An Empirical Analysis of Search Engine Advertising: 
Sponsored Search and Cross-Selling in Electronic Markets

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September 2007

NET Institute Working Paper

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1We thank the NET Institute www.NETinst.org for financial support.
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Abstract

The phenomenon of sponsored search advertising – where advertisers pay a fee to Internet search engines to be displayed alongside organic (non-sponsored) web search results – is gaining ground as the largest source of revenues for search engines. Using a unique panel dataset of several hundred keywords collected from a large nationwide retailer that advertises on Google, we empirically model the relationship between different metrics such as click-through rates, conversion rates, bid prices and keyword ranks. Our paper proposes a novel framework and data to better understand what drives these differences. We use a Hierarchical Bayesian modeling framework and estimate the model using Markov Chain Monte Carlo (MCMC) methods. We empirically estimate the impact of keyword attributes on consumer search and purchase behavior as well as on firms’ decision-making behavior on bid prices and ranks. We find that the presence of retailer-specific information in the keyword increases click-through rates, and the presence of brand-specific information in the keyword increases conversion rates. Our analysis provides some evidence that advertisers are not bidding optimally with respect to maximizing the profits. We also demonstrate that as suggested by anecdotal evidence, search engines like Google factor in both the auction bid price as well as prior click-through rates before allotting a final rank to an advertisement. Finally, we conduct a detailed analysis with product level variables to explore the extent of cross-selling opportunities across different categories from a given keyword advertisement. We find that there exists significant potential for cross-selling through search keyword advertisements. Latency (the time it takes for consumer to place a purchase order after clicking on the advertisement) and the presence of a brand name in the keyword are associated with consumer spending on product categories that are different from the one they were originally searching for on the Internet.

Keywords: Online advertising, Search engines, Hierarchical Bayesian modeling, Paid search, Click-through rates, Conversion rates, Keyword ranking, Bid price, Electronic commerce, Cross-Selling, Internet economics.

JEL: C33, C51, D12, L10, M31, M37, L81
1. Introduction

The Internet has brought about a fundamental change in the way consumers obtain information, thereby facilitating a paradigm shift in consumer search and purchase patterns. In this regard, search engines are able to leverage the value as information location tools by selling advertising linked to search terms entered by online users and referring them to the advertisers. Indeed, the phenomenon of sponsored search advertising – where advertisers pay a fee to Internet search engines to be displayed alongside organic (non-sponsored) web search results – is gaining ground as the largest source of revenues for search engines. The global paid search advertising market is predicted to have a 37 percent compound annual growth rate, to more than $33 billion in 2010 and has become a critical component of firm’s marketing campaigns. This is not surprising given that 94% of consumers use search engines to find information on the Web, and 81% who use search engines find the information they are looking for every time (Nielsen-Net Ratings).

Search engines like Google, Yahoo and MSN have discovered that as intermediaries between users and firms, they are in a unique position to try new forms of advertisements without annoying consumers. In this regard, sponsored search advertising has gradually evolved to satisfy consumers’ penchant for relevant search results and advertisers' desire for inviting high quality traffic to their websites. These keyword advertisements are based on customers’ own queries and are thus considered far less intrusive than online banner advertisements or pop-ups. In many ways, one could imagine that this enabled a shift in advertising from ‘mass’ advertising to more ‘targeted’ advertising. How does this mechanism work? In sponsored search, firms who wish to advertise their products or services on the Internet submit their product information in the form of keyword listings to search engines. Bid values are assigned to each individual keyword to determine the placement of each listing among search results when a user performs a search. When a consumer searches for that term on a search engine, the advertisers’ web page appears as a sponsored link next to the organic search results that would otherwise be returned using the neutral criteria employed by the search engine. By allotting a specific value to each keyword, an advertiser only pays the assigned price for the people who click on their listing to visit its website. Because listings appear when a keyword is searched for, an advertiser can reach a more targeted audience on a much lower budget.

Despite the growth of search advertising, we have little understanding of how consumers respond to contextual and sponsored search advertising on the Internet. In this paper, we focus on previously
unexplored issues: How does sponsored search advertising affect consumer search and purchasing behavior on the Internet? More specifically, what features of a sponsored keyword advertisement do consumers respond to most during web search in terms of click-through rates and conversions? How do keyword attributes influence the advertiser’s actual and optimal bidding decisions, and the search engine’s ad ranking decision? Is there any potential for cross-selling products using sponsored search advertising? While an emerging stream of theoretical literature in sponsored search has looked at issues such as mechanism design in auctions, no prior work has empirically analyzed these kinds of questions. Given the shift in advertising from traditional banner advertising to search engine advertising, an understanding of the determinants of conversion rates and click-through rates in search advertising is essential for both traditional and Internet retailers.

Using a unique panel dataset of several hundred keywords collected from a large nationwide retailer that advertises on Google, we study the effect of sponsored search advertising on consumer search, click and purchase behavior in electronic markets. We propose a Hierarchical Bayesian modeling framework in which we model consumers’ behavior jointly with the advertiser’s and search engine’s decisions. To the best of our knowledge, our paper is the first empirical study that models and documents the impact of search advertising on consumer’s click-through, conversion and purchase behavior in electronic markets. Our findings and contributions can be summarized as follows.

First, we build a model to empirically estimate the impact of various attributes of sponsored search advertisements (such as the ranking, the presence of retailer information, brand information and the length of the ad in words) on consumer click-through rates, and purchase propensities. We find that the ranking is negatively associated with the click-through rates and conversion rates, the presence of retailer-specific information in the keyword increases click-through rates, the presence of brand-specific information in the keyword increases conversion rates, while the length of the keyword is associated with a decrease in click-through rates. By quantifying the magnitude of these effects in the domain of online advertising, we extend the existing literature that had examined the impact of traditional media advertising on consumer behavior. Further, by examining the differential impact of ‘retailer-specific’ advertising versus ‘brand-specific’ advertising on consumer and firm decision-making processes, our research contributes towards the extant literature in marketing that has examined the implications of retail store advertising vis-à-vis national brand advertising in a channel context.
Second, we analyze the impact of these covariates on the decisions of the firms involved in the sponsored advertising process—the bid price of the advertiser and the rank allotted by the search engine to the advertiser. We show that while the advertiser is exhibiting some learning behavior over time by deciding their bid prices in accordance with past performance, they are not bidding optimally. A vast majority (94%) of the bids involve bidding above the optimal value, with the average deviation being 23.3 cents. We conduct policy simulations to assess the relative profit impact from placing optimal bid prices, and find that it can make substantial improvements in its expected profits. Finally, we also demonstrate that as postulated by the popular press, search engines are indeed taking into account both the bid price of the advertiser as well as the quality metrics such as prior click-through rates before setting the final rank of an advertisement. Our findings thus contribute towards providing empirical evidence about the bidding behavior and auction mechanism in search engines.

Third, we present analysis with product level variables to explore the extent of cross-selling opportunities across different categories from a given keyword advertisement. By examining purchase incidence across categories, we find that there exists significant potential for cross-selling through paid search advertisements. Moreover, latency (the time it takes for consumers to place a purchase order after clicking on the advertisement) and the presence of a brand name in the advertisement play an important role in influencing the extent to which consumers spend on different product categories. Our research extended the existing marketing literature by investigating consumers’ acquisition decisions for multiple products when exposed to online advertising. To the best of our knowledge, this is the first study of this kind in an online context.

The remainder of this paper is organized as follows. Section 2 gives an overview of the different streams of literature from marketing and computer science related to our paper. Section 3 describes the data and gives a brief background into some different aspects of sponsored search advertising that could be useful before we proceed to the empirical models and analyses. In Section 4, we present a model to study the click-through rate, conversion rate and keyword ranking simultaneously, and discuss our empirical findings. In Section 5, we study the cross-selling potential of paid advertisements by modeling the impact of ranking and keyword characteristics on consumer spending in the searched product category as well as in the non-searched product categories. In Section 6, we discuss some implications of our findings and then conclude the paper.
2. Prior Literature

Our paper is related to several streams of research. It contributes to recent research in online advertising by providing the first known empirical analysis of sponsored search keyword advertising. Much of the existing academic (e.g., Cho, Lee and Tharp 2001, Gallagher, Foster and Parsons 2001, Dreze and Hushsherr 2003) on advertising in online world has focused on measuring changes in brand awareness, brand attitudes, and purchase intentions as a function of exposure. This is usually done via field surveys or laboratory experiments using individual (or cookie) level data. Sherman and Deighton (2001) and Ilfeld and Winer (2002), show using aggregate data that increased online advertising leads to more site visits. In contrast to other studies which measure (individual) exposure to advertising via aggregate advertising dollars (e.g., Mela, Gupta and Jedidi 1998, Ilfeld and Winer 2002), we use data on individual search keyword advertising exposure. Manchanda et al. (2006) look at online banner advertising. Because banner ads have been perceived by many consumers as being annoying, traditionally they have had a negative connotation associated with it. Moreover, it was argued that since there is considerably evidence that only a small proportion of visits translate into final purchase (Sherman and Deighton 2001, Moe and Fader 2003, Chatterjee, Hoffman and Novak 2003), click-through rates may be too imprecise for measuring the effectiveness of banners served to the mass market. Interestingly however, Manchanda et al. (2006), found that banner advertising actually increases purchasing behavior, in contrast to conventional wisdom. These studies therefore highlight the importance of investigating the impact of other kinds of online advertising such as search keyword advertising on actual purchase behavior, since the success of keyword advertising is also based on consumer click-through rates.

A large literature in economics sees advertising as necessary to signal some form of quality (for example, Grossman and Shapiro 1984). Chen and He (2006) build a model of a market where there is only paid search and no organic search. Their model looks at paid search as an information signaling tool. There is also an emerging theoretical stream of literature exemplified by Edelman, Ostrovsky and Schwartz (2007) that examines auction price and mechanism design in keyword auctions.

Despite the emerging theory work, very little empirical work exists in online search advertising. This is primarily because of difficulty for researchers to obtain such advertiser-level data. Existing work has so far focused on search engine performance (Telang Boatwright, and Mukhopadhyay 2004, Bradlow and Schmittlein 2000). Moreover, the handful of studies that exist in search engine marketing have
typically analyzed publicly available data from search engines. Animesh, Ramachandran and Viswanathan (2006) look at the presence of quality uncertainty and adverse selection in paid search advertising on search engines. Goldfarb and Tucker (2007) examine the factors that drive variation in prices for advertising legal services on Google. In a paper related to our work, Rutz and Bucklin (2007) studied the conversion rates of hotel marketing keywords to analyze the profitability of different campaign management strategies.

To summarize, our research is distinct from extant online advertising research as it has largely been limited to the influence of banner advertisements on attitudes and behavior. We extend the literature by empirically comparing the impact of different keyword characteristics on the performance of online search advertising in paid search towards understanding the larger question of analyzing how keyword characteristics drive consumers’ search and purchase behavior, as well as firms’ optimal bid prices and ranking decisions.

Our paper is also related to the stream of work in cross-selling. Amongst the first papers that formally model sequential ordering and the cross-selling opportunities is Kamakura, Ramaswami and Srivastava (1991). Their research applies latent trait analysis to position financial services and investors along a common continuum. Knott, Hayes and Neslin (2002) present next product-to-purchase models that can be used to predict what is to be purchased next and when. Li, Sun and Wilcox (2005) model consumers’ sequential acquisition decisions for multiple products and services, a behavior that is common in service and consumer technology industries. We thus contribute to the literature by demonstrating the cross-selling potential of paid search advertising in an online context, thereby supplementing the existing stream of work on cross-selling.

3. Data

3.1 Data Description

We first describe the data generation process for paid search advertisement since it differs on many dimensions from traditional offline advertisement. Once the advertiser gets a rank allotted (based on the bid price) to display its textual ad, these sponsored ads show up on the top left, right and bottom of the computer screen in response to a query that a consumer types on the search engine. The textual ad typically consists of headline, a word or a limited number of words describing the product or service and a hyperlink that refers the consumer to the advertiser’s website after a click. The serving
of a text ad in response to a query for a certain keyword is denoted as an impression. If the consumer clicks on the ad, he is led to the landing page of the advertiser’s website. This is recorded as a click, and advertisers usually pay on a per click basis. In the event that the consumer ends up purchasing a product from the advertiser, this is recorded as a conversion. The time between a click and an actual purchase is known as latency. This is usually measured in days. In the majority of cases the value of this variable is 0, denoting that the consumer placed an order at the same time as when they landed on a firm’s website.

Our data contains weekly information on paid search advertising from a large nationwide retail chain, which advertises on Google. The data span all keyword advertisements by the company during a period of three months in the first quarter of 2007, specifically for the 13 calendar weeks from January 1 to March 31. Unlike most datasets used to investigate on-line environments which usually comprise of browsing behavior only, our data are unique in that we have individual level stimulus (advertising) and response (purchase incidence).

Each keyword in our data has a unique advertisement ID. The data consists of the number of impressions, number of clicks, the average cost per click (CPC) which represents the bid price, the rank of the keyword, the number of conversions, the total revenues from conversion and the average order value for a given keyword for a given week. While an impression often leads to a click, it may not lead to an actual purchase (defined as a conversion). The product of CPC and number of clicks gives the total costs to the firm for sponsoring a particular advertisement. Thus the difference in total revenues and total costs gives the total profits accruing to the retailer from advertising a given keyword in a given week.

Our dataset includes 5147 observations from a total of 1799 unique keywords that had at least one positive impression. Note that our main interest in this empirical investigation is to examine various factors that drive differences in click-throughs, conversions and transaction value during a purchase after conversion. Towards this, we proceed with two studies. In the first study presented in Section 4, we analyze click-through, conversion, bid price, and ranks based on the whole sample by jointly modeling the consumers’ search and purchase behavior, the advertiser’s bid pricing behavior, and the

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3 The firm is a large Fortune-500 retail store chain with several hundred retail stores in the US but due to the nature of the data sharing agreement between the firm and us, we are unable to reveal the name of the firm.
4 Note that not all keyword advertisements had a positive impression across all weeks.
search engine’s keyword rank allocating behavior. In the second study presented in Section 5, we use a subset of this data to study the impact of ranking, latency and keyword characteristics on consumer spending in the searched as well as in the non-searched categories.

Table 1a reports the summary statistics of our main dataset. As shown, the average weekly number of impressions is 383 for one keyword, among which around 33 lead to a click-through, and 0.48 lead to a purchase. Our data suggest the conversion rate conditional on a click-through (0.013) is almost twice as high as the click-through rate (0.008). Moreover, the average bid price is about 30 cents, and the average rank of these keywords is about 5.2. Finally, we have information on three important keyword characteristics, which we next briefly discuss with a focus on the rationale of analyzing them.

### 3.2 Keyword Characteristics

Prior work in computer science (Broder 2002, Jansen and Spink 2007) have analyzed the goals for users’ web searches and classified user queries in search engines into three classes: navigational (for example, searching for a specific firm or retailer), transactional (for example, searching for a specific product) or informational (for example, longer keywords). In recognition of these electronic marketplace realities, search engines not only sell non-branded generic keywords such as advertisements, but also well-known brand names that can be purchased by any third-party advertiser in order to attract consumers to its Web site. Hence, we focus on the three important keyword specific characteristics for a firm (the advertiser) when it advertises on a search engine. This includes whether the keyword has (i) retailer-specific information (for example, "Retailername", Retailer Name", RetailerName.com"), (ii) brand-specific information (for example, a product or manufacturer brand name), (iii) and the length (in words) of the keyword. As shown in Table 1a, about 5.7% of the keyword advertisements in our data include the retailer’s name, and approximately 40% include a brand name. By focusing on retailer and brand information in the keywords, we gain insights into the implications of searches coming from consumers who are aware of the advertiser and are likely to buy from the specific firm (Retailer-specific keywords) relative to those consumers who are aware of a nationally known product or manufacturer brand (brand specific keywords) and are likely to be more vulnerable to competition from other retailers. We discuss further implications in Section 6.

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5 For example, a consumer seeking to purchase a digital camera is as likely to search for a popular brand name such as NIKON, CANON or KODAK on a search engine as searching for the generic phrase “digital camera” on the same search engine. Similarly, the same consumer may search for a retailer such as “BEST BUY” or “CIRCUIT CITY” on the search engine.
The length of the keyword is also an important determinant of search and purchase behavior but anecdotal evidence on this varies across trade press reports. Some studies have shown that the percentage of searchers who use a combination of keywords is 1.6 times the percentage of those who use single-keyword queries (Kilpatrick 2003). In contrast, another study on data generated by ‘natural’ search listings found that single-keywords have on average the highest number of unique visitors (Oneupweb 2005). To investigate the impact of the length of a keyword, we constructed a variable that indicates the number of words in a keyword that a user queried for on the search engine (and in response to which the paid advertisement was displayed to the user). In our data, the average length of a keyword is about 2.6.

We enhanced the dataset by introducing keyword-specific characteristics such as Brand, Retailer and Length. For each keyword, we constructed two dummy variables, based on whether they were (i) branded or unbranded keywords and (ii) retailer-specific or non-retailer specific advertisements. To be precise, for creating the variable in (i) we looked for the presence of a brand name (either a product-specific or a company specific) in the keyword, and labeled the dummy as 1 or 0, with 1 indicating the presence of a brand name. For (ii), we looked for the presence of the advertising retailer’s name in the keyword, and then labeled the dummy as 1 or 0, with 1 indicating the presence of the retailer’s name.

4. A Simultaneous Model of Click-through, Conversion, Bid Price and Keyword Rank

We cast our model in a hierarchical Bayesian framework and estimate it using Markov chain Monte Carlo methods (see Rossi and Allenby 2003 for a detailed review of such models). We postulate that the decision of whether to click and purchase in a given week will be affected by the probability of advertising exposure (for example, through the rank of the keyword) and individual differences (both observed and unobserved). We simultaneously model consumers’ click-through and conversion behavior, the advertiser’s keyword pricing behavior, and the search engine’s keyword rank allocating behavior.

Assume for search keyword \( i \) at week \( j \), there are \( n_{ij} \) click-throughs among \( N_{ij} \) impressions (the number of times an advertisement is displayed by the retailer), where \( n_{ij} \leq N_{ij} \) and \( N_{ij} > 0 \). Suppose that among the \( n_{ij} \) click-throughs, there are \( m_{ij} \) click-throughs that lead to purchases, where \( m_{ij} \leq n_{ij} \). Let us further assume that the probability of having a click-through is \( p_{ij} \) and the probability of having a purchase
conditional on a click-through is \( q_{ij} \). In our model, a consumer faces decisions at two levels – one, when she sees a keyword advertisement, she makes decision whether or not to click it; two, if she clicks on the advertisement, she can take any one of the following two actions – make a purchase or not make a purchase.

Thus, there are three types of observations. First, a person clicked through and made a purchase. The probability of such an event is \( p_{ij} \). Second, a person clicked through but did not make a purchase. The probability of such an event is \( p_{ij}(1 - q_{ij}) \). Third, an impression did not lead to a click-through or purchase. The probability of such an event is \( 1 - p_{ij} \). Then, the probability of observing \((n_{ij}, m_{ij})\) is given by:

\[
f(n_{ij}, m_{ij}, p_{ij}, q_{ij}) = \frac{N_{ij}}{m_{ij}! (n_{ij} - m_{ij})! (N_{ij} - n_{ij})!} \left\{ p_{ij} q_{ij} \right\}^{m_{ij}} \left\{ p_{ij} (1 - q_{ij}) \right\}^{n_{ij} - m_{ij}} \left\{ 1 - p_{ij} \right\}^{N_{ij} - n_{ij}}
\]

(4.1)

### 4.1 Modeling the Consumer’s Decision: Click-through

The click-through probability is likely to be influenced by the position of the ad (\( \text{Rank} \)), how specific or broad the keyword is (\( \text{Length} \)), and whether it contains any retailer-specific (\( \text{Retailer} \)) or brand-specific information (\( \text{Brand} \)). Hence, in equation (4.1), \( p_{ij} \) the click-through probability is modeled as:

\[
p_{ij} = \frac{\exp(\beta_{i0} + \beta_{i1} \text{Rank}_{ij} + \alpha_{1} \text{Retailer}_{i} + \alpha_{2} \text{Brand}_{i} + \alpha_{3} \text{Length}_{i} + \varepsilon_{ij})}{1 + \exp(\beta_{i0} + \beta_{i1} \text{Rank}_{ij} + \alpha_{1} \text{Retailer}_{i} + \alpha_{2} \text{Brand}_{i} + \alpha_{3} \text{Length}_{i} + \varepsilon_{ij})}
\]

(4.2)

We capture the unobserved heterogeneity with a random coefficient on the intercept by allowing \( \beta_{i0} \) to vary along its population mean \( \bar{\beta}_0 \) as follows:

\[
\beta_{i0} = \bar{\beta}_0 + \xi_{i0}^\beta
\]

(4.3)

We also allow the rank coefficient of the \( i^{th} \) keyword to vary along the population mean \( \bar{\beta}_1 \) and the keywords’ characteristics as follows:

\[
\begin{align*}
\beta_{i1} &= \bar{\beta}_1 + \gamma_1 \text{Retailer}_{i} + \gamma_2 \text{Brand}_{i} + \gamma_3 \text{Length}_{i} + \xi_{i1}^\beta \\
\begin{bmatrix}
\xi_{i0}^\beta \\
\xi_{i1}^\beta
\end{bmatrix} &\sim \text{MVN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{11}^\beta & \Sigma_{12}^\beta \\ \Sigma_{21}^\beta & \Sigma_{22}^\beta \end{bmatrix}\right)
\end{align*}
\]

(4.5)
4.2 Modeling the Consumer’s Decision: Conversion

The click-through probability is likely to be influenced by the position of the ad (Rank), how specific or broad the keyword is (Length), and whether it contains any retailer-specific (Retailer) or brand-specific information (Brand). In addition, the click-through rate (CTR) will also have an impact on conversion rates. Hence, in equation (4.1), $q_{ij}$, the conversion probability is modeled as follows:

$$
q_{ij} = \frac{\exp(\theta_{i0} + \theta_{i1} \text{Rank}_{ij} + \delta_1 \text{CTR}_{ij} + \delta_2 \text{Retailer}_{ij} + \delta_3 \text{Brand}_{ij} + \delta_4 \text{Length}_{ij} + \eta_{ij})}{1 + \exp(\theta_{i0} + \theta_{i1} \text{Rank}_{ij} + \delta_1 \text{CTR}_{ij} + \delta_2 \text{Retailer}_{ij} + \delta_3 \text{Brand}_{ij} + \delta_4 \text{Length}_{ij} + \eta_{ij})}
$$

(4.6)

As before, we capture the unobserved heterogeneity with a random coefficient specified on both the intercept and the rank coefficient, as follows:

$$
\theta_{i0} = \bar{\theta}_0 + \varsigma_{i0}^\theta
$$

(4.7)

$$
\theta_{i1} = \bar{\theta}_1 + \kappa_i \text{Retailer}_{ij} + \kappa_j \text{Brand}_{ij} + \kappa_k \text{Length}_{ij} + \varsigma_{i1}^\theta
$$

(4.8)

$$
\begin{bmatrix}
\varsigma_{i0}^\theta \\
\varsigma_{i1}^\theta
\end{bmatrix} \sim MVN
\left(
\begin{bmatrix}
0 \\
0
\end{bmatrix},
\begin{bmatrix}
\Sigma_{11}^\theta & \Sigma_{12}^\theta \\
\Sigma_{21}^\theta & \Sigma_{22}^\theta
\end{bmatrix}
\right)
$$

(4.9)

Thus, equations (4.1) - (4.9) model the demand for a keyword, i.e. consumer’s decision.

4.3 Modeling the Advertiser’s Decision – Bid Price

Next, we model the advertiser’s (i.e., the firm’s) strategic behavior. The advertiser decides its bidding strategy in terms of how much to bid for each keyword at week $j$. Since the firm optimizes its advertising strategies based on learning from past performances, we take into account two types of learning. The first is the most naïve learning that involves bidding sufficiently high so as to secure a good rank. This kind of learning is based on the outcome from the keyword’s rank in the previous period. The second kind is the more sophisticated kind of learning that will be based on the keyword’s profit in the previous time period where profit is defined as revenues from sponsored search advertising minus the costs of placing that advertisement for the firm (the cost is equal to the total number of clicks times cost per click).\(^6\)

These learning mechanisms can be expressed as follows:

$$
\ln(\text{BidPrice}_{ij}) = \omega_{i0} + \omega_{i1} \text{Rank}_{i,j-1} + \omega_{i2} \text{Profit}_{i,j-1} + \lambda_i \text{Retailer}_{ij} + \lambda_j \text{Brand}_{ij} + \lambda_k \text{Length}_{ij} + \mu_{ij}
$$

(4.10)

$$
\omega_{i0} = \bar{\omega}_0 + \varsigma_{i0}^\omega
$$

(4.11)

\(^6\) To normalize the distribution of this variable, we took the log (Profit). Since the profit value can also be less than 0 in some cases, we took the absolute value of the profit, and then assigned the correct sign before the transformed value.
\[
\omega_{i1} = \omega_1 + \rho_{i1}Retailer_i + \rho_{i2}Brand_i + \rho_{i3}Length_i + \xi_{i1}^\alpha \\
\omega_{i2} = \omega_2 + \rho_{21}Retailer_i + \rho_{22}Brand_i + \rho_{23}Length_i + \xi_{i2}^\alpha
\] (4.12)

\[
\omega_{i1} = \omega_1 + \rho_{i1}Retailer_i + \rho_{i2}Brand_i + \rho_{i3}Length_i + \xi_{i1}^\alpha \\
\omega_{i2} = \omega_2 + \rho_{21}Retailer_i + \rho_{22}Brand_i + \rho_{23}Length_i + \xi_{i2}^\alpha
\] (4.13)

The error terms in equations (4.11) – (4.13) are distributed as follows:

\[
\begin{bmatrix}
\xi_{i0}^\alpha \\
\xi_{i1}^\alpha \\
\xi_{i2}^\alpha
\end{bmatrix} \sim MVN
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\Sigma_{i1}^\alpha & \Sigma_{i2}^\alpha & \Sigma_{i3}^\alpha \\
\Sigma_{i1}^\alpha & \Sigma_{i2}^\alpha & \Sigma_{i3}^\alpha \\
\Sigma_{i1}^\alpha & \Sigma_{i2}^\alpha & \Sigma_{i3}^\alpha
\end{bmatrix}
\] (4.14)

4.4 Modeling the Search Engine’s Decision – Keyword Rank

Next, we model the search engine’s strategic behavior. The search engine decides on the ranking of each search keyword base on the submitted bid price from the advertiser and its previous click-through rate.

\[
\ln(Rank_{ij}) = \phi_{i0} + \phi_{i1}BidPrice_{i,j-1} + \bar{\phi}_2CTR_{i,j-1} + \tau_1Retailer_i + \tau_2Brand_i + \tau_3Length_i + \nu_{ij}
\] (4.15)

\[
\phi_{i0} = \bar{\phi}_0 + \xi_{i0}^\phi
\] (4.16)

\[
\phi_i = \bar{\phi}_i + \pi_iRetailer_i + \pi_2Brand_i + \pi_3Length_i + \xi_i^\pi
\] (4.17)

The error terms in equations (4.16) and (4.17) are distributed as follows:

\[
\begin{bmatrix}
\xi_{i0}^\phi \\
\xi_{i1}^\phi
\end{bmatrix} \sim MVN
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\Sigma_{i1}^\phi & \Sigma_{i2}^\phi \\
\Sigma_{i1}^\phi & \Sigma_{i2}^\phi
\end{bmatrix}
\] (4.18)

Finally, to model the unobserved co-variation among click-through, conversions, bid price and the keyword ranking, we let the four error terms to be correlated in the following manner:

\[
\begin{bmatrix}
\xi_{ij} \\
\eta_{ij} \\
\mu_{ij} \\
\nu_{ij}
\end{bmatrix} \sim MVN
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\
\Omega_{12} & \Omega_{22} & \Omega_{23} & \Omega_{24} \\
\Omega_{13} & \Omega_{23} & \Omega_{33} & \Omega_{34} \\
\Omega_{14} & \Omega_{24} & \Omega_{34} & \Omega_{44}
\end{bmatrix}
\] (4.19)

A couple of clarifications are useful to note here. First, the three characteristics of a keyword (Retailer, Brand, Length) are all mean centered. This means that \( \bar{\beta}_i \) is the average effect of \( \beta_i \) in equation (4.4). A similar interpretation applies to the parameters \( \theta_{i1}, \omega_{i1}, \omega_{i2} \) and \( \phi_i \). Second, in equations (4.6) and (4.15), the coefficient of click-through rate (CTR) is modeled as a fixed effect rather as a random
coefficient in order to facilitate empirical identification. Due to the fact that we have a large number of observations with zero click-through rates, empirical identification is difficult if we were to model CTR with a random coefficient specification.

To ensure that the model is fully identified even with sparse data (data in which a large proportion of observations are zero), we conduct the following simulation. We picked a set of parameter values, and generated the number of click-throughs, the number of purchases, bid price, and ranking for each keyword, which mimicked their actual observed values in the data according to the model and the actual independent variables observed in our data. We then estimated the proposed model with the simulated dataset and found that we were able to recover the true parameter values. This relieves a potential concern on empirical identification of the model due to the sparseness of the data.

4.5 Results

Next, we discuss our empirical findings. We first discuss the effects of various keyword characteristics and keyword ranking on click-through rates of the sponsored search advertisements. The coefficient of Retailer in Table 2a, $\alpha_1$, is positive and significant indicating that keyword advertisements that contain retailer-specific information lead to a significant increase in click-through rates. Specifically, this is correspondent to 26.16% increase in click-through rates with the presence of retailer information. This result is useful for managers because it confirms that keyword advertisements that explicitly contain information identifying the advertiser lead to higher click-through rates than other kinds of keywords which lack such information.

= = Insert Tables 2a and 2b = =

On the other hand, the coefficient of Length in Table 2a is negative suggesting that longer keywords typically tend to experience lower click-through rates. Specifically, we find that all else equal an increase in the length of the keyword by one word decreases the click-through rates by 3.6%. Intuitively, this result has an interesting implication if one were to tie this result with those in the literature on consideration sets in marketing. A longer keyword typically tends to suggest a more ‘directed’ or ‘specific’ search whereas a shorter keyword typically suggests a more generic search. That is, the shorter the keyword is, the less information it likely carries and the larger context should be supplied to focus the search (Finkelstein et al. 2001). This implies that the consideration set for the consumer is likely to shrink as the search term becomes ‘narrower’ in scope. Danaher and Mullarkey (2003) show that user involvement during search (goal-directed versus surf mode) plays a crucial role
in the effectiveness of online banner ads. Since the consumers in our data get to see the ads displayed by all the retailers who are bidding for that keyword at the time of the search, the probability of a goal-directed consumer clicking on the retailer’s advertisement decreases unless the retailer carries the specific product that the consumer is searching for. In contrast, a consumer who does not have a goal-directed search (has a wider consideration set) and is in the surfing mode, is likely to click on several advertising links before she finds a product that induces a purchase.

Perhaps surprisingly, we find that the presence of a brand name in the search keyword (either a product-specific brand or a manufacturer-specific brand) has no statistically significant effect on click-through rates although it does affect the conversion rates (we discuss more on this later).

Some additional substantive results are exactly as expected. Rank has an overall negative relationship with CTR in Table 2a. This implies that lower the rank of the advertisement (i.e., higher the location of the sponsored ad on the computer screen), higher is the click-through rate. The position of the advertisement link on the search engine page clearly plays an important role in influencing click-through rates. This kind of primacy effect has also been seen in other empirical studies of the online world. Ansari and Mela (2003) suggested a positive relationship between the serial position of a link in an email and recipients' clicks on that link. Similarly, Drèze and Zufryden (2004) implied a positive relationship between a link's serial position and site visibility. Thus, ceteris paribus, website designers and online advertising managers would place their most desirable links toward the top of a web page or email and their least desirable links toward the bottom of the web page or email. Brooks (2004) showed that the higher the link’s placement in the results listing, the more likely a searcher is to select it. The study reports similar results with non-sponsored listings.

When we consider the interaction effect of these variables on the impact that Rank has on click-through rates, we find that keywords that contain retailer-specific or brand-specific information lead to an increase in the negative relationship between Rank and click-through rates. That is, for keywords that contain retailer-specific or brand-specific information, a lower rank (better placement) leads to even higher click-through rates. On the other hand, we find that the coefficient of Length is insignificant suggesting that longer keywords do not have any impact on the negative relationship between click-through rates and Rank.
As shown in Table 2b, the estimated unobserved heterogeneity covariance is significant including all of its elements. This suggests that the baseline click-through rates and the way that keyword ranking predicts the click-through rates are different across keywords, driven by unobserved factors beyond the three observed keyword characteristics.

Next consider Tables 3a and 3b with findings on conversion rates. Our analysis reveals that the coefficient of *Brand*, $\delta_2$, is positive and significant indicating that keywords that contain information specific to a brand (either product-specific or manufacturer-specific) experience higher conversion rates on an average. Specifically, the presence of brand information in the keyword increases conversion rates by 23.76%. This suggests that ‘branded’ keywords are indeed more valuable to an advertiser than ‘non-branded’ ones.

In contrast neither *Length*, nor *Retailer* is statistically significant in their overall effect on conversion rates. As expected, *Rank* has a negative relationship with conversion rates. Lower the *Rank* (i.e., higher the sponsored keyword on the screen), higher is the Conversion Rate. Also as expected, CTR has a positive relationship with conversion rates. Higher the CTR, higher the conversion rate. To be precise, an increase in click-through rate from 0 (min) to 1 (max) increases conversion by as much as 126.1% while a decrease in the rank from the maximum possible position or worst case scenario (which is 64 in our data) to the minimum position or best case scenario (which is 1 in our data) increases conversion by 99.8%. These analyses suggest that in terms of magnitude, the rank of a keyword on the search engine has a smaller impact on conversion rates than CTR.

When we consider the effect of these keyword characteristics on the impact of *Rank* on the conversion rate, we find that keywords that are specific to a brand do not have a statistically significant effect on the relationship between rank and conversion rates. However, keywords that have retailer information in them do moderate the relationship between Rank and conversion rate. The length of a keyword typically has no significant effect on the relationship between Conversion Rate and Rank. Recall that because we model the coefficient of CTR, $\bar{\delta}_2$, as a fixed effect for the empirical identification purpose, there are no coefficients for *Retailer*, *Brand* and *Length* in its case.
As shown in Table 3b, the estimated unobserved heterogeneity covariance is significant including all of its elements. This suggests that the baseline conversion rates and the way that keyword ranking predicts the click-through rates are different across keywords, driven by unobserved factors.

Next, we turn to firms’ behavior. Interestingly, the analysis of bid prices reveals that there is a negative relationship between Bid Price and Retailer as well as between Bid Price and Brand, whereas there is a positive relationship between Bid Price and Length. This implies that the firm places lower bids for advertisements that contain retailer or brand information and higher bids for those advertisements that are narrow in scope. Further, there is a negative relationship between Bid Price and Lag Rank as well Lag Profit. These results are indicative of the fact that while there is some naïve learning behavior exhibited by the firm, it is certainly not bidding optimally. Towards investigating the extent to which the firm is deviating from optimal bid prices, we conduct some policy simulations. These details are presented in Section 4.6.

Finally, on the analysis of Rank, we find that all three covariates—Retailer, Brand and Length have a statistically significant and negative relationship with Rank, suggesting that the search keywords that have retailer-specific information or brand-specific information or are more specific in their scope generally tend to have lower ranks (i.e., they are listed higher up on the screen).

How do search engines decide on the final rank? Anecdotal evidence and public disclosures by Google suggest that it incorporates a performance criterion along with bid price when determining the ranking of the advertisers. The advertiser in the top position might pay more per click than the advertiser in the second position, but there is no guarantee that it will be displayed in the first slot. This is because past performance such as click-through rates are factored in by Google before the final ranks are published. Like Google, MSN and Yahoo also decide on the final ranks based on both max bid price and previous click-through rate. The coefficients of Bid Price and Lag CTR are negative and statistically significant in our data. Thus, our results from the estimation of the Rank equation confirms that the search engine is indeed incorporating both bid prices and previous click-through rates in determining the final rank of a keyword. Note from Table 5a that the coefficient of Bid Price is more than twice the coefficient of Lag CTR, suggesting that bid price has a much larger role to play in determining the final rank.

= = Insert Tables 5a and 5b = =
Finally, it is worth noting in Table 6 that the unobserved covariance between (i) click-through propensity and bid price, (ii) between click-through propensity and keyword rank, and (iii) between conversion propensity and bid price all turn out to be statistically significant. This suggests that keyword ranking is endogenous and the firm’s bids are likely to be based on the same keyword’s past performance. Therefore, it is important to simultaneously model the consumer’s click-through and purchase behavior, and the advertiser’s and search engine’s decisions.

4.6 Policy Simulations

A primary goal of research in marketing is to evaluate and recommend optimal policies for marketing actions. Towards this, we estimate the optimal bid price for each keyword and assess how much the advertiser’s decision (actual bid price) deviates from the optimal bid price based on our model estimates. Using the parameter estimates from the click-through, conversion and rank models and data on CTR, conversion rates, revenues and actual bid price of each advertisement, we estimated the expected profit of the firm.

We assume the advertiser determines the optimal bid price for each keyword to maximize the expected profit ($\Pi$) from each consumer impression of the advertisement:

$$\Pi_{ij} = p_{ij} (q_{ij} r_{ij} - BidPrice_{ij})$$

In equation (4.20), $p_{ij}$ is the expected click through rate for keyword i at week j, $q_{ij}$ is the expected conversion rate conditional on a click through, $r_{ij}$ is the expected revenue from a conversion that is observed from our data, and $BidPrice_{ij}$ is the actual cost per click (bid price) paid by the advertiser to the search engine for each keyword. $p_{ij}$, $q_{ij}$ and $Rank_{ij}$ are predicted based on equations (4.2), (4.6) and (4.15) respectively, using the estimates obtained from the proposed model. Note that both the click-through rate $p_{ij}$ and the conversion rate $q_{ij}$ are functions of $Rank_{ij}$ which is a function of the $BidPrice_{ij}$.

We conduct the optimization routine to maximize the expected profit from each consumer impression of the advertisement for each keyword at each week, using the grid search. Our simulation results highlight that there is a considerable amount of difference in the optimal bid prices and the actual bid prices, with the average deviation being 23.3 cents per bid. In terms of bid prices, we find that a vast majority of the bids actually highlight that the firm is overbidding. Specifically, 6% of the bids are below the optimal bid prices with the average difference being 67 cents, while the remaining 94% of the bids are above the optimal bid price with the average difference being 28.7 cents. We also
examined the deviation from the optimal bid prices based on whether the advertisement had retailer or brand information. On an average, the firm was underbidding by 11.2 cents for each ad that had retailer information in it and was overbidding by 16.4 cents for each ad that had brand information in it. For those keywords that did not have retailer or brand information in them the firm was generally overbidding with the range going from 25.4 cents to 27.7 cents. These results are very intuitive: the lack of competition for retailer-specific keywords is likely to be driving the underbidding behavior while the presence of intense competition in branded or generic keywords would be driving the overbidding behavior.

Consequently, there is significant amount of divergence between optimal expected profits and actual profits accruing to the firm from their current bid prices, with the average difference being 1.14 times the expected profits with actual bid prices. Next we examined the sample based on overbidding or underbidding behavior. We found that the average difference in profits is 1.15 times the expected profits with actual bid prices when the firm is overbidding. When the firm is underbidding, the ratio is 1.05. Figure 1a and 1b highlight the differences from the use of optimal and actual bid prices.

In order to investigate how the three keyword level covariates are associated with optimal bid prices, we ran some OLS regressions with keyword level random effects. The dependent variable was the optimal bid price. Our analysis reveals that the presence of retailer-specific information (Retailer) or brand-specific (Brand) information leads to an increase in the optimal bid price, while longer keywords (Length) is associated with a lower optimal bid price. Specifically, the presence of retailer and brand information should lead to an increase in the optimal bid prices by 21.5% and 3.9%, respectively while an increase in the length of the keyword by one word should lead to a decrease in the bid price by 2.3%. Note that this is in contrast to the results from equation (4.10) wherein using actual bid prices we found that the firm is actually decreasing bid prices when it has either retailer or brand information in the keywords, and increasing bid prices for longer keywords.

To summarize, while the firm is exhibiting some learning behavior over time in terms of deciding on bid prices based on its rank and profit in the previous period, our simulations suggest that it can improve its profits dramatically by bidding optimally. Further, it would be better off by placing higher bids on keyword advertisement that either have retailer or brand information in them, and lower bids as keywords become longer. Moreover, we also find that expected profits from retailer-specific
keywords are likely to be much higher than those from brand-specific keywords. We discuss the implications of these findings in Section 6.

5. Empirical Analysis: Impact of Sponsored Search Advertisement on Cross-Selling

In this section, we investigate the impact of sponsored search advertising in a given category on consumer’s propensity to buy products across other categories. Our dataset has detailed information on the various categories of products that were eventually purchased by consumers after they had clicked on any given paid advertisement. There are six product categories in our data: bath, bedding, electrical appliances, home décor, kitchen and dining. Due to the confidentiality agreement with the firm that gave us the data, we are not able to reveal any more details about the individual products within these categories. Since, our analysis is about the cross-selling potential of a given product-based advertisement, we exclude advertisements that only have the retailer information in them but no product information. Hence, we focus on the 801 observations from 166 keywords that have some product or product category information imbedded in them. Table 1b reports the summary statistics of the data. As shown, the average spending is 79 dollars on the searched product category, and 21.8 dollars on the non-searched product category. The average latency is about a day. These statistics provide some evidence suggesting that keyword advertising can lead to purchases on a non-searched product category, and consumers may wait for a while after starting the search to complete an order.

![Insert Table 1b = =](image)

Each order can lead to a purchase from the searched product category and/or from any of the other five non-searched product categories. We model the consumer purchase behavior as a two-stage decision process. In the first stage, the consumer decides on how much to spend on the searched product category. We adopt the Tobit model specification to account for a large number of zeros in consumer spending on either the searched product category or non-searched product categories. Let’s denote $y_{ij}^{own}$ as the money spent on the searched product category in order $j$ for the searched keyword $i$. We assume there is latent spending intention ($z_{ij}^{own}$) that determines how much to spend on the searched product category, that is,

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7 In the estimation, both $y_{ij}^{own}$ and $y_{ij}^{cross}$ are rescaled by dividing the actual amount by 10.
\[ y_{ij}^{own} = z_{ij}^{own} \quad \text{if } z_{ij}^{own} > 0 \] (5.1)
\[ y_{ij}^{own} = 0 \quad \text{if } z_{ij}^{own} \leq 0 \] (5.2)

We model the latent buying intention of the searched category as:
\[ z_{ij}^{own} = \alpha_{ij}^{own} + \sum_{k=1}^{K-1} y_{k}^{own} \text{Search}_{ik} + \beta_1^{own} \text{Latency}_{ij} + \beta_2^{own} \text{Rank}_{ij} + \beta_3^{own} \text{Brand}_{i} + \beta_4^{own} \text{Length}_{i} + \epsilon_{ij}^{own} \] (5.3)

where \( \text{Search}_{ik} = 1 \) if the searched category is the \( k^{th} \) product category for keyword \( i \), and \( \text{Search}_{ik} = 0 \) if the searched category is not the \( k^{th} \) product category for keyword \( i \). \( \text{Latency}_{ij} \) is the time duration in number of days between the search and the order \( j \) for keyword \( i \). \( \text{Rank}_{ij} \) is the average rank of keyword \( i \) for order \( j \). \( \text{Brand}_{i} \) is a dummy variable indicating whether a brand name is included in the search keyword \( i \). \( \text{Length}_{i} \) is the number of words included in the search keywords \( i \).

We have a total of 6 product categories, that is, \( K=6 \) and without loss of generality, we use category 6 as the baseline. To complete the model specification, we assume the following distributions regarding the error term and intercept term:
\[ \epsilon_{ij}^{own} \sim N(0, \sigma_{own}^2) \] (5.4)
\[ \alpha_{i}^{own} \sim N(\alpha_{own}, \tau_{own}^2) \] (5.5)

In the second stage, the consumer decides on how much to spend on the non-searched product categories in total conditional on the spending on the searched product category. Let’s denote \( y_{ij}^{cross} \) as the money spent on the non-searched product category in order \( j \) for the searched keyword \( i \). We assume there is latent spending intention \( (z_{ij}^{cross}) \) that determines how much to spend on the non-searched product category, that is,
\[ y_{ij}^{cross} = z_{ij}^{cross} \quad \text{if } z_{ij}^{cross} > 0 \] (5.6)
\[ y_{ij}^{cross} = 0 \quad \text{if } z_{ij}^{cross} \leq 0 \] (5.7)

We model the latent buying intention of the non-searched category as follows:
To complete the model specification, we assume the following distributions regarding the error term and intercept term:

\[ \varepsilon_{ij}^{cross} \sim N(0, \sigma^{2}_{cross}) \]  
\[ \alpha_{i}^{cross} \sim N(\alpha^{cross}, \tau^{2}_{cross}) \]  

Equations (5.1) – (5.3), and (5.6) – (5.8) lead to a non-linear fully non-recursive simultaneous equations model. Note that \( \gamma_{k}^{own}, \gamma_{k}^{cross} \) as well as \( \beta_{1}^{own} - \beta_{3}^{own} \) are modeled as fixed effects due to the empirical identification with our data.

### 5.1 Results

We next discuss the findings from our analysis. In table 7a, the coefficient, \( \gamma_{1}^{own} \) is negative and significant suggesting that consumer average spending on the searched category is lower in category 1 than category 6. On the other hand, the coefficient, \( \gamma_{2}^{own} \) is positive and significant suggesting that the consumer average spending on the searched category is higher in category 2 than category 6. The coefficients, \( \gamma_{3}^{own}, \gamma_{4}^{own}, \) and \( \gamma_{5}^{own} \) are statistically insignificant suggesting that on an average, and consumers spend the same amount in each of these categories (3, 4 and 5) as they do in category 6 when they search for a product in each of these categories.

What are the main factors that affect this kind of consumer behavior? Based on the estimates in Table 7a and 7b, we find that \textit{Latency} tends to decrease consumer spending on the searched category, but increase their average spending on the non-searched category. Recall that latency is the time between when consumers click on an advertisement and when they actual purchase the product from the website. Intuitively, this result suggests that if consumers delay the final purchase of the product after the initial click on the ad, they are likely to digress from their original spending intention in the searched category and increasing their purchase of products in other non-searched categories. Note also that the coefficient of \( \gamma^{own} \) is negative suggesting that if a consumer has already spent a lot on the category that they had originally searched for, then they are likely to spend less on the other categories.

\[ = = \text{Insert Tables 7a and 7b} = = \]
Interestingly, we find that the presence of Brand information in the search keyword advertisement does not affect the amount that consumers spend on the category that they originally searched for on the search engine. However, note from Table 7b that it does significantly increase consumers’ spending in the other categories. This implies that the presence of a brand name in a keyword advertisement can have a strong switching effect on consumer’s purchasing propensities. It has a similar flavor to the bait and switch strategies used by retailers, when they attract consumers to their stores based advertisements in one category and then induce them to buy a product in a different category addition to the original product, perhaps through some marketing promotion. Thus, our analysis indicates a strong cross-selling potential of a sponsored search advertisement that contains a brand name in it. The statistically significant estimates of $\gamma_{1}^{\text{cross}}$, $\gamma_{2}^{\text{cross}}$, and $\gamma_{3}^{\text{cross}}$ in Table 7b indicate that there are complementary demands for three product categories at each purchase incidence. In particular, we see in Table 7b that categories 1, 2, and 3 (bath, bedding and electrical appliances) exhibit the strongest opportunities for cross-selling.\(^8\)

We find that neither Rank nor the Length has any impact on consumers’ spending either on the searched category or the non-searched category. This is not too surprising. Both these attributes are likely to influence consumer click-through behavior but are unlikely to affect their latent spending intention once they have already landed on the retailer’s web page. As a robustness check, we also fit a model that controls for the potential endogeneity in Rank. We found similar results on the coefficient estimates. We also included dummies for different categories of landing pages such as search page, shop, home page, information page, product page and category page. This did not affect the qualitative nature of the results, and moreover the estimates on the dummies were not statistically significant.

6. Managerial Implications and Conclusion

The phenomenon of sponsored search advertising is gaining ground as the largest source of revenues for search engines. However, we have little understanding of how consumers respond to sponsored search advertising on the Internet, and how what factors drive firms’ decision on bid prices and ranks. In this research, we focus on understanding how sponsored search advertising affects consumer

\(^{8}\)Note that our model can only capture the contemporaneous complementary relationship among products on the same purchase occasion. We do not have sufficient information to discuss the exact acquisition sequence amongst categories.
search and purchasing patterns on the Internet. Specifically, we focus on analyzing the impact of different keyword level covariates on different metrics of sponsored search advertisement performance taking both consumer and firm behavior into account. Finally, we analyze the cross-selling potential from sponsored search advertising.

Using a unique panel dataset of several hundred keywords collected from a nationwide retailer that advertises on Google, we empirically model the relationship between different metrics such as click-through rates, conversion rates and keyword ranks. We use a Hierarchical Bayesian modeling framework and estimate the model using Markov Chain Monte Carlo (MCMC) methods. We began our research with an investigation of how keyword specific characteristics affect click-through rates, conversion rates and ranks, and found considerable differences across keywords. Since the ultimate aim of sponsored search advertisement is to increase demand, we also aim to analyze the profitability of such ads using different metrics of performance. Towards this, we compare the cross-selling potential of keywords across different categories in paid search advertisement. Our data reveals that there is a considerable amount of heterogeneity in terms of the revenues that accrue from different keywords as well as significant differences in the performance metrics.

Arguably, the mix of retailer-specific and brand-specific keywords in an online advertiser's portfolio has some analogies to other kinds of marketing mix decisions faced by firms in many markets. For instance, typically it is the retailer who engages in 'retail store' advertising that has a relatively 'monopolistic' market. In contrast, typically it is the manufacturer who engages in advertising 'national-brands'. From the retailer's perspective, these advertisements are likely to be relatively more 'competitive' since national brands are likely to be stocked by its competitors too. Retailer-name searches are navigational searches, and are analogous to a customer finding the retailer's phone number or address in the White Pages. These searches are driven by brand awareness generated by catalog mailings, TV ads, etc, and are likely to have come from more 'loyal' consumers. Even though the referral to the retailer’s website came through a search engine, the search engine had very little to do with generating the demand in the first place. On the other hand, searches on product or manufacturer specific brand names are analogous to consumers going to the Yellow Pages—they know they need a product or service, but don't yet know where to buy it (Kaufman 2007). These are likely to be “competitive” searches. Even for loyal buyers, a “branded” search means the searcher is surveying the market and is vulnerable to competition. If the advertiser wins the click and the order, that implies they have taken market share away from a competitor. Thus, retailer-specific keywords
are likely to be searched and clicked by 'loyal' consumers who are inclined towards buying from that retailer whereas brand-specific keywords are likely to be searched and clicked by the 'shoppers or searchers' who can easily switch to competition. Our policy simulations show that average profitability from conversions generated by 'retailer' keywords is much higher than that from 'brand' keywords. Our results thus provide some managerial insights for an advertiser of sponsoring such retail store keywords (retailer-specific keywords) with national-brand keywords (brand-specific keywords).

Most firms who sponsor online keyword advertisements set a daily budget, select a set of keywords, determine a bid price for each keyword, and designate an ad associated with each selected keyword. If the company’s spending has exceeded its daily budget, however, its ads will not be displayed. With millions of available keywords and a highly uncertain click-through rate associated with the ad for each keyword, identifying the most profitable set of keywords given the daily budget constraint becomes challenging for companies wishing to promote their goods and services via search-based advertising (Rusmevichientong and Williamson 2006). In this regard, our analysis reveals that while retailer-specific information is more important than brand-specific information in predicting click-through rates, the opposite holds true in predicting conversion rates. Sponsored advertisements that contain retailer or brand information, or are more specific in their scope generally tend to have lower ranks (i.e., they are listed higher up on the screen). Since the search engine accounts for both bid price and previous click-through rates in deciding on the final rank, these results can have useful implications for a firm’s Internet paid search advertising strategy by shedding light on what the most “attractive” keywords from a firm’s perspective are, and how it should optimally bid in search engine advertising campaigns. The analysis of these keyword attributes on conversion rates also provide insights into what kind of keyword advertisers should bid on in the event that search engines migrate from a pay-per-click model to a pay-per-action model as Google has recently claimed it will do.

Finally, we have shown some evidence that although the average click-through and conversion rates are typically very low in sponsored search, there are other benefits from such advertising. Specifically, retailers can not only refine their keyword purchases on search engines, but also set up relevant cross-selling opportunities on their own websites by advertising ‘brand-specific’ keywords. The strategy is that when a consumer searches for a specific product and lands deep within the retailer’s website by clicking on its keyword advertisement, the retailer can pair that product with other products that sell well with that keyword and prominently feature them on its website. This provides a retailer with an opportunity to not only convert someone on the product they had searched for, but also get other
opportunities for cross-selling in a sponsored search environments. From the retailer’s perspective, there could be synergies in promoting both categories simultaneously rather than separately. Indeed anecdotal evidence suggests that retailers are engaging in the practice of looking up the most-searched and the top-converting keywords on their websites, and bidding for them on search engines. They are taking cross-selling reports from other marketing mix campaigns and putting up the top cross-selling product for the searched product on the same page (Squire 2003). Consumers who display high cross-selling potential in paid search advertising can also be targeted with coupons customized to induce such bundled purchases, not only in the online world but also in the offline world. This becomes important in light of the fact that 79% of users who search on Google end up purchasing offline at a retail store location.9

Interestingly, we find that latency in purchases is not necessarily detrimental for a firm that is sponsoring the ad. While it is in general associated with a reduction the purchases of the category that the consumer was searching for, it increases consumers’ spending in other product categories. In a way, it has an impact similar to a bait and switch strategy. This effect is particularly strong in keywords that have a brand name in it, since consumers who click on branded keywords typically tend to spend more on other categories than the one they were originally searching for. Thus, online advertisers can focus on investing more often in such keywords relative to the generic keywords, especially if the cannibalization effect of drawing out consumers from one category is smaller relative to revenue expansion effect. From the point of view of the manufacturer, such dependencies across categories may be exploited by running cooperative promotions within brands but across categories. Of course, such decisions would need a detailed profitability analysis based not only on the potential from cross-selling in other product categories but also the performance of the keyword in its own category.

To conclude, our paper is the first known empirical study that estimates the effect of sponsored search advertising at a keyword level on consumer search and purchase behavior in electronic markets by empirically estimating the impact of keyword attributes on consumer actions. We also analyze the impact of these covariates on the decisions of the firms involved in the sponsored advertising process—the bid price of the advertiser and the rank allotted by the search engine to the advertiser. We conduct simulations to assess the relative profit impact from changes in bid prices, and find that despite some learning, the advertiser is not bidding optimally. Our findings also confirm the opinions

9 2005 Home and Garden Survey, conducted by Media-Screen and GMI (April 2005).
postulated by the popular press that search engines factor in both the bid price of the advertiser as well as the performance metrics such as prior click-through rates before allotting the final rank to a given ad. Finally, using data on product-level variables, we have demonstrated that there exists significant potential for cross-selling through search keyword advertisements.

Our paper has several limitations. These limitations arise primarily from the lack of information in our data. For example, we do not have data on competition. That is, we do not know the keyword ranks or other performance metrics such as click-through rates and conversion rates of the keyword advertisements of the competitors of the firm whose data we have used in this paper. Further, we do not have sufficient data to estimate category specific cross-selling effects. Using larger datasets, future work can investigate the extent of cross-selling by product category in order to predict what is likely to be purchased next and when (for example, in Knott, Hayes and Neslin 2002). Further, we do not have any knowledge of other information that was mentioned in the textual description in the space following a paid advertisement during consumers’ queries. Future work could integrate that information with our modeling approach to have more precise estimates. We hope that this study will generate further interest in exploring this important emerging area in marketing.
References


Appendix: The MCMC Algorithm

We ran the MCMC chain for 40,000 iterations, and used the last 20,000 iterations to compute the mean and standard deviation of the posterior distribution of the model parameters, in both applications presented in the paper. Due to space constraint, we only report below the MCMC algorithm for the simultaneous model of click-through rate, conversion rate, bid price and keyword rank. The MCMC algorithm for the cross-selling model is available from the authors upon request.

1. Draw \( c_{ij}^p \) and \( c_{ij}^q \)

As specified, the likelihood function of the number of clicks \( (n_{ij}) \) and number of purchases \( (m_{ij}) \) is

\[
 l(c_{ij}^p, c_{ij}^q | n_{ij}, m_{ij}) \propto \{p_y q_y \}^{n_{ij}} \{p_y (1-q_y)\}^{n_{ij} - n_y} \{1 - p_y\}^{n_y - n_{ij}}
\]

where

\[
 p_y = \frac{\exp(c_{ij}^p)}{1 + \exp(c_{ij}^p)}, \quad q_y = \frac{\exp(\lambda_y)}{1 + \exp(\lambda_y)}.
\]

\[
 c_{ij}^p = m^p_{ij} + \epsilon_y, \quad m^p_{ij} = \beta_{i0} + \beta_{i1} \text{Rank}_{ij} + \alpha_1 \text{Retailer}_{ij} + \alpha_2 \text{Brand}_{ij} + \alpha_3 \text{Length}_{ij},
\]

\[
 c_{ij}^q = m^q_{ij} + \eta_y, \quad m^q_{ij} = \theta_{i0} + \theta_{i1} \text{Rank}_{ij} + \theta_{i2} \text{CTR}_{ij} + \delta_1 \text{Retailer}_{ij} + \delta_2 \text{Brand}_{ij} + \delta_3 \text{Length}_{ij}.
\]

We further define the following notations:

\[
 D = \Omega^*_1 - \Omega^*_2 \Omega^*_2 \Omega^*_1 \Omega^*_2
\]

\[
 \Omega^*_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}, \quad \Omega^*_2 = \begin{bmatrix} \Omega_{33} & \Omega_{34} \\ \Omega_{43} & \Omega_{44} \end{bmatrix}, \quad \Omega^*_2 \Omega^*_1 = \begin{bmatrix} \Omega_{13} & \Omega_{14} \\ \Omega_{23} & \Omega_{24} \end{bmatrix}
\]

\[
 u_{ij} = \ln(\text{BidPrice}_{ij}) - (\omega_{i0} + \omega_{i1} \text{Rank}_{i,j-1} + \omega_{i2} \text{Profit}_{i,j-1} + \lambda_1 \text{Retailer}_{ij} + \lambda_2 \text{Brand}_{ij} + \lambda_3 \text{Length}_{ij})
\]

\[
 u_{ij} = \ln(\text{CTR}_{ij}) - (\phi_{i0} + \phi_{i1} \text{BidPrice}_{i,j-1} + \phi_{i2} \text{CTR}_{i,j-1} + \tau_1 \text{Retailer}_{ij} + \tau_2 \text{Brand}_{ij} + \tau_3 \text{Length}_{ij})
\]

\[
 E_{ij} = \Omega^*_2 \Omega^*_2 \Omega^*_1 \Omega^*_2
\]

We use Metropolis-Hastings algorithm with a random walk chain to generate draws of \( c_{ij} = (c_{ij}^p, c_{ij}^q) \)

(see Chib and Greenberg 1995, p330, method 1). Let \( c_{ij}^{(p)} \) denote the previous draw, and then the next draw \( c_{ij}^{(n)} \) is given by:

\[
 c_{ij}^{(n)} = c_{ij}^{(p)} + \Delta
\]

with the accepting probability \( \alpha \) given by:

\[
 \min \left[ \frac{\exp[-1/2(c_{ij}^{(n)} - m_{ij} - E_{ij})' \Omega^{-1} c_{ij}^{(n)} - m_{ij} - E_{ij})]}{\exp[-1/2(c_{ij}^{(p)} - m_{ij} - E_{ij})' \Omega^{-1} c_{ij}^{(p)} - m_{ij} - E_{ij})]} \right]^{1/2} \Omega^{1/2}
\]

\( \Delta \) is a draw from the density Normal(0, 0.015I) where I is the identity matrix.
2. Draw $b_i = [\beta_i', \theta_i', \omega_i', \phi_i']'$

$y_{g_1} = c_g^\rho - (\alpha_i:\text{Retailer}_i + \alpha_2:\text{Brand}_i + \alpha_3:\text{Length}_i)$

$y_{g_2} = c_g^\rho - (\overline{\delta}_1:\text{CTR}_i + \delta_2:\text{Retailer}_i + \delta_3:\text{Brand}_i + \delta_4:\text{Length}_i)$

$y_{g_3} = \ln(\text{BidPrice}_{i,j}) - (\lambda_1:\text{Retailer}_i + \lambda_2:\text{Brand}_i + \lambda_3:\text{Length}_i)$

$y_{g_4} = \ln(\text{Rank}_{i,j}) - (\overline{\phi}_2:\text{CTR}_{i,j-1} + \tau_1:\text{Retailer}_i + \tau_2:\text{Brand}_i + \tau_3:\text{Length}_i)$

$y_{xy} = \begin{bmatrix} x_{y1}' & 0 & 0 & 0 \\ 0 & x_{y2}' & 0 & 0 \\ 0 & 0 & x_{y3}' & 0 \\ 0 & 0 & 0 & x_{y4}' \end{bmatrix}$, $\Sigma = \begin{bmatrix} \Sigma^\beta & 0 & 0 & 0 \\ 0 & \Sigma^\theta & 0 & 0 \\ 0 & 0 & \Sigma^\omega & 0 \\ 0 & 0 & 0 & \Sigma^\phi \end{bmatrix}$

$x_{y1} = x_{y2} = [1, \text{Rank}_{i,j}]$, $x_{y3} = [1, \text{Rank}_{i,j-1}, \text{Profit}_{i,j-1}]$, $x_{y4} = [1, \text{Profit}_{i,j-1}]$

$\overline{b}_1 = \beta_0$, $\overline{b}_2 = \beta_1 + \gamma_1:\text{Retailer}_i + \gamma_2:\text{Brand}_i + \gamma_3:\text{Length}_i$

$\overline{b}_3 = \overline{\theta}_0$, $\overline{b}_4 = \overline{\theta}_1 + \kappa_1:\text{Retailer}_i + \kappa_2:\text{Brand}_i + \kappa_3:\text{Length}_i$

$\overline{b}_5 = \omega_0$, $\overline{b}_6 = \omega_1 + \rho_1:\text{Retailer}_i + \rho_2:\text{Brand}_i + \rho_3:\text{Length}_i$

$\overline{b}_7 = \omega_2 + \rho_{11}:\text{Retailer}_i + \rho_{22}:\text{Brand}_i + \rho_{23}:\text{Length}_i$

$\overline{b}_8 = \overline{\phi}_0$, $\overline{b}_9 = \overline{\phi}_1 + \pi_1:\text{Retailer}_i + \pi_2:\text{Brand}_i + \pi_3:\text{Length}_i$

Then $b_i \sim \text{MVN}(A_i, B_i)$

$B_i = [(x_i'\Omega_i^{-1}x_i)^{-1} + \Sigma^{-1}]^{-1}$, $A_i = B_i[x_i'\Omega_i^{-1}y_i + \Sigma^{-1}b_i]$

3. Draw $a = [\alpha', \overline{\delta}', \lambda', \overline{\phi}', \tau']'$

$y_{g_1} = c_g^\rho - (\overline{\beta}_0 + \overline{\beta}_1:\text{Rank}_{i,j})$

$y_{g_2} = c_g^\rho - (\overline{\theta}_0 + \overline{\theta}_1:\text{Rank}_{i,j})$

$y_{g_3} = \ln(\text{BidPrice}_{i,j}) - (\overline{\omega}_0 + \omega_1:\text{Rank}_{i,j-1} + \omega_2:\text{Profit}_{i,j-1})$

$y_{g_4} = \ln(\text{Rank}_{i,j}) - (\overline{\phi}_0 + \overline{\phi}_1:\text{BidPrice}_{i,j-1})$

$x_{y} = \begin{bmatrix} x_{y1}' & 0 & 0 & 0 \\ 0 & x_{y2}' & 0 & 0 \\ 0 & 0 & x_{y3}' & 0 \\ 0 & 0 & 0 & x_{y4}' \end{bmatrix}$

$x_{y1} = x_{y2} = [\text{Retailer}_i, \text{Brand}_i, \text{Length}_i]$, $x_{y3} = x_{y4} = [\text{CTR}_{i,j-1}, \text{Retailer}_i, \text{Brand}_i, \text{Length}_i]$

$\overline{a} = 0_{8 \times 1}$, $\Sigma_0 = 100I$

Then $a \sim \text{MVN}(A, B)$

$B = [(X'\Omega^{-1}X)^{-1} + \Sigma^{-1}]^{-1}$, $A = B[X'\Omega^{-1}Y + \Sigma^{-1}a_0]$
4. Draw $\Omega$

$$y_{ij1} = c_{ij}^\nu - (\beta_{i0} + \beta_{ij} \text{Rank}_{ij} + \alpha_i \text{Retailer}_i + \alpha_2 \text{Brand}_i + \alpha_3 \text{Length}_i)$$

$$y_{ij2} = c_{ij}^\nu - (\theta_{i0} + \theta_{ij} \text{Rank}_{ij} + \theta_2 \text{CTR}_{ij} + \theta_3 \text{Retailer}_i + \delta_2 \text{Brand}_i + \delta_3 \text{Length}_i)$$

$$y_{ij3} = \ln(\text{BidPrice}_{ij}) - (\omega_{i0} + \omega_i \text{Rank}_{i,j-1} + \omega_{i2} \text{Profit}_{i,j-1} + \lambda_i \text{Retailer}_i + \lambda_2 \text{Brand}_i + \lambda_3 \text{Length}_i)$$

$$y_{ij4} = \ln(\text{Rank}_{ij}) - (\phi_{i0} + \phi_i \text{BidPrice}_{i,j-1} + \phi_2 \text{CTR}_{i,j-1} + \tau_i \text{Retailer}_i + \tau_2 \text{Brand}_i + \tau_3 \text{Length}_i)$$

$$\Omega \sim IW \left( \sum_i \sum_j y_{ij} \cdot y_{ij} + Q_0, N + q_0 \right); \quad Q_0 = 10I \text{ and } q_0 = 10; N = \# \text{ of observations}$$

5. Draw $\Sigma^\beta$, $\Sigma^\theta$, and $\Sigma^\omega$

$$\Sigma^\beta \sim IW \left( \sum_i (\beta_i - \bar{\beta})' (\beta_i - \bar{\beta}) + Q_0, N + q_0 \right); \quad Q_0 = 10I \text{ and } q_0 = 10; n = \# \text{ of keywords}$$

$$\Sigma^\theta \sim IW \left( \sum_i (\theta_i - \bar{\theta})' (\theta_i - \bar{\theta}) + Q_0, N + q_0 \right); \quad Q_0 = 10I \text{ and } q_0 = 10; n = \# \text{ of keywords}$$

$$\Sigma^\omega \sim IW \left( \sum_i (\omega_i - \bar{\omega})' (\omega_i - \bar{\omega}) + Q_0, N + q_0 \right); \quad Q_0 = 10I \text{ and } q_0 = 10; n = \# \text{ of keywords}$$

where IW stands for the Inverted Wishart Distribution.

6. Draw $f_1 = [\overline{\beta}_0, \overline{\beta}_1, \gamma_1, \gamma_2, \gamma_3]'$

$$x_i = \begin{bmatrix} \overline{\beta}_0 & 0 & 0 & 0 \\ 0 & \overline{\beta}_1 & \gamma_1 & \gamma_2 \\ \tilde{a} = 0_{5 \times 1}, \quad \Sigma_0 = 100I \\ \text{Then} \quad f_1 \sim MVN(A, B) \implies B = [(X'\Sigma^{-1}X)^{-1} + \Sigma_0^{-1}]^{-1}, \quad A = B[X'\Sigma^{-1}\beta + \Sigma_0^{-1}\tilde{a}_0]$$

7. Draw $f_2 = [\overline{\theta}_0, \overline{\theta}_1, \kappa_1, \kappa_2, \kappa_3]'$ similar to step 6

8. Draw $f_3 = [\overline{\omega}_0, \overline{\omega}_1, \rho_{11}, \rho_{12}, \rho_{13}, \overline{\omega}_2, \rho_{21}, \rho_{22}, \rho_{23}]'$ similar to step 6

9. Draw $f_4 = [\overline{\phi}_0, \overline{\phi}_1, \pi_1, \pi_2, \pi_3]'$ similar to step 6
### Table 1a: Summary Statistics of the Paid Search Data (N=5147)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impressions</td>
<td>383.376</td>
<td>2082.086</td>
<td>1</td>
<td>97424</td>
</tr>
<tr>
<td>Clicks</td>
<td>32.915</td>
<td>519.555</td>
<td>0</td>
<td>33330</td>
</tr>
<tr>
<td>Orders</td>
<td>0.483</td>
<td>8.212</td>
<td>0</td>
<td>527</td>
</tr>
<tr>
<td>Click-through Rate (CTR)</td>
<td>0.008</td>
<td>0.059</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Conversion Rate</td>
<td>0.013</td>
<td>0.073</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Bid Price</td>
<td>0.294</td>
<td>0.173</td>
<td>0.005</td>
<td>1.410</td>
</tr>
<tr>
<td>Lag Rank</td>
<td>4.851</td>
<td>6.394</td>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>Log (Lag Profit)</td>
<td>0.106</td>
<td>1.748</td>
<td>-5.160</td>
<td>10.710</td>
</tr>
<tr>
<td>Rank</td>
<td>5.179</td>
<td>7.112</td>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>Lag CTR</td>
<td>0.007</td>
<td>0.053</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Retailer</td>
<td>0.057</td>
<td>0.232</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Brand</td>
<td>0.398</td>
<td>0.490</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Length</td>
<td>2.588</td>
<td>0.734</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

### Table 1b: Summary Statistics of the Cross-Selling Data (N=801)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order Value – Own ($)</td>
<td>79.007</td>
<td>100.812</td>
<td>0</td>
<td>930</td>
</tr>
<tr>
<td>Order Value – Cross ($)</td>
<td>21.805</td>
<td>78.534</td>
<td>0</td>
<td>1249</td>
</tr>
<tr>
<td>Latency</td>
<td>1.062</td>
<td>3.527</td>
<td>0</td>
<td>29</td>
</tr>
<tr>
<td>Rank</td>
<td>1.257</td>
<td>1.999</td>
<td>1</td>
<td>40.25</td>
</tr>
<tr>
<td>Brand</td>
<td>0.883</td>
<td>0.322</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Length</td>
<td>2.410</td>
<td>0.956</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 2a: Coefficient Estimates on Click-through Rate

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Retailer</th>
<th>Brand</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$\beta_0$</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
</tr>
<tr>
<td></td>
<td>-2.062</td>
<td>2.031</td>
<td>-0.105</td>
<td>-0.109</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.155)</td>
<td>(0.090)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>(\bar{\beta}_1)</td>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
<td>$\gamma_3$</td>
<td></td>
</tr>
<tr>
<td>Rank</td>
<td>-0.251</td>
<td>-0.251</td>
<td>-0.056</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.061)</td>
<td>(0.022)</td>
<td>(0.014)</td>
</tr>
</tbody>
</table>

Table 2b: Unobserved Heterogeneity Estimates in the Click-through Model (\(\Sigma^\beta\))

<table>
<thead>
<tr>
<th>(\beta_{i0}) (Intercept)</th>
<th>(\beta_{i1}) (Rank)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_{i0}) (Intercept)</td>
<td>0.905</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
</tr>
<tr>
<td>(\beta_{i1}) (Rank)</td>
<td>-0.085</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
</tr>
</tbody>
</table>

Note: Posterior means and posterior standard deviations (in the parenthesis) are reported, and estimates that are significant at 95% are bolded in Tables 2a - 7.
Table 3a: Coefficient Estimates on Conversion Rate

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Retailer</th>
<th>Brand</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-4.812</td>
<td>-0.481</td>
<td>0.469</td>
<td>-0.130</td>
</tr>
<tr>
<td></td>
<td>(0.213)</td>
<td>(0.339)</td>
<td>(0.138)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Rank</td>
<td>-0.099</td>
<td>0.293</td>
<td>0.049</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.106)</td>
<td>(0.035)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>CTR</td>
<td>0.822</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.368)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3b: Unobserved Heterogeneity Estimates in the Conversion Model ($\Sigma^\theta$)

<table>
<thead>
<tr>
<th></th>
<th>$\theta_{i0}$ (Intercept)</th>
<th>$\theta_{i1}$ (Rank)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{i0}$ (Intercept)</td>
<td>0.503</td>
<td>-0.051</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>$\theta_{i1}$ (Rank)</td>
<td>-0.051</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.007)</td>
</tr>
</tbody>
</table>
### Table 4a: Coefficient Estimates on Bid Price

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Retailer</th>
<th>Brand</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega_0$</td>
<td>$\lambda_1$</td>
<td>$\lambda_2$</td>
<td>$\lambda_3$</td>
</tr>
<tr>
<td>Intercept</td>
<td>-1.285</td>
<td>-1.036</td>
<td>-0.171</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.089)</td>
<td>(0.043)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>LagRank</td>
<td>$\omega_1$</td>
<td>$\rho_{11}$</td>
<td>$\rho_{12}$</td>
<td>$\rho_{13}$</td>
</tr>
<tr>
<td></td>
<td>-0.027</td>
<td>0.110</td>
<td>0.013</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.039)</td>
<td>(0.013)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>LagProfit</td>
<td>$\omega_2$</td>
<td>$\rho_{21}$</td>
<td>$\rho_{22}$</td>
<td>$\rho_{23}$</td>
</tr>
<tr>
<td></td>
<td>-0.020</td>
<td>-0.049</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.033)</td>
<td>(0.022)</td>
<td>(0.013)</td>
</tr>
</tbody>
</table>

### Table 4b: Unobserved Heterogeneity Estimates in the Bid Price Model ($\Sigma^\omega$)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_0$ (Intercept)</th>
<th>$\omega_i$ (LagRank)</th>
<th>$\omega_i$ (LagProfit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$ (Intercept)</td>
<td>0.255</td>
<td>-0.027</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.004)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\omega_i$ (LagRank)</td>
<td>-0.027</td>
<td>0.015</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\omega_i$ (LagProfit)</td>
<td>0.009</td>
<td>0.0005</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>
### Table 5a: Coefficient Estimates on Keyword Rank

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Retailer</th>
<th>Brand</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$\bar{\phi}_0$</td>
<td>$\tau_1$</td>
<td>$\tau_2$</td>
<td>$\tau_3$</td>
</tr>
<tr>
<td></td>
<td>2.119</td>
<td>-0.636</td>
<td>-0.434</td>
<td>-0.109</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.152)</td>
<td>(0.076)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Bid Price</td>
<td>$\bar{\phi}_1$</td>
<td>$\pi_1$</td>
<td>$\pi_2$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td></td>
<td>-3.025</td>
<td>1.787</td>
<td>0.307</td>
<td>0.455</td>
</tr>
<tr>
<td></td>
<td>(0.353)</td>
<td>(0.390)</td>
<td>(0.179)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>CTR</td>
<td>$\bar{\phi}_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1.328</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 5b: Unobserved Heterogeneity Estimates in the Keyword Rank Model ($\Sigma^\theta$)

<table>
<thead>
<tr>
<th></th>
<th>$\bar{\phi}_0$ (Intercept)</th>
<th>$\bar{\phi}_1$ (Rank)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\phi}_0$ (Intercept)</td>
<td>1.289</td>
<td>-2.007</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.146)</td>
</tr>
<tr>
<td>$\bar{\phi}_1$ (Bid Price)</td>
<td>-2.007</td>
<td>3.886</td>
</tr>
<tr>
<td></td>
<td>(0.146)</td>
<td>(0.334)</td>
</tr>
</tbody>
</table>
Table 6: Estimated Covariance across Click-through, Conversion, Bid Price and Rank ($\Omega$)

<table>
<thead>
<tr>
<th></th>
<th>Click-through</th>
<th>Conversion</th>
<th>Bid Price</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Click-through</td>
<td>0.461</td>
<td>-0.077</td>
<td>0.015</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.062)</td>
<td>(0.007)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Conversion</td>
<td>-0.077</td>
<td>0.254</td>
<td>-0.043</td>
<td>-0.054</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.045)</td>
<td>(0.019)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>Bid Price</td>
<td>0.015</td>
<td>-0.043</td>
<td>0.170</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.019)</td>
<td>(0.004)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Rank</td>
<td>0.279</td>
<td>-0.054</td>
<td>-0.012</td>
<td>0.250</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.043)</td>
<td>(0.006)</td>
<td>(0.008)</td>
</tr>
</tbody>
</table>
**Table 7a: Estimates on Consumer Spending on the Searched Product Category**

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Latency</th>
<th>Rank</th>
<th>Brand</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{own}$</td>
<td>$\beta_1^{own}$</td>
<td>$\beta_2^{own}$</td>
<td>$\beta_3^{own}$</td>
<td>$\beta_4^{own}$</td>
</tr>
<tr>
<td>8.349</td>
<td>-0.410</td>
<td>0.024</td>
<td>-1.756</td>
<td>-1.061</td>
</tr>
<tr>
<td>(2.974)</td>
<td>(0.079)</td>
<td>(0.145)</td>
<td>(1.496)</td>
<td>(0.900)</td>
</tr>
</tbody>
</table>

Search$_1$  Search$_2$  Search$_3$  Search$_4$  Search$_5$

<table>
<thead>
<tr>
<th>$\gamma_1^{own}$</th>
<th>$\gamma_2^{own}$</th>
<th>$\gamma_3^{own}$</th>
<th>$\gamma_4^{own}$</th>
<th>$\gamma_5^{own}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-17.845</td>
<td>6.569</td>
<td>4.619</td>
<td>-0.252</td>
<td>-4.739</td>
</tr>
<tr>
<td>(4.255)</td>
<td>(2.250)</td>
<td>(2.658)</td>
<td>(2.263)</td>
<td>(3.100)</td>
</tr>
</tbody>
</table>

$\sigma^2_{own}$  $\tau^2_{own}$

114.361  12.167

(6.910)  (4.740)

**Table 7b: Estimates on Consumer Spending on Non-Searched Product Category**

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Latency</th>
<th>Rank</th>
<th>Brand</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{cross}$</td>
<td>$\beta_1^{cross}$</td>
<td>$\beta_2^{cross}$</td>
<td>$\beta_3^{cross}$</td>
<td>$\beta_4^{cross}$</td>
</tr>
<tr>
<td>-9.973</td>
<td>0.583</td>
<td>-0.311</td>
<td>7.256</td>
<td>1.770</td>
</tr>
<tr>
<td>(4.926)</td>
<td>(0.131)</td>
<td>(0.327)</td>
<td>(2.345)</td>
<td>(1.486)</td>
</tr>
</tbody>
</table>

Search$_1$  Search$_2$  Search$_3$  Search$_4$  Search$_5$

<table>
<thead>
<tr>
<th>$\gamma_1^{cross}$</th>
<th>$\gamma_2^{cross}$</th>
<th>$\gamma_3^{cross}$</th>
<th>$\gamma_4^{cross}$</th>
<th>$\gamma_5^{cross}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.718</td>
<td>-11.600</td>
<td>-17.056</td>
<td>-3.576</td>
<td>-2.714</td>
</tr>
<tr>
<td>(4.767)</td>
<td>(3.478)</td>
<td>(4.486)</td>
<td>(3.319)</td>
<td>(4.128)</td>
</tr>
</tbody>
</table>

$\sigma^2_{cross}$  $\tau^2_{cross}$

260.199  7.779

(27.040)  (3.236)
Figure 1a: Distribution of the Difference between Optimal and Actual Bids

Figure 1b. Distribution of the Log of Difference in Expected Profits using Optimal and Actual Bids