_1(p) \), \( \tilde{\pi}_1(p) \) and \( \tilde{\pi}_2(p) \) all intersect at \( p = 1 \).
1/9) and high types become more likely, the entrepreneur gains by extracting some rent from high types by raising \( K \) above \( 2v_L \), effectively committing not to implement production unless at least one consumer bids strictly above the minimum. When \( p \) rises further (above 1/3), the entrepreneur is tempted to extract more rents by raising the minimum bid \( b \) to \( v_H = 3 \). In this illustrative example with \( C = 1.5 \), it is never optimal to raise \( K \) above 4 to \( K = 2v_H = 6 \). However, as can be observed from Figure 1, it is optimal for high levels of fixed cost.

[Figure 2 about here.]

3 General binomial model (N buyers)

There are \( N \) potential consumers, whom we call buyers, though they can also be called the funders or crowdfunders. Each buyer’s valuation is an independent draw from the same binomial distribution as before, having a probability \( 0 < p < 1 \) on high valuation \( v_H \) and \( 1 - p \) on low valuation \( v_L \).

Let \( P(n) \) denote the probability that exactly \( n \) out of \( N \) buyers have high valuation. That is,

\[
P(n) = \frac{N!}{(N-n)!n!} p^n (1-p)^{N-n}.
\]

Similarly, let \( \hat{P}(n) \) denote the probability that \( n \) out of \( N-1 \) buyers have high valuation.

\[
\hat{P}(n) = \frac{(N-1)!}{(N-1-n)!n!} p^n (1-p)^{N-1-n}.
\]

Let us first consider the case where the entrepreneur intends to exclude low valuation buyers and sells to high valuation buyers (at price \( v_H \)) only when there are at least \( n \) of these types. It is helpful to characterize the entrepreneur’s strategy, \( (b, K) \), in terms of the minimal number of high types required for implementation, which we denote by \( \hat{n} \) or simply \( n \) when not ambiguous, and the exclusion or non-exclusion of low value buyers. In each case, we calculate the optimal \( (b, K) \) associated with each such value \( n \). Under exclusion, \( \hat{n} \) is best implemented by choosing \( b = v_H \) and any \( K \in (\hat{n}v_H, n v_H] \). Using a tilde to denote exclusion of low types, the profit obtained by this type of strategy equals:

\[
\tilde{\pi}_n = \sum_{j=n}^{N} P(j)(jv_H - C).
\]

To maximize profit in this class of strategies, the critical number of high types \( n \) is set so
that \( nv_H \geq C \geq (n - 1)v_H \).

Next, let us consider the case where low valuation buyers are not excluded. That is, \( b = v_L \). Consider the strategy where production and sales take place if at least \( n \) buyers have high valuation and high valuation buyers pay \( b \), for some \( n \geq 1 \). This can be implemented by choosing \( K_n = (N - n)v_L + nb \). An incentive constraint needs to be imposed on \( b \): a high valuation buyer should weakly prefer to bid \( b \) than \( v_L \). This translates into the following condition:

\[
(v_H - b) \sum_{j=n}^{N-1} \hat{P}(j) \geq (v_H - v_L) \sum_{j=n}^{N-1} \hat{P}(j).
\]

That is,

\[
b \leq \bar{b}_n := v_H - (v_H - v_L) \frac{\sum_{j=n}^{N-1} \hat{P}(j)}{\sum_{j=n-1}^{N-1} \hat{P}(j)}.
\]

Since the entrepreneur seeks to maximize profits, we will focus attention on equilibria where buyers with high valuation pay exactly \( \bar{b}_n \). Observe that \( \bar{b}_n \) does not depend on \( C \). Note that the profit earned by the entrepreneur with this strategy equals

\[
\pi_n = \sum_{j=n}^{N} P(j)(j\bar{b}_n + (N - j)v_L - C),
\]

for any \( n \geq 1 \). The case of \( n = 0 \) where the entrepreneur commits to sell even in the absence of any high type cannot induce any payment above the minimum price of \( v_L \), so \( \bar{b}_0 = v_L \), \( K_0 = Nv_L \) and \( \pi_0 = Nv_L - C \).

Here we are neglecting two types of incentive constraints. First, we should, in principle, also impose that no \( H \)-type wants to deviate to some bid \( b \in (v_L, \bar{b}_n) \). However, this is automatically implied by the more stringent incentive constrained. Second, we would need to impose that bidders with high valuations do not prefer to bid higher than \( \bar{b}_n \). We conjecture that this constraint will not bind for the optimal threshold \( \hat{n} \) chosen by the entrepreneur. In any case, if the entrepreneur restricts the set of possible bids to \( \{v_L, \bar{b}_n\} \) there is no such potential problem. Moreover, restricting bids is a feature of some crowdfunding platforms and in the current case (where entrepreneurs can commit to only produce if the threshold

\[\text{Only in the knife-edge case where } v_H/C \text{ is an integer } \hat{n}, \text{ both } \hat{n} \text{ and } \hat{n} + 1 \text{ generate the same optimal profit.}\]

\[\text{Note that in this case, there is a unique optimal value of } K, \text{ not a range of } Ks \text{ that will do as above. Namely, if } K \text{ is slightly lower than this value, a buyer with a high valuation would bid strictly less than } b \text{ if he expects all low valuation buyers to pay } v_L \text{ and all high valuation buyers other than himself to pay } b.\]

\[\text{If we define } \hat{P}(-1) = 0, \text{ equation (1) is valid for all } n \geq 0.\]
is reached), there is no loss of generality in restricting the number of possible bids to the number of buyer types, which is two.\footnote{However, in the case of non-commitment (see next section), the distinction between restricted and unrestricted offers turns out to be crucial.}

Figure 3 shows the regions in the \((C, p)\)-space where the different strategies are optimal for \(N = 5\), \(v_L = 1\) and \(v_H = 1.6\). Note that \(\pi_5 = \tilde{\pi}_5\) as in both cases one only sells when all buyers have high valuation. For \(\hat{p} = v_L/v_H = 0.625\) and \(C \in [(i - 1)v_H, iv_H]\), \(\pi_i = \tilde{\pi}_i\) (for \(i = 1, ..., 4\)). For \(p > \hat{p}\), it is optimal to sell only at high price \(v_H\); for \(p < \hat{p}\), it is not optimal to exclude low valuation buyers. For a given \(p < \hat{p}\), the minimal amount asked goes up in steps as \(C\) increases. For example, for \(p = 0.5\): \(K_0 = 5\), \(K_1 = 5.04\), \(K_2 = 5.32\), \(K_3 = 5.98\), \(K_4 = 6.92\), \(K_5 = 8\). For a given fixed cost \(C\), the minimal number of high valuation buyers goes up in steps as \(p\) increases. The corresponding minimal amount asked makes upward jumps as the optimal strategy switches from selling and producing when there are at least \(n\) to \(n + 1\) high valuation buyers, but decreases in \(p\) in the interior region where selling and producing when there are at least \(n\) high type valuation buyers. In any case, \(K > C\) when \(p > 0\) and \(C < Nv_H\). See Figure 4 for an illustration.

\textbf{Proposition 1} For \(p > \hat{p} = v_L/v_H\), the profit-maximizing crowdfunding mechanism consists in setting a minimum price of \(b = v_H\) and requesting a funding threshold \(K = \hat{n}v_H \geq C\) where \(\hat{n}\) is the smallest integer larger or equal to \(C/v_H\).

For \(p < \hat{p} = v_L/v_H\) the profit-maximizing crowdfunding mechanism consists in setting a minimum price of \(b = v_L\), a maximum price of \(b_{\hat{n}}\), and requesting a funding threshold \(K_{\hat{n}} = \hat{n}b_{\hat{n}} + (N - \hat{n})v_L \geq C\) where \(b_{\hat{n}}\) is defined in (1) and \(\hat{n} = \arg\max_n\{\pi_n\}\).

This result is proved in the Appendix. (Appendix provided upon request.) The optimal value \(\hat{n}\) does not have a neat closed-form solution, but we can prove that it satisfies the intuitive properties of increasing with \(C\) and with \(p\).

\textbf{Proposition 2} For \(p > \hat{p}\), \(\hat{n}\) is weakly increasing in \(C\) and independent of \(p\).

For \(p < \hat{p}\), \(\hat{n}\) is weakly increasing in both \(C\) and \(p\).
4 The role of commitment

We assumed that the entrepreneur is able to commit not to produce unless the total amount raised exceeds the threshold $K$. In particular, he will not produce if this threshold is not reached even if the total amount raised is more than enough to cover the cost $C$. Suppose the deadline of the crowdfunding platform is about to expire and the threshold is not made by just a small margin. In this case the entrepreneur would have an incentive to make a fake bid sufficient for the aggregate to exceed the threshold and to permit production to take place. Let us consider what happens if we relax the commitment power of the entrepreneur and assume that the entrepreneur will produce if and only if the total amount raised is weakly above the fixed cost $C$. Naturally, removing commitment power weakly reduces the entrepreneur’s profit. We now analyze when the entrepreneur’s profit is strictly lower and how this limited commitment affects consumer and total welfare. We always allow the entrepreneur to commit to a minimal bid (which will again be either $v_L$ or $v_H$).

Let us first consider $p \geq \hat{p}$. From Proposition 1 we know that with commitment the entrepreneur optimally chooses minimum bid $b = v_H$ and threshold $K = \hat{n}v_H$ where $\hat{n}$ is the smallest integer larger than or equal to $C/v_H$. It is therefore clear that the same profit $\tilde{\pi}_{\hat{n}}$ is obtained under non-commitment by just setting the minimum price at $v_H$. Hence, neither consumer nor producer welfare is affected by the commitment power when $p \geq \hat{p}$.

Consider now $p < \hat{p}$ and let $\pi_{\hat{n}}(p)$ denote the maximal profit obtained under commitment. If $\hat{n} = 0$, it must be the case that $C < Nv_L$ since $K_0 = Nv_L$ and $\pi_0 = Nv_L - C$. This maximal profit can also be obtained under non-commitment by setting the minimum bid equal to $b = v_L$. All buyers will bid $v_L$, knowing they will obtain the good for sure because $C < Nv_L$. In this case, commitment is again irrelevant for producer and consumer welfare. If $\hat{n} = N$, then it must be the case that $C \geq (N - 1)v_H$ and commitment is again irrelevant for producer and consumer welfare because $\pi_N = \tilde{\pi}_N$. The same outcome can again be obtained under non-commitment by setting minimal price equal to $v_H$.

Next, we need to consider the cases where under commitment $\pi_{\hat{n}}$ is optimal with $0 < \hat{n} < N$. It proves critical to distinguish the case where buyers are unrestricted in their bids from the case where the entrepreneur can further restrict the set of permissible bids.

4.1 Unrestricted bids

It is immediately clear that non-commitment leads to strictly lower profits. Under commitment high types bid $b_{\hat{n}}$ and the project is implemented only if the threshold $K_{\hat{n}} > C$ is reached. This is no longer an equilibrium under non-commitment when bids are unrestricted because a high type will then strictly prefer to bid slightly below $b_{\hat{n}}$. The only (pure) strategy
equilibria in which high types pay \( b > v_L \) and where the goods are produced when at least \( n \) buyers have high type must satisfy \( nb + (N - n)v_L = C \). Notice that in such an equilibrium the entrepreneur only makes a strictly positive profit when at least \( n + 1 \) types are high. Let us denote by \( \pi_n \) the profit obtained in such an equilibrium. Observe that \( \pi_0 = \pi_0 \) and \( \pi_N = 0 \). Furthermore, \( \pi_{N-1} < \pi_N \). The entrepreneur will then compare \( \max_n \{ \pi_n \} \) and \( \max_n \{ \bar{\pi}_n \} \) and choose the optimal one. Consumer and total surplus may be higher or lower under non-commitment.

For example, consider the case \( N = 2 \). Then commitment is only relevant for the parameter range where \( \pi_1 \) is optimal under commitment (see Fig. 1). For this range of parameters, it follows from the discussion above that under non-commitment only \( \pi_0 \), \( \bar{\pi}_1 \) and \( \bar{\pi}_2 \) are potential optimal payoffs for the entrepreneur.

In particular, when \( C < \min\{2v_L, v_H\} \), \( \pi_0 \) is optimal when \( p \) is relatively small and \( \bar{\pi}_1 \) is optimal when \( p \) is close to \( \hat{p} \). In the first case, consumer surplus and total welfare are higher under non-commitment because production always takes place, which is efficient, whereas under commitment production only happens when at least one buyer has high type. In the second case, consumer surplus and total welfare are lower under non-commitment because consumer surplus equals zero, whereas under commitment high type buyers enjoy positive expected surplus.

Similarly, when \( v_H < C < 2v_L \), \( \pi_0 \) is optimal when \( p \) is relatively small and \( \bar{\pi}_2 \) is optimal when \( p \) is relatively high. Again, consumer surplus and total welfare are higher under non-commitment only in the first case, for exactly the same reasons as given above.

When \( 2v_L < C < v_H \), \( \bar{\pi}_1 \) is optimal under non-commitment. This reduces profits and lowers consumer surplus to zero. Commitment is unambiguously welfare improving in this region of the parameter space.

Finally, when \( C > \max\{2v_L, v_H\} \), \( \bar{\pi}_2 \) is optimal. Again, in this case consumer surplus and total welfare are strictly higher under commitment.

### 4.2 Restricted bids

Suppose first that \( C < Nv_L \). Then it is clear that without commitment power the entrepreneur cannot induce \( b > v_L \) from the \( H \)-types, while obtaining \( v_L \) from the \( L \)-types. This is because any \( H \)-type would deviate to paying \( v_L \), knowing that the total amount received will always weakly exceed \( Nv_L \). So the entrepreneur can only choose between selling at \( v_L \) to all and raising \( Nv_L - C \) or setting a minimal price of \( v_H \). If \( p \) is relatively low, the first option is optimal whereas if \( p \) is relatively high, the second option will be optimal. When the first option is optimal, production takes place for sure and all consumers enjoy
it. This is the efficient outcome. Hence, in this case the commitment power provided by the
crowdfunding platform reduces consumer and total welfare but increases profits. Whenever
the second option is optimal, consumer surplus equals zero and profit is strictly lower than
with commitment. Hence, in this case commitment power is welfare enhancing and improves
consumer and producer surplus.

Let us now consider $C \geq N v_L$. Whereas in the case of unrestricted bids the entrepreneur’s
profit is strictly reduced under non-commitment, this is not necessarily true when bids can
be restricted. In some cases the entrepreneur does not need the commitment power, because
she can implement exactly the same outcome (and thus realize the same profit) without
affecting consumers. The easiest way to see this is in the example with $N = 2$, $p < \hat{p}$
and $C > \max\{2v_L, v_H\}$. (See Fig. 1.) With unrestricted bids we saw that only $\tilde{\pi}_2$ can
be implemented. However, by restricting the bids to $\{v_L, \bar{b}_1\}$ the entrepreneur can simply
obtain $\pi_1$, the maximal payoff under commitment.

There are further differences between the cases of restricted and unrestricted bids, but
we need to go beyond the example of two buyers to illustrate this. Let us therefore consider
again the example with $N = 5$, $v_L = 1$ and $v_H = 1.6$, and let $p = 0.6$ and $C = 6.4$. With
commitment the entrepreneur would choose feasible bids $\{v_L = 1, \bar{b}_4 \approx 1.436\}$ and threshold
$K = 4\bar{b}_4 + v_L \approx 6.744$. It can be easily verified that, even without commitment, no high
type would want to deviate to $v_L$ because $3\bar{b}_4 + 2v_L \approx 6.3 < C$. A deviation would only yield
a positive outcome if all other four bidders turn out to have type $H$, and this probability is
too small to make it worthwhile.

However, if fixed cost equals $C' = 6.2$, $H$-types would want to deviate to $v_L$ because then
$3\bar{b}_4 + 2v_L \approx 6.3 > C'$ and only three out of the remaining four bidders should have type
$H$. Hence, the entrepreneur cannot obtain payoff $\pi_4(p)$ and will have to change the set of
permissible bids in order to maximize her profits under non-commitment. For the particular
example at hand, the optimal mechanism for the entrepreneur without commitment is to
exclude low type consumers and set minimal price at $v_H$, yielding a profit of $\tilde{\pi}_4(p) = 0.1918$
(whereas $\pi_4(p) = 0.2177$ and $\pi_3(p) < 0$). Hence, commitment power enhances consumer and
producer welfare in this case.

However, for other parameter values, the entrepreneur without commitment power may
switch mechanism in a different manner. Rather than excluding low type buyers she can
change the set of permissible bids, and in particular the price to be paid by the $H$-types. For
example, fix now $p = 0.3$ and let $6.3 \leq C \leq 6.62$. With commitment, the entrepreneur would
set admissible bids $\{v_L = 1, \bar{b}_4 \approx 1.542\}$ and threshold $K = 4\bar{b}_4 + v_L \approx 7.168$. Note that
$3\bar{b}_4 + 2v_L > 6.62$, so that $H$-types would certainly want to deviate and bid $v_L$. Hence, under
non-commitment payoff $\pi_4(0.3)$ is not feasible. If $C = 6.3$ (where $\pi_3(0.3) = 0.026$ is relatively
close to \( \pi_4(0.3) = 0.028 \) the entrepreneur will switch to setting the set of feasible bids to \( \{v_L = 1, \bar{b}_3 \approx 1.456\} \) (and the project would be implemented if at least 3 bidders pay the higher bid\(^{17}\)). Hence, consumer and total welfare are strictly higher under non-commitment. However, if \( C' = 6.62 \) the entrepreneur does better by setting feasible bids to \( \{v_L = 1, \tilde{b}\} \) where \( \tilde{b} = (C' - 2v_L) / 3 - \varepsilon = 1.54 - \varepsilon \) and the project is implemented if at least 4 bids are high\(^{18}\). This yields approximately a profit of \( \hat{\pi}_4(0.3) = 0.0179 \) whereas under commitment maximal profit would be equal to \( \pi_4(0.3) = 0.0182 \) and \( \pi_3(0.3) < 0 \). Hence, in this case, total welfare is the same as under commitment (because the project is implemented if and only if at least 4 buyers have valuation \( v_H \)). Consumer surplus is increased because high valuation buyers pay a slightly lower price.

Figure 5 illustrates how optimal selling strategies need to be adapted when the entrepreneur cannot commit against producing when the amount pledged exceeds the fixed cost of production. Obviously, the entrepreneur’s profit decreases whenever he needs to adjust his selling strategy. In the (larger) orange area where minimal price is set at \( v_H \), consumer surplus is also reduced. Hence, in this case commitment is unambiguously good. On the other hand, in the areas marked \( \hat{\pi}_2, \hat{\pi}_3, \) and \( \hat{\pi}_4 \) total surplus is the same under commitment and non-commitment. Under non-commitment the entrepreneur leaves a higher rent to \( H \)-types through lower prices without affecting the probability of production.

[Figure 5 about here.]

5 Concluding remarks

We have analyzed the optimal design of crowdfunding when entrepreneurs are profit-maximizers and consumers have standard motivations. When the crowdfunding platform enables the entrepreneur to commit to only produce if the funding threshold is reached, the entrepreneur will choose this threshold strategically. The optimal strategy trades off higher prices for high type buyers (when setting a high threshold) against a higher probability of production (when setting a low threshold). The threshold will be strictly above cost, so that production never takes place at a loss. This feature of optimal crowdfunding design is in contrast with the optimal general mechanism of selling and producing a good with fixed cost, as analyzed by Cornelli (1996). When the entrepreneur lacks this commitment power (for example, because the crowdfunding platform cannot prevent the entrepreneur from making fake bids to reach the threshold), the entrepreneur’s profit will be reduced. However, this reduction in profit is mild (or sometimes even non-existent) when the entrepreneur can restrict the set of permiss-

\(^{17}\)This is so because \( 2\bar{b}_3 + 3v_L < C < 3\bar{b}_3 + 2v_L \).

\(^{18}\)Observe that \( 3\tilde{b} + 2v_L = C' - 3\varepsilon < C' < 4\tilde{b} + v_L \) for small \( \varepsilon > 0 \).
sible bids. Our result that without commitment power social welfare may go up points out that the entrepreneur’s interest may conflict that of consumers and social welfare. So there is scope for policy investigation and intervention.

Our analysis is just a first step in better understanding the motives of entrepreneurs in crowdfunding. In future research, we are planning to examine more general distributions of valuations, investment incentives by entrepreneurs and platforms, and the role of banks in financing risky projects.

References


Figure 1: Optimal selling strategies in \((C,p)\)-space \((N = 2, v_L = 1, v_H = 3)\).
Figure 2: Profits for the four different selling strategies ($v_L = 1$, $v_H = 3$, $C = 1.5$).
Figure 3: Optimal selling strategies in \((C,p)\)-space \((N = 5, v_L = 1, v_H = 1.6)\).
Figure 4: Funding threshold as function of $p$ for $C = 4.9$, $N = 5$, $v_L = 1$, $v_H = 1.6$. 
Figure 5: Optimal selling strategies in \((C, p)\)-space under no commitment \((N = 5, v_L = 1, v_H = 1.6)\).