Design of Consumer Review Systems and Product Pricing

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Abstract

Consumer review systems have become an important marketing communication tool through which consumers share and learn product information. Although there is abundant evidence that consumer reviews have significant impact on consumer purchasing decisions, the design of consumer review systems and its impact on review outcomes and product sales have not yet been well examined. This paper analyzes firms’ review system design and product pricing strategies. We formally model two design features of consumer review systems – rating scale and disclosure of specific product attribute information. We show that firms’ optimal strategies critically depend on contextual characteristics such as product quality, product popularity, and consumer misfit cost. Our results suggest that firms should choose a low rating scale for niche products and a high rating scale for popular products. Firms should disclose specific product attribute information to attract the desired consumer segment when product quality is low relative to misfit cost, and the resulting optimal size of the targeted consumer market increases in product popularity and product quality. Different pricing strategies should be deployed during the initial sale period for different product types. For niche products, firms are advised to adopt lower-bound pricing for high-quality products to take advantage of the positive word of mouth. For popular products, firms are advised to adopt upper-bound pricing for high-quality products to enjoy the direct profit from the initial sale period, even after taking into account the negative impact of high price on consumer reviews.

Keywords: economic modeling, e-commerce, consumer reviews, online word of mouth, product uncertainty

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Introduction

With the prevalence of the internet and the success of e-commerce, consumers increasingly resort to the Web to gather information about the products of interest before making their purchasing decisions. Common online sources for product information include merchant-provided product descriptions, consumer reviews, professional/expert reviews, community forums, product Q&A sites, and so on. Among them, consumer reviews (user-generated product reviews) represent one of the most popular and influential information sources in shaping consumers’ purchasing behavior. According to a recent study by the E-tailing Group (2010), 71% of online shoppers stated that consumer reviews have the greatest impact on their product researching experience.

Several factors contribute to consumers’ preference for reviews from fellow consumers when researching products online. Information presented in consumer product reviews is more credible, trustworthy, and relevant than merchant-provided product descriptions (Bickart and Schindler 2001). Consumer reviews are more user-oriented while merchant-provided descriptions and professional reviews are more product-oriented (Chen and Xie 2008). Consumer reviews generate empathy and a sense of community among readers (Bickart and Schindler 2001, Mudambi and Schuff 2010). There is abundant empirical evidence that consumer reviews affect product sales in various domains including movies (Liu 2006, Dellarocas et al. 2007), books (Chevalier and Mayzlin 2006, Sun 2012), beer (Clemons et al. 2006), TV shows (Godes and Mayzlin 2004), video games (Zhu and Zhang 2010), and so on.

In response, many firms have adopted consumer review systems as a marketing communication tool to facilitate consumer sharing and learning about their products (Chen and Xie 2008). These consumer review systems are usually integrated with firms’ e-commerce sites.
Customers are invited to rate the products after their purchases and promotional incentives are often provided to encourage these post-purchase evaluations. Firms often present rating statistics and aggregate consumer reviews on their product pages. Both firms and consumers benefit from such consumer review systems. Prospective customers can read these product reviews from fellow consumers to learn more about the products in order to make more informed purchasing decisions. Consumer review systems also help firms improve customer service, lower return rate, increase conversion from browsers to buyers, and better respond to consumer needs through product improvement, logistic planning, assortment, and price adjustment (Mangalindan 2007, Fowler 2009).

Advanced Web technologies enable firms to obtain precise control over consumer review systems as they design, host, and monitor such review systems. Firms make design decisions regarding what type of information to solicit from customers, what rating scale to use, how to aggregate product reviews, and what type of information to display in consumer review systems. Different review system design choices are observed in popular online consumer review systems. For example, IMDb allows customers to rate a movie on a scale of one to ten. Many retailers such as Amazon, L.L.Bean, Macy’s, Target, and Wal-Mart let customers rate a product on a scale of one to five. These firms also ask customers if they would like to recommend the product to other customers or not. Similarly, YouTube’s “like/dislike” button enables a binary positive or negative rating. Other than the overall rating, reviews about various product attributes help potential consumers to better evaluate how well the product might fit their personal preferences. For example, depending on products, review systems hosted by L.L.Bean, Macy’s, Target, and Wal-Mart invite customers to evaluate a pre-selected set of product attributes such as appearance, comfort, easy to use, features, style, true to color, true to size, etc. The collected product attribute
data are then aggregated and summarized on the product pages. The resulting product reviews not only provide consumers’ overall ratings, but also reveal their assessments of relevant product attributes. Such product attribute summaries help improve potential consumers’ pre-purchase judgment of the products so that they will have less post-purchase “surprises” about product color, size, and other attributes. Interestingly, not all sites offer product-specific attribute summaries in their review systems, and even for the sites that do offer such a review system design feature, attribute summaries are not provided for all products.

Several interesting questions arise as we observe the diversified design features in existing consumer review systems. What accounts for these design differences? How do review system design choices affect consumer rating behavior and review outcomes? How do users of the review systems interpret review outcomes and make their purchasing decisions accordingly? What are the optimal review system design choices?

In this paper, we study the design of firm-managed consumer review systems and examine the impact of review system design features on current customer ratings and future customers’ purchasing decisions by investigating the interaction of a firm’s pricing and review system design decisions. We propose a formal analytical model to address a series of research questions: How do consumers rate a product after purchase? How do prospective customers interpret the product reviews and make their purchasing decisions accordingly? What is the impact of a firm’s review system design and pricing decisions on consumer reviews? What are the firm’s optimal design choices for its online consumer review system and how do these design choices interact with its pricing decision? How do product and consumer characteristics moderate the firm’s design and pricing strategies?
Figure 1 presents our conceptual model of online consumer review systems. This conceptual model builds upon the consumer choice framework proposed in (Hansen 1976) and highlights the information role of consumer review systems in reducing consumers’ product uncertainty. We examine the impact of a firm’s review system design choices and pricing decisions on the outcomes of consumer reviews, accounting for contextual factors such as product and consumer characteristics. In our model, we formally analyze two interrelated processes: product rating process and rating interpretation process. In the product rating process, product and consumer characteristics influence a firm’s product pricing and review system design decisions. These factors collectively determine review outcomes such as review volume, mean rating, and product attribute summary. In the rating interpretation process, the result of consumer reviews influences future consumers’ expectations and their purchasing decisions.

--- Insert Figure 1 here ---

Consumer review systems often perform three key functions – the collection, aggregation, and display of product reviews from consumers. The review collection function determines who can provide product reviews and what information to solicit from them. The collected reviews may take various forms such as numerical rating scores, textual comments, and video recordings. These product reviews usually contain two types of information – overall rating and product attribute evaluations. The review aggregation function computes summary statistics from numerical ratings and summarizes product attribute evaluations. The review display function determines what to present to users, as well as where and how to display the aggregated and detailed reviews.

Firms’ review system design choices determine the operationalization of these three key functions. In this paper, we model two types of design features of consumer review systems – the
scale of overall rating and the disclosure of specific product attribute information. Once a firm decides the rating scale for its review system, the corresponding available rating levels are embedded in the user review page from which a reviewer can choose the one that best matches her evaluation of the product. Summary statistics such as the mean rating and the rating volume can then be computed through the aggregation function and presented through the display function of the review system.

In addition to the overall rating score, a firm can also choose to explicitly disclose specific product attribute information through the review system. The system can ask reviewers to voice their opinions on a set of preselected product attributes on the user review page in addition to soliciting the overall rating and open-ended comments. For example, on the user review page, reviewers can evaluate the product’s appearance, features, style, and other attributes. Either by aggregating such attribute specific evaluations or by aggregating and mining open-ended user comments, the review system can generate summary of specific attribute evaluations and present it on review pages. Compared to lengthy open-ended comments, product attribute information disclosed and highlighted on the product review pages is convenient to access and easy to comprehend for consumers (Archak et al. 2011). Due to consumers’ diversified personal tastes and preferences, a review on product attributes perceived as revealing negatives about the product by one person may be considered as enlightening positives by another. By controlling what specific product attributes and how much attribute information to be disclosed in the review system, a firm can effectively influence how the product will be perceived by prospective customers. Through the design of consumer review system, the firm can convey specific product information to consumers by explicitly soliciting, aggregating, and displaying certain product attribute information on the review pages and attract the desired consumer segment.
Both the scale of user ratings and the disclosure of specific product attribute information in the review system have a direct impact on user ratings. Based on the resulting product ratings, prospective customers update their beliefs about the product and further make their purchasing decisions. In addition, contextual factors such as product popularity, consumers’ misfit cost, and product quality moderate the impact of online review system design on consumers’ purchasing decisions and therefore the firm’s optimal pricing and review system design decisions are contingent on these factors.

We model consumer review systems as a marketing communication tool, which reduces consumer uncertainty about both the product quality and the product fit. We simultaneously analyze a firm’s product pricing and review system design decisions considering the impact of contextual factors. Our results suggest that firms should choose a low rating scale for niche products and a high rating scale for popular products. Different pricing strategies should be deployed during the initial sale period for different product types. For niche products, firms are advised to adopt lower-bound pricing for high-quality products to take advantage of the positive word of mouth. For popular products, firms are advised to adopt upper-bound pricing for high-quality products to enjoy the direct profit from the initial sale period even after taking into account of the negative impact of high price on consumer reviews. We show that review system design enables firms to communicate additional product information to consumers and effectively segment the consumer market through strategically soliciting and disclosing certain product attributes in the review system. Firms should disclose specific product attribute information to attract the desired consumer segment when product quality is low relative to misfit cost, and the resulting optimal size of the targeted consumer market increases in product popularity and product quality.
Our paper contributes to the literature of consumer product reviews in multiple ways. First, we formally model a firm’s two review system design decisions: rating scale and disclosure of product attributes, and analyze the impact of these two review system design features on consumer ratings and the firm’s profit. Second, while most of the review literature focuses on the rating interpretation process, our paper takes a holistic approach and analyzes both the product rating and the rating interpretation processes. Third, we explicitly formulate the information role of consumer reviews. Reviews from past consumers serve as an imperfect signal for product quality and product fit to facilitate learning for future consumers. Fourth, we analyze the interaction between the firm’s pricing and its review system design choices and simultaneously solve for its optimal decisions. Fifth, we identify three contextual factors—popularity, misfit cost, and quality—and examine how they moderate the firm’s pricing and review system design decisions.

This paper proceeds as follows: we review related literatures in the subsequent section. We next propose a model of online consumer review systems and then analyze and discuss the interaction of review systems design and a firm’s pricing strategy. Finally, we conclude with managerial implications and directions for future research.

Literature Review

There is an extensive literature studying online word of mouth in the fields of information systems and marketing. Researchers have identified different roles of online word-of-mouth systems. One research stream views online word-of-mouth systems as a reputation mechanism (Dellarocas 2003, Bakos and Dellarocas 2011) and papers in this research stream focus on building trust and reducing seller uncertainty. Another research stream views online word-of-mouth systems as a marketing communication tool (Godes and Mayzlin 2004, Chen and Xie
and papers in this research stream focus on communicating product information to consumers and reducing their uncertainty about products. This paper takes the product information view of online word-of-mouth systems and studies one particular type of such systems – *online product review systems*.

Based on the provider of the reviews, online product review systems can be categorized into *professional reviews* and *consumer reviews*. Professional reviews and consumer reviews exhibit different features, and prospective customers respond to these two types of reviews differently. Compared to professional reviews, consumer reviews are more user-oriented as opposed to product-oriented and are considered more credible and trustworthy by prospective customers (Bickart and Schindler 2001, Chen and Xie 2008). This paper studies the design of consumer review systems, and thus we focus on reviewing literature in consumer review systems.

Consumer review systems have been studied both analytically and empirically in the literature. We review the literature of consumer product reviews from three perspectives – contextual characteristics, review outcomes, and design of review systems. Table 1 presents a detailed literature review of consumer review systems.

--- Insert Table 1 here ---

Prior studies have identified important contextual factors in determining how online consumer reviews affect consumers’ purchasing decisions and product sales. These factors include product popularity (Zhu and Zhang 2010, Sun 2012), reviewer identity (Forman et al. 2008), consumer internet experience (Zhu and Zhang 2010), consumer expertise (Chen and Xie 2008), product age (Archak et al. 2011), firm age and growth (Clemons et al. 2006, Kuksov and Xie 2010), product type (Dellarocas et al. 2007, Dellarocas et al. 2010), marketing effort (Dellarocas et al. 2007, Duan et al. 2008), professional reviews (Liu 2006, Dellarocas et al. 2007,
Duan et al. 2008, Gao et al. 2011), etc. Empirical studies have shown that many contextual factors like product and consumer characteristics moderate the impact of consumer reviews on sales. However, firms’ strategic responses to specific contextual conditions remain unanswered. In this paper, we formally model three contextual characteristics and study their impact on consumer reviews as well as a firm’s pricing and review system design decisions.

In terms of review outcomes, online consumer reviews are captured in numerical rating scores and textual review content in the existing literature. Most studies utilize mean rating and rating volume to measure consumer reviews. Only a few papers look at the distribution of available customer reviews. Specifically, the variability of consumer rating scores is captured by standard deviation (Clemons et al. 2006), variance (Sun 2012), and coefficient of variation (Zhu and Zhang 2010). Only recently have researchers started studying the impact of textual review content (Archak et al. 2011). In this study, we consider mean rating, rating volume, and summary of users’ product attribute evaluations, all determined by the hosting firm’s review system design features.

From the system design perspective, this paper is related to the literature of information systems design. Design of information systems has been shown to have significant impact on firms’ operations and performance for business-to-business electronic markets (Basu and Hevner 1992), procurement auction systems (Greenwald et al. 2010), keyword auction systems (Liu et al. 2010), bundle trading markets (Guo et al. 2011), and software development systems (Ji et al. 2005, Ji et al. 2011). This paper expands the IS design literature to the domain of online consumer review systems.

Our work builds upon the existing literature of consumer reviews and addresses the design issues of consumer review systems that have not yet been answered in the literature.
Existing literature in online consumer reviews either does not consider the design of consumer
review systems or treats the system design as exogenously given. In this paper, we endogenize
the firm’s design choice by modeling two review system design features: rating scale and
disclosure of product attribute information. We systematically study the impact of different
review system design features on consumer rating and learning behavior and derive the firm’s
integrated optimal review system design and pricing decisions while accounting for effects of
different product and consumer characteristics.

The Model

Firm and Consumers

Consider a firm selling a product through the online channel. Consumers value both the quality
of the product and the fit of the product (e.g., how well the product fits their tastes). The quality
of the product \( v \) is the firm’s private information. Before purchase, consumers are uncertain
about the true value of \( v \) but they share a common belief that the product quality \( v \) is uniformly
distributed on \([v, \overline{v}]\), where \( \overline{v} > v > 0 \). Therefore consumers’ pre-purchase expectation of the
product quality is \( \hat{v} = (v + \overline{v})/2 \). From the consumers’ perspective, the difference between the
two bounds of perceived product quality \( (\overline{v} - v) \) measures the degree of quality uncertainty.

Consumers are heterogeneous in terms of their tastes for the product. The firm knows its
product information but not individual consumer’s taste preference. Consumers, on the other
hand, know their taste preferences, but do not know product information and how the product fits
their own tastes before consumption. We use a unit line to represent consumer taste preference
and denote \( t \) as the misfit cost parameter which represents consumer unit misfit cost. Without
loss of generality, we assume that the product is located at point zero. Thus, a consumer’s taste
location on the unit line also represents the misfit of the product for the focal consumer. For example, a consumer located at \( x \in [0,1] \) has a product misfit of \( x \) and incurs a misfit cost of \( tx \). A consumer with a higher \( x \) incurs a higher misfit cost. Without product information, consumers are uncertain about their product misfit and their belief of the product misfit can be represented by a random variable with a density function \( f(x) \).

Consumers’ belief of the product misfit is consistent with the true consumer taste distribution. We assume that the true consumer taste distribution has density function of

\[
f(x) = \theta + 2x(1 - \theta), \quad x \in [0,1]
\]

where \( x \in [0,1] \) represents a consumer’s taste location on the unit line and \( \theta \in [0,2] \) represents the popularity of the product. This general density function \( f(x) \) represents a series of products with different popularity levels. Figure 2 illustrates three representative examples of user taste distributions, which correspond to three different product types. When \( \theta \in [0,1] \), there are relatively fewer consumers located close to the offered product and therefore these cases correspond to niche products. When \( \theta \in (1,2] \), there are relatively more consumers located close to the offered product and therefore these cases correspond to popular (or mass) products. When \( \theta = 1 \), the density function \( f(x) = 1 \) represents a uniform distribution and corresponds to neutral products. Thus, \( \theta \) is the product popularity parameter, with a higher \( \theta \) indicating a higher popularity.

--- Insert Figure 2 here ---

We assume the density function \( f(x) \) is public knowledge. Before purchase consumers do not know exactly how well the product fits their own tastes and they share a common belief that the product misfit follows the density function of \( f(x) \). Therefore consumers’ pre-purchase
expectation of the product misfit is \( \hat{x} = \int_0^1 x f(x) dx = \frac{4 - \theta}{6} \). Consequently, consumers have a higher expected misfit cost for niche products and a lower expected misfit cost for popular products. The firm charges a price \( p \) for the product. Therefore consumers’ pre-purchase expected net utility is \( \hat{u} = \hat{v} - \hat{t} - p \).

In summary, there are two types of product uncertainty – quality uncertainty and fit uncertainty. In the next subsection, we discuss how consumer review systems can help reduce these two types of product uncertainties.

**Consumer Rating and Interpretation Processes through Review Systems**

The firm hosts an online product rating system to facilitate information sharing among its customers. We study the firm’s review system design choice and its pricing strategy in a two-period model. Consumers arrive independently in each period and each consumer has unit demand for the product. The total number of consumers in each period is normalized to 1. At the beginning of the first period, the firm makes its pricing and review system design decisions. In the first period, consumers are uncertain about their valuations of the product quality and how well the product will fit their tastes. First-period consumers make their purchasing decisions based on their expected valuation of the product quality and their expected misfit cost. After consuming the product, consumers learn the true product quality and the true product fit. Based on their realized net utility, first-period consumers rate the product in the review system. In the second period, consumers learn more about the product quality and the product fit from the posted reviews. Second-period consumers then update their beliefs for the product and make their purchasing decisions accordingly.
We use $p_i$ to represent the product price, $\hat{v}_i$ to represent consumers’ expected valuation of the product quality, and $\hat{x}_i$ to represent their expected misfit in period $i$, where $i = 1, 2$. In the first period, consumers’ expected misfit is $\hat{x}_1 = 2/3 - \theta/6$ and their expected valuation on product quality is $\hat{v}_1 = (\bar{v} + \nu)/2$. The first-period consumers’ pre-purchase expected utility is given by $\hat{u}_i = \hat{v}_i - t\hat{x}_i - p_1$. After purchase, consumers learn the true product quality and how well the product fits their tastes, i.e., their true product misfit. The realized utility for a customer located at $x$ is $u(x) = v - tx - p_1$. Table 1 summarizes the notations of the paper.

--- Insert Table 1 here ---

After purchase, first-period consumers rate the product in the review system. There are two key modeling components regarding online consumer product rating systems. The first component is the product rating process. In other words, after first-period customers consume the product, how do they rate the product? The second component is the rating interpretation process. In other words, how do product ratings affect consumers’ expectations of the product in the second period?

We start with the product rating process. We use $s$ to denote the rating scale of the review system. To simplify the exposition, we normalize product ratings to a vertical unit line segment where the highest rating is 1, the lowest rating is 0, and other rating levels are evenly spaced out along the unit line segment. Thus, in a system with $s$ rating levels, the available rating levels correspond to points $0, \frac{1}{s-1}, \ldots, \frac{s-2}{s-1}$, and 1 on the vertical line. We next introduce a rating function $R(\cdot)$, which maps a customer’s post-purchase utility to one of the available rating levels. We assume that consumers rate the product truthfully based on their post-
purchase net utility\(^2\). We first transform consumers’ post-purchase utility \(u(x) \in R\) to a utility score \(w(x) \in (0,1)\) according to a logistic function \(w(x) = \frac{e^{u(x)}}{1 + e^{-u(x)}}\). This type of logistic transformation is used widely in modeling consumer choices (Malhotra 1984, Franses and Paap 2001). The logistic transformation converts consumers’ post-purchase utility to a utility score which has the same scale as the product ratings. Consumers then rate the product by matching the converted utility score to a rating level according to the rating function \(R(x)\) defined below:

\[
R(x) = \begin{cases} \frac{r}{s-1}, & \text{if } \min \left\{ \frac{i}{s-1} - w(x) \right\}, i = 0, \ldots, s-1 \leq \varepsilon \\ \text{does not rate}, & \text{otherwise} \end{cases}
\]

where \(r = \arg\min_i \left\{ \frac{i}{s-1} - w(x) \right\}, i = 0, \ldots, s-1\) is the rating level chosen by consumer \(x\),

\(w(x) = \frac{1}{1 + e^{e_x p - v}}\) is the utility score for consumer \(x\), and parameter \(\varepsilon\) measures consumers’ propensity to review the product.

The rating function \(R(x)\) is defined such that the consumer selects the rating level closest to her utility score. When \(\varepsilon < \frac{1}{2(s-1)}\), some consumers choose not to rate the product because none of the available rating levels closely reflect their evaluations of the product. When \(\varepsilon \geq \frac{1}{2(s-1)}\), all customers will rate. As a result, this rating function creates a mapping between consumer misfit and product rating. Figure 3 demonstrates the mapping from consumers’ post-purchase utility to product rating for two rating scales with \(s = 2\) and 5. For example, a consumer

\(^2\) We do not model how to elicit honest feedback from consumers in online review systems. See (Dellarocas 2006, Mayzlin 2006, Sullivan 2008) for discussions about potential manipulation of product reviews.
with misfit $x$ generates a utility score $w(x)$ based on her net utility $u(x)$, and she will rate

\[ \frac{i}{s-1}, \text{ if the utility score } w(x) \text{ falls within } \left[ \frac{i}{s-1}-\varepsilon, \frac{i}{s-1}+\varepsilon \right]. \]

--- Insert Figure 3 here ---

In this paper, we capture the overall rating results by the mean $\mu(p_1,s)$ and volume $n(p_1,s)$ of customer ratings. Given a utility score $w(x)$, the inverse function is

\[ w^{-1}(w) = \left( \frac{1}{t} \right) \left[ \ln \left( \frac{1-w}{w} \right) + v - p_1 \right], \]

which represents the customer’s misfit. Consider the case in which there are customer ratings for each of the rating scales $\left\{ 0, \frac{1}{s-1}, \ldots, \frac{s-2}{s-1}, 1 \right\}$. The mapping from customer rating score $w(x)$ to customer misfit is as follows: consumers who give the highest rating (1) have a misfit value in $\left[ 0, w^{-1}(1-\varepsilon) \right]$, where

\[ w^{-1}(1-\varepsilon) = \left( \frac{1}{t} \right) \left[ v - p_1 - \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) \right]; \]

consumers who give the lowest rating (0) have a misfit value in $\left[ w^{-1}(\varepsilon), 1 \right]$, where $w^{-1}(\varepsilon) = \left( \frac{1}{t} \right) \left[ v - p_1 + \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) \right];$

consumers who rate $\frac{i}{s-1}$ have a misfit value in $\left[ w^{-1}\left( \frac{i}{s-1} + \varepsilon \right), w^{-1}\left( \frac{i}{s-1} - \varepsilon \right) \right]$, where

\[ w^{-1}\left( \frac{i}{s-1} + \varepsilon \right) = \left( \frac{1}{t} \right) \left[ \ln \left( \frac{1-\varepsilon}(s-1)-i \right) + v - p_1 \right] \]

and

\[ w^{-1}\left( \frac{i}{s-1} - \varepsilon \right) = \left( \frac{1}{t} \right) \left[ \ln \left( \frac{1+\varepsilon}(s-1)-i \right) + v - p_1 \right]. \]

The case when all ratings exist implies that $w^{-1}(1-\varepsilon) \geq 0$ and $w^{-1}(\varepsilon) \leq 1$, i.e.,

\[ p_1 \leq v - \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) \quad \text{and} \quad p_1 \geq v - t + \ln \left( \frac{1-\varepsilon}{\varepsilon} \right). \]

In the following analysis, we assume the above conditions hold so that we can focus on the all ratings existing case. A discussion of the special
cases in which only some of the rating scales are rated is included in Appendix A. The resulting review volume and mean rating can be characterized as:

\[ n(p_1, s) = \int_0^{w^{-1}(1-s)} f(x) \, dx + \sum_{i=1}^{s-2} \left[ \int_{w^{-1}\left(1 - \frac{1}{i+1} \right)}^{w^{-1}\left(1 - \frac{1}{i+2} \right)} f(x) \, dx \right] + \int_{w^{-1}(1-s)}^{1} f(x) \, dx \]  

(2)

\[ \mu(p_1, s) = \frac{1}{n(p_1, s)} \left[ (1) \int_0^{w^{-1}(1-s)} f(x) \, dx + \sum_{i=1}^{s-2} \left( \frac{i}{s-1} \right) \int_{w^{-1}\left(1 - \frac{1}{i+1} \right)}^{w^{-1}\left(1 - \frac{1}{i+2} \right)} f(x) \, dx \right] \]  

(3)

where \( f(x) = \theta + 2x(1-\theta) \). These two key rating results, \( n(p_1, s) \) and \( \mu(p_1, s) \), can be further simplified and detailed derivations are provided in Appendix B.

The product rating process as described above exhibits the following two properties. First, consumers rate the product based on their post-purchase evaluations of the product quality and how well the product fits their tastes, as well as the price paid for the product. Consumers give neutral/positive/negative ratings for the product if their post-purchase utilities are zero/positive/negative. Similar modeling approach has been adopted in modeling consumer rating behavior (Kuksov and Xie 2010, Sun 2012). Second, consumers with extremely high and extremely low net utility are more likely to rate. Let us take a review system with a 5-star rating scale as an example. Given first-period product price \( p_1 \) and review propensity parameter \( \varepsilon \), more customers give 1-star or 5-star ratings than 3-star ratings, i.e., the 1-star volume \( 1 - w^{-1}(\varepsilon) \) and the 5-star volume \( w^{-1}(1-\varepsilon) \) are greater than the 3-star volume \( w^{-1}(1/2-\varepsilon) - w^{-1}(1/2+\varepsilon) \).

This property of the product rating process is consistent with empirical findings (Dellarocas and Narayan 2006, Hu et al. 2009, Gao et al. 2011). Hu et al. (2009) refer to consumers’ tendency to rate the product when they are extremely satisfied (rate to brag) or unsatisfied (rate to moan) as the brag and moan effect which results in the under-reporting bias of the customer with moderate views.
Next, we model the rating interpretation process and demonstrate how second-period consumers learn from the product review results. In the second period, customers observe the first-period ratings and update their beliefs on quality $v$ accordingly. The numerical overall rating serves as a signal of the product quality. Given a review scale level $s$ and observing the mean rating $\mu(p_1, s)$ and the volume of reviews $n(p_1, s)$, the second-period consumers form their expected valuation on product quality as follows:

$$\hat{v}_2 = n(p_1, s)[v + \mu(p_1, s)(\bar{v} - v)] + [1 - n(p_1, s)](\frac{\bar{v} + v}{2})$$

(4)

where $\mu(p_1, s)$ and $n(p_1, s)$ are given by equations (2) and (3).

Second-period consumers’ updated belief of the product quality is a weighted average of the review-based belief and the no-review belief. Without considering the review results, consumers’ expected product quality is $(\bar{v} + v)/2$. If consumers form their belief of the product quality purely based on the review results, then the perceived product quality is $v + \mu(p_1, s)(\bar{v} - v)$, which increases in the mean product rating $\mu(p_1, s)$. When the mean rating is low, consumers’ perceived product quality is close to $v$. When the mean rating is high, consumers’ perceived product quality is close to $\bar{v}$. We assume that consumers only partially depend on product reviews to learn product quality. Specifically, when updating their belief, the weight that consumers put on the review results is the review volume. In other words, consumers rely on the review results more if there are more reviews. Prior studies have identified several reasons for this type of consumer behavior such as awareness (Liu 2006, Duan et al. 2008), as well as credibility and trust (Fowler 2009).
**Design of Consumer Review Systems**

In this subsection, we model the firm’s choice for two design features of online product rating systems – rating scale and disclosure of specific product attribute information. We use \( s \in \mathbb{Z} \) and \( s \geq 2 \) to denote the number of *rating scale* levels. For example, \( s = 2 \) corresponds to the “like/dislike” or “recommend/not recommend” case and \( s = 5 \) corresponds to the 5-star rating case. As the number of scale levels increases, it becomes more costly for consumers to compare and select the most appropriate rating level that reflects their evaluations of the product and it eventually becomes overwhelming for consumers to rate. Thus, we only observe limited options of rating scales in practice. In other words, rating levels are bounded by consumers’ capacity to evaluate the product\(^3\). Therefore we examine the firm’s optimal choice of the rating scale from a finite set \( s \in \{2, 3, ..., \overline{s}\} \), where \( \overline{s} \) is the maximum number of rating levels.

We next consider the second design choice of consumer review systems – disclosure of specific product attribute information. In addition to displaying an overall rating score about the product, a review system can also publish an aggregated summary of product attributes on the product pages. The product attribute summary can be generated either by directly inviting consumers to rate preselected product attributes or by mining and aggregating consumers’ open-ended comments about the product. By controlling the number of product attributes and what attributes to collect and display in the review system, the firm can strategically reveal the product’s goodness-of-fit information and influence second-period consumers’ perception on how well the product might fit their tastes.

To model this second design feature, we use \( 0 \leq k \leq 1 \) to represent the firm’s design choice of *product attribute disclosure*. When the firm chooses to disclose specific product

\(^3\) Commonly observed scale levels include 2, 5, and 10.
attribute information through the review system, second-period consumers are better informed of the goodness-of-fit of the product compared to the first-period consumers. As a result, they form an updated belief on the misfit of the product. Some consumers believe that their misfit values fall within $[0,k]$ and the product is a better fit with the updated expected misfit of

$$ \frac{1}{F(k)} \int_{0}^{k} xf(x) dx , $$

which is lower than their original expected misfit. The other consumers believe that their misfit values fall within $(k,1]$ and the product is a worse fit with the updated expected misfit of

$$ \frac{1}{1-F(k)} \int_{k}^{1} xf(x) dx , $$

which is higher than their original expected misfit. The consumer segment with the reduced expected misfit is favorable to the firm, whereas the consumer segment with the increased expected misfit is unfavorable to the firm. While attribute-specific reviews help both consumer groups improve their beliefs on product fit, such consumer learning is imperfect as consumers still don’t know the exact product misfit before purchasing the product. As second-period consumers self-select into these two consumer segments, the firm is better off to serve only the favorable consumer segment $[0,k]$. By strategically disclosing selected product attribute information through consumer reviews, the firm is able to communicate specific product fit information, influence consumers’ perception of the product misfit, and implement different product positioning strategy, all characterized by the design decision $k$.

A review system with $k = 1$ corresponds to the general appeal disclosure strategy in which the firm does not facilitate the disclosure of product attribute information in the review system so that second-period consumers have the same pre-purchase uncertainty regarding product fit as first-period consumers. A review system with $k < 1$ corresponds to the special
appeal disclosure strategy in which the firm selectively discloses product attribute information and serves only the favorable consumer segment. A lower $k$ value indicates a more concentrated market strategy, which can be implemented by disclosing highly differentiating attributes or more attribute information. The firm’s product attribute disclosure strategy directly influences consumers’ perception of the product fit. By selectively disclosing specific product attribute information, the firm is able to implement different market segmentation strategies or complement its existing market segmentation strategies. The general appeal disclosure strategy corresponds to the full market coverage strategy and the special appeal disclosure strategy corresponds to the single-segment concentration strategy.

A Review System with Overall Rating Only

In this section, we analyze a review system that only solicits and displays consumers’ overall product ratings, assuming the firm adopts the general appeal disclosure strategy ($k = 1$). In this review system, the firm's only design decision is to choose the scale of consumer ratings.

Formulation

In the first period, anticipating consumers’ pre-purchase expected utility function, the firm sets its price $p_1 \leq \bar{p}_1 = \hat{v}_i - t\hat{x}_i = \frac{\bar{v} + v}{2} - \left(\frac{2}{3} - \frac{\theta}{6}\right)t$ to achieve a positive sale, where $\bar{p}_1$ is the maximum price the firm can charge such that first-period consumers will purchase. The maximum price, $\bar{p}_1$, is determined by consumers’ pre-purchase expected gross utility. We assume that the expected quality is higher than the expected misfit cost, i.e.,

$$\bar{p}_1 = \frac{\bar{v} + v}{2} - \left(\frac{2}{3} - \frac{\theta}{6}\right)t \geq 0.$$ This assumption ensures that it is feasible for the firm to set a
positive price and make a positive sale. As a result, all consumers purchase the product in the first period and the firm’s profit is $\pi_1 = p_1$. The result of serving all consumers in the first period is due to the assumption that first-period consumers share the same belief about the quality and fit of the product. This assumption simplifies the model analysis and helps us to focus on the impact of the firm’s review system design and pricing decisions on past consumers’ review behavior and future consumers’ purchasing behavior.

After purchase, customers learn about their realized utility given by $u(x) = v - tx - p_1$ and rate the product according to the rating function $R(x)$ defined in (1). Note that the firm may charge a price lower than $\bar{p}_1$ such that customers will have higher post-purchase utility, which in turn will positively impact the reviews by first-period customers. Lemma 1 summarizes the properties of consumers’ rating results – rating volume $n(p_1, s)$ and mean rating $\mu(p_1, s)$. The proofs of all lemmas and propositions are delegated to Appendix C.

**Lemma 1 (Properties of rating volume and mean rating):**

(a) Mean rating, $\mu(p_1, s)$, decreases in the first-period price ($p_1$) and increases in the product quality ($v$) regardless of the product type.

(b) For niche products, rating volume, $n(p_1, s)$, increases in the first-period price ($p_1$) and decreases in the product quality ($v$); for popular products, rating volume, $n(p_1, s)$, decreases in the first-period price ($p_1$) and increases in the product quality ($v$); and for neutral products, rating volume, $n(p_1, s)$, is independent of the first-period price ($p_1$) and product quality ($v$).
When the first-period price decreases, or the product quality increases, consumers’ post-purchase utility increases and therefore the mean rating $\mu(p_1, s)$ increases. As a result, a higher mean rating signals a better-quality product to second-period consumers.

Lowering the first-period price and increasing the product quality have similar impacts on the review volume and their impacts depend on the product type. Lowering the first-period price and increasing the product quality both result in higher post-purchase utilities for all consumers. As a result, more consumers located close to the product on the taste line give the rating 1, and fewer consumers located close to 1 on the taste line give the rating 0. For popular products, because more consumers are located close to the product and fewer are located close to 1, the net result is that the total rating volume increases. In contrast, for niche products, more consumers are located close to 1, and the net result is that the total rating volume decreases with fewer consumers giving low ratings. For neutral products, because the consumer taste density is a constant, the first-period price and product quality have no impact on rating volume for neutral products.

In the rating interpretation process, second-period consumers update their beliefs on product quality based on rating results. Lemma 2 describes the properties of consumers’ updated belief on quality.

**Lemma 2 (Properties of the second-period consumers’ expected valuation on quality):**

The second-period consumers’ expected valuation on product quality ($\hat{v}_2$):

(a) increases in the true product quality ($v$) and decreases in the first-period price ($p_1$);

(b) increases in the rating scale ($s$) for popular products, decreases in the rating scale ($s$) for niche products, and is independent of the rating scale ($s$) for neutral products; and

(c) increases in the product popularity ($\theta$) and decreases in the unit misfit cost ($t$).
As shown in Lemma 1, a higher product quality or a lower price leads to a higher mean rating which signals a higher product quality. Lemma 2 shows that second-period consumers observe this signal and their perception of the product quality increases. The firm can manipulate the first-period price to influence first-period consumer reviews, and therefore, second-period consumers’ expected valuation on quality.

Interestingly, for a given product quality level and a given price, consumers’ perception of the product quality is negatively related to the unit misfit cost. This relationship is due to the fact that consumer ratings reflect their evaluations of both the quality and the goodness-of-fit of the product. When unit misfit cost decreases, first-period customers enjoy a higher post-purchase net utility and thus rate the product better. As a result, new consumers observe an increased overall rating and therefore form a higher expectation for the product quality. Since consumers incur a lower overall misfit cost for popular products, their perception of the product quality is higher for popular products when everything else remains the same.

Since we consider a review system that only publishes the overall rating and does not disclose any product attribute information, second-period consumers face the same level of fit uncertainty as first-period consumers, i.e., \( \hat{x}_2 = \hat{x}_1 = 2/3 - \theta/6 \). Thus the second-period consumers’ pre-purchase expected utility is \( \hat{u}_2 = \hat{v}_2 - t\hat{x}_2 - p_2 \). In response, the firm sets the second-period price at \( p_2 = \hat{v}_2 - t\hat{x}_2 \). We focus on the more interesting case in which the true quality is high enough that \( \hat{v}_2 > t\hat{x}_2 \). As a result, all consumers will purchase in the second period and the second-period profit is given by \( \pi_2 = p_2 = \hat{v}_2 - t\hat{x}_2 \).
Therefore the firm’s overall decision problem can be specified as:

\[
\max_{p_1,s} \pi(p_1, s) = \pi_1 + \pi_2 = p_1 + \hat{v}_2 - t\hat{\epsilon}_2
\]

s.t. \[0 \leq p_1 \leq \bar{p}_i\]
\[2 \leq s \leq \bar{s}, \; s \in \mathbb{Z}\] (5)

The firm sets the first-period price and selects the rating scale for the consumer review system to maximize its total profit of the two periods.

**Optimal Design of the Rating Scale \( s \)**

Proposition 1 delineates the firm’s optimal design choice for the rating scale of the review system.

**Proposition 1 (Optimal rating scale level):**

(a) For a popular product (\( \theta > 1 \)), it is optimal for the firm to offer \( s^* = \bar{s} \), the maximum number of rating levels;

(b) For a niche product (\( \theta < 1 \)), it is optimal for the firm to offer \( s^* = 2 \), the minimum number of rating levels;

(c) For a neutral product (\( \theta = 1 \)), rating levels have no impact on the firm’s profit.

We find that the firm’s optimal design for rating scale is contingent on the popularity of the product. For a popular product, a higher rating level, \( s \), has a positive effect on the second-period consumers’ perception of the quality of the product (as shown in Lemma 2), which leads to a higher overall profit for the firm for a given first-period price. Therefore, it is optimal for the firm to offer the maximum number of rating levels. In contrast, for a niche product, a higher rating level, \( s \), has a negative effect on the second-period consumers’ perception of the quality of the product. Therefore it is optimal for the firm to offer the minimum number of rating levels.
For neutral products, the firm’s profit is independent of the rating level, and therefore the optimal rating level could be any integer between 2 and $\bar{r}$.

**Pricing Strategy**

Second-period consumers learn about the quality of the product through the review system. Since the review system does not disclose product attribute information, second-period consumers’ uncertainty about the product’s goodness-of-fit remains the same. In other words, consumers have the same expected misfit cost in both periods. This review system only reduces consumer product quality uncertainty and has no impact on product fit uncertainty.

The firm’s first-period pricing has two countervailing effects on its overall profit. Increasing the first-period price directly increases the firm’s first-period profit but indirectly decreases its second-period profit through its impact on consumer reviews. The consumer review system provides a mechanism for the firm to manipulate second-period consumers’ perception of the product quality. Specifically, the second-period consumers’ updated belief on the quality of the product is at its maximum when the firm offers the product for free ($p_1 = 0$), and it is at its minimum when the firm sets the price to the maximum price such that first-period consumers will purchase ($p_1 = \bar{p}_1$).

The firm aims to balance these two effects of first-period pricing to maximize its total profit and the resulting optimal pricing strategy depends on product quality, product popularity, and misfit cost.

**Proposition 2 (Optimal pricing in the first period):**

(a) For a popular product ($\theta > 1$), if the true product quality is high with

\[ v > \bar{p}_1 + \frac{t^2 - t\theta(v - \bar{v})}{2(1 - \theta)(v - \bar{v})}, \]

it is optimal for the firm to charge $p_1^* = \bar{p}_1$ in the first period; if
the true product quality is medium with \( \frac{t^2 - t\theta(v - \nu)}{2(1 - \theta)(v - \nu)} \leq \nu \leq \bar{p}_1 + \frac{t^2 - t\theta(v - \nu)}{2(1 - \theta)(v - \nu)} \), it is optimal to charge \( p_1^* = \nu - \frac{t\theta(v - \nu) - t^2}{2(\theta - 1)(v - \nu)} \); if the true product quality is low with \( \nu < \frac{t^2 - t\theta(v - \nu)}{2(1 - \theta)(v - \nu)} \), it is optimal to offer it for free in the first period.

(b) For a niche product (\( \theta < 1 \)), if the true product quality is high with
\[
v > \frac{\bar{p}_1 + \frac{t^2 - t\theta(v - \nu)}{2(1 - \theta)(v - \nu)}}{2},
\]
it is optimal for the firm to offer it for free in the first period; if the true product quality is low with \( \nu \leq \frac{\bar{p}_1 + \frac{t^2 - t\theta(v - \nu)}{2(1 - \theta)(v - \nu)}}{2} \), it is optimal to charge
\[
p_1^* = \bar{p}_1.
\]

(c) For a neutral product (\( \theta = 1 \)), if the misfit cost is high relative to the product uncertainty \( t > v - \nu \), it is optimal for the firm to charge \( p_1^* = \bar{p}_1 \); otherwise, it is optimal to offer the product for free in the first period.

The direct effect of the first-period price on the first-period profit is straightforward – the first-period profit increases linearly in the first-period price for all products (\( \partial \pi_1 / \partial p_1 = 1 \)). The indirect effect of the first-period price on the firm’s second-period profit is more nuanced. Overall the second-period profit decreases in the first-period price for all products (\( \partial \pi_2 / \partial p_1 < 0 \)). However, product characteristics (product popularity \( \theta \) and product true quality \( \nu \)) moderate the magnitude of this indirect effect (\( |\partial \pi_2 / \partial p_1| \)). Specifically, increasing the true product quality amplifies the indirect effect for niche products (\( \frac{\partial |\partial \pi_2 / \partial p_1|}{\partial \nu} > 0 \) when \( \theta < 1 \)) while it diminishes
the indirect effect for popular products \( \frac{\partial \pi_2}{\partial p_1} < 0 \) when \( \theta > 1 \). For neutral products, the true product quality has no impact on the indirect effect \( \frac{\partial \pi_2}{\partial p_1} = 0 \) when \( \theta = 1 \) and the magnitude of the indirect effect is determined by product misfit and quality uncertainty \( \frac{\partial \pi_2}{\partial p_1} = (v - v_1) / t \). The firm balances the direct and indirect effects of the first-period price on the firm’s profit and adopts three possible pricing strategies.

The lower-bound pricing strategy refers to offering the product for free either through charging zero price or through providing coupons, rebates, and other promotional benefits. It is optimal for the firm to adopt lower-bound pricing for high-quality niche products, low-quality popular products, and low-misfit neutral products since the negative indirect effect of the first-period price on the second-period profit dominates the positive direct effect of the first-period price on the first-period profit. The upper-bound pricing strategy refers to charging the maximum price \( \bar{p}_1 \) at which consumers still participate. It is optimal for the firm to adopt upper-bound pricing for low-quality niche products, high-quality popular products, and high-misfit neutral products since the positive direct effect of the first-period price on the first-period profit dominates the negative indirect effect of the first-period price on the second-period profit. The interior pricing strategy refers to charging a price between the lower and upper bounds. It is optimal for the firm to adopt interior pricing for medium-quality popular products only and the price is set at such a level that the positive direct effect equals the negative indirect effect.

We find that the firm’s pricing strategies serve different objectives. Through upper-bound pricing, the firm pursues the maximum first-period profit. Here the firm takes advantage of the information asymmetry on quality by charging the maximum possible price in the first period. Through lower-bound pricing, the firm aims to maximize second-period profit by sacrificing its
first-period profit. Specifically, the firm manipulates the first-period price to its lowest possible level to signal a higher quality to future consumers through the review system.

Another interesting finding is that the firm’s optimal design of rating scale and optimal pricing strategy are different for popular and niche products. The firm utilizes a high rating scale for popular products, but a low rating scale for niche products. When the product quality is relatively high, the firm selects upper-bound pricing for popular products, but lower-bound pricing for niche products. When the product quality is relatively low, the firm selects lower-bound pricing for popular products, but upper-bound pricing for niche products.

A Review System with Both Overall Rating and Product Attribute Summary

In this section, we consider a review system that reveals both overall rating score and product attribute information.

**Formulation**

We consider a review system that not only sets the rating scale $s$ but also discloses preselected product attributes. The choice of soliciting and aggregating reviews of product attributes through the review system enables the firm to segment their consumers. There are two product attribute disclosure strategies. The general appeal disclosure strategy ($k = 1$) involves no disclosure of product attributes and serving all consumers. The special appeal disclosure strategy ($k < 1$) involves disclosing specific product attributes in the review system and serving only the favorable consumer segment.

Second-period consumers update their beliefs about the product quality ($\hat{v}_2$) based on the mean and volume of consumer ratings, which is affected by the first-period price and the review system rating scale level as shown in equation (4). If the firm chooses the special appeal
disclosure strategy \((k < 1)\), then only the consumers whose perceived misfit falls within \([0,k]\) are served and they have an expected misfit of \(\hat{x}_2(k) = \frac{1}{F(k)} \int_{0}^{k} xf(x) dx\). In response, the firm sets the second-period price to \(p_2(k) = \hat{v}_2 - t\hat{x}_2(k)\). If the firm chooses the general appeal disclosure strategy \((k = 1)\), then no product attribute information is provided and all consumers share the same expected misfit of \(\hat{x}_2(1) = 2/3 - \theta/6\). In response, the firm sets the second-period price to \(p_2(1) = \hat{v}_2 - t\hat{x}_2(1)\) and all consumers are served.

Compared to the general appeal disclosure strategy, the special appeal disclosure strategy enables the firm to disclose a set of preselected product attributes to reduce consumer product fit uncertainty, i.e., \(\hat{x}_2(k) < \hat{x}_2(1)\). As a result, the firm serves only the favorable consumer segment and charges a higher price, i.e., \(p_2(k) > p_2(1)\).

The firm simultaneously decides the two design choices \((k \text{ and } s)\) and pricing to maximize its total profit. The firm’s decision problem can be formulated as:

\[
\max_{p_1,k,s} \pi(p_1,k,s) = p_1 + F(k)\left[\hat{v}_2(p_1,s) - t\hat{x}_2(k)\right]
\]

s.t. \(0 \leq p_1 \leq \bar{p}_1\)

\(0 \leq k \leq 1\)

\(s \in \{2, 3, \ldots, \bar{s}\}\)  

We next explore the properties of the two design choices in Lemma 3.

**Lemma 3 (Properties of rating scale and product attribute disclosure):**

(a) Given the firm’s first-period price \(p_1\) and the design choice \(k\), the optimal rating scale is \(s^* = \bar{s}\) for a popular product \((\theta > 1)\); \(s^* = 2\) for a niche product \((\theta < 1)\); and the rating level has no impact on the firm’s profit for a neutral product \((\theta = 1)\).
(b) Given the firm’s first-period price $p_1$ and the design choice $s$, if the product quality is relatively low ($v < C_0 + p_1$ for niche products; $v < C_1 + p_1$ for neutral products; and $v < C_2 + p_1$ for popular products), it is optimal for the firm to choose the special appeal disclosure strategy, $k^* = \hat{v}_2 / t < 1$, and only serve consumers in $[0, k^*]$ in the second period; otherwise, it is optimal for the firm to choose the general appeal disclosure strategy, $k^* = 1$, and serve all consumers in the second period.

The values of $C_0$, $C_1$, and $C_2$ are defined in the proof of Lemma 3 in Appendix C.

Lemma 3 demonstrates that the firm’s review system design decisions interact with its pricing decisions. Given any price and attribute disclosure levels, the firm selects the optimal rating scale of the review system based on product popularity. Given any price and rating scale levels, the firm discloses product attribute information through the review system when the true product quality is less than a threshold which is dependent on product and consumer characteristics. Therefore, the firm has to take into account the interaction between review system design and pricing when making its design choices.

**Optimal Design of the Review System and Pricing**

In this subsection, we consider the firm’s simultaneous decisions on the first-period price and the review system design choices $s$ and $k$. We are interested in determining the firm’s optimal decisions and deriving their corresponding market conditions defined by contextual characteristics such as the unit misfit cost $t$, the product quality $v$, and the product popularity $\theta$.

For niche and popular products, the optimal solutions become overly complex and no closed-form analytical expression for the separating market conditions can be obtained. Thus we
present the firm’s optimal solutions for neutral products here in Proposition 3 and we will then illustrate the results for niche and popular products through numerical analysis.

**Proposition 3 (Optimal pricing and design of review system for neutral products):**

For neutral products ($\theta = 1$), the firm’s optimal first-period price and review system design solution is:

(a) $p^*_1 = 0$ and $k^* = \frac{tv + v(\bar{v} - v)}{t^2}$, if $C_1 \leq v \leq C_1$;

(b) $p^*_1 = 0$ and $k^* = 1$, if $\max\{C_1, C_4\} \leq v$ and $t \leq \bar{v} - v$;

(c) $p^*_1 = \frac{\bar{v} + v - t}{2}$ and $k^* = \frac{(t - \bar{v} + v)(\bar{v} + v) + 2v(\bar{v} - v)}{2t^2}$, if $v \leq \min\{C_1, C_3\}$ or $C_1 \leq v \leq \min\{C_1 + \bar{p}_1, C_4\}$;

(d) $p^*_1 = \frac{\bar{v} + v - t}{2}$ and $k^* = 1$, if $v \geq C_1 + \bar{p}_1$ and $t \geq \bar{v} - v$;

(e) Rating level $s$ has no impact on the firm’s profit and $s^* \in \{2, 3, \ldots, \bar{s}\}$.

The value of $C_1$ is defined in the proof of Lemma 3 in Appendix C and the values of $C_3$ and $C_4$ are defined in the proof of Proposition 3 in Appendix C.

Proposition 3 describes the firm’s optimal pricing and review system design decisions for neutral products, which critically depend on the quality of the product and consumers’ misfit cost. Figure 4 visualizes the results for neutral products.

--- Insert Figure 4 here ---

Figure 4 demonstrates the market conditions for the firm’s optimal decisions based on two contextual characteristics with the unit misfit cost as the horizontal dimension and the true product quality as the vertical dimension. When the unit misfit cost is low relative to the true product quality (region A in Figure 4), the firm adopts the general appeal disclosure strategy.
When the unit misfit cost is high relative to the true product quality (regions B and C in Figure 4), the firm adopts the special appeal disclosure strategy. For a given misfit cost $t$, the firm’s pricing strategy changes from upper-bound pricing (region C in Figure 4) to lower-bound pricing (region B in Figure 4) as the product quality increases. When the firm adopts the special appeal disclosure strategy, the targeted consumer segment in the second period under lower-bound pricing is bigger than that under upper-bound pricing.

The firm’s review system design decision, $k$, interacts with its pricing decision. Compared to a review system that only solicits the rating scores, a review system with the added feature of product attribute disclosure reduces consumer uncertainty about the product misfit. By selectively disclosing specific product attributes in the review system, the firm gains the capability of segmenting the second-period consumer market and better managing its second-period profit. This gained capability alleviates the negative effect of the first-period price on the second-period profit. As a result, the firm adopts the upper-bound pricing strategy in a greater parameter space when it discloses product attribute information ($k < 1$) in the review system.

For niche ($0 \leq \theta < 1$) and popular ($1 < \theta \leq 2$) products, we demonstrate the firm’s optimal strategies through numerical analysis, since closed-form analytical expressions of the separating market conditions cannot be obtained. Figure 5 presents the case for niche products. The firm’s optimal pricing strategy and design choices for niche products are similar to those for neutral products. When product popularity $\theta$ increases, consumers’ perception of product quality increases (as shown in Lemma 2). As a result, the special appeal disclosure strategy is optimal for a smaller parameter region and the corresponding optimal size of the targeted consumer segment increases. The negative effect of the first-period price on the second-period profit
declines as the product popularity increases, and consequently, the upper-bound pricing strategy is optimal in a larger parameter region (as indicated by double arrows in Figure 5).

--- Insert Figure 5 here ---

Figure 6 presents the case for popular products. Since there are three possible pricing strategies (lower-bound, upper-bound, and interior pricing) for popular products and two possible product attribute disclosure solutions (general appeal and special appeal disclosure strategies), there are six possible solutions for popular products.

For popular products, the moderating effect of contextual characteristics on the firm’s product attribute disclosure decision is the same as that for niche products while the moderating effect of contextual characteristics on the firm’s pricing decision is the opposite of that for niche products. For a given true product quality, when the unit misfit cost increases, the firm moves from the general appeal disclosure strategy to special appeal disclosure strategy (e.g., regions A→B in Figure 6). For a given unit misfit cost, when the true product quality increases, the firm moves from the lower-bound pricing to interior pricing, and to upper-bound pricing (e.g., regions A→E→D in Figure 6). For popular products, when product popularity $\theta$ increases, the firm is more likely to offer the product for free and serve all the second-period consumers (as indicated by double arrows in Figure 6).

--- Insert Figure 6 here ---

For popular products with medium-level quality, the firm adopts the interior pricing strategy. At the optimal interior price, the marginal gain in the first-period profit from increasing $p_1$ equals the marginal loss in the second-period profit from the reduced consumers’ perception of product quality due to the increased $p_1$. We find that when everything else remains the same, the optimal interior price increases in the true product quality such that the difference between
the product quality and the price \((v - p_1^*)\) is a constant. As a result, for a given product popularity level and consumer unit misfit, the corresponding targeted market size does not change in product quality under the interior pricing strategy. When the firm adopts the lower-bound or upper-bound pricing strategy, however, the size of the targeted consumer segment in the second period increases in product quality, which is similar to that of the niche and neutral products. For all product types, the size of the targeted consumer segment increases in product popularity and decreases in misfit cost.

**Concluding Remarks**

Consumer review systems have become an important marketing communication tool for firms to facilitate consumer sharing and learning about their products. To fully benefit from managing such review systems, firms need to understand the mechanism of consumer reviews, actively participate in the design of review systems, and most importantly, integrate the review system design choices with other operational decisions such as logistics and pricing. This paper formally models review system design as a firm’s strategic decision. To explore the information role of consumer review systems, we explicitly depict both the product rating and the rating interpretation processes and study two types of product uncertainty. Before purchase, consumers are uncertain about the product quality as well as the product fit. The product rating score provided in the review system serves as an imperfect signal for product quality. Consumers rely on the product rating score to learn the product quality. This paper models rating scale as a review system design decision, demonstrates the impact of rating scale on consumer review outcomes, and makes recommendation of the optimal rating scale for different product types.

Consumer review systems may also selectively solicit and display product attribute information. Consumers resort to the reviews of described product attributes to learn the product
fit. The firm knows its product information but not individual consumers’ tastes. Product attribute disclosure helps the firm target a specific consumer segment as the desired consumers self-select into the segment after learning from reviews. Potential benefits of disclosing product attribute information include reducing consumers’ product fit uncertainty, serving the desired consumer segment at higher prices, achieving higher browser-to-buyer conversion rates, reducing operational costs such as returns (Mangalindan 2007, Fowler 2009), etc. Different from most of the literature, this paper explicitly models product attribute disclosure choice as a review system design decision which provides an additional instrument for the firm to manage the appeal of the product to prospective customers. This strategic system design enables the firm to execute the single-segment concentration strategy and serve only the favorable consumer segment.

Existing literature has shown consumer product reviews have significant impacts on consumers’ purchasing decisions and product sales. However, little is known about the impact of the design of consumer review systems. Our research contributes to consumer reviews literature by systematically modeling the product rating and rating interpretation processes, examining the impact of product and consumer characteristics, and studying the impact of a firm’s pricing and review system design decisions on consumer review outcomes and future consumer learning about a given product. Our results have important implications for the design of online consumer review systems and firms’ corresponding strategic responses.

In addition, we investigate the interaction of a firm’s review system design choices and its pricing strategies. We show that the firm’s pricing decision and review system design decision are mutually dependent and should be made simultaneously. We find that firms’ optimal pricing and review system design decisions are contingent on contextual characteristics. Firms
should carefully evaluate market conditions such as how the true product quality matches their consumers’ perception, whether their product appeals to a mass market or a niche market, and how much consumers value the fit of the product. We find that a review system with low scale levels such as “like/dislike” is optimal for niche products and a review system with high scale levels such as “1 to 10” is optimal for popular products. When consumer valuation for the product fit is low, the firm is advised to adopt a general appeal disclosure strategy by offering only numerical ratings and revealing no specific product attribute information in the review system. When consumer valuation for the product fit is high, the firm is advised to adopt a special appeal disclosure strategy by soliciting and revealing highly differentiating product attribute information in the product review system to reduce consumer uncertainty on product fit and attract the desired consumer segment. When implementing the product attribute disclosure strategy, firms could introduce multi-dimensional ratings based on the selected product attributes and customize the set of attributes based on specific product categories to attract desired consumers.

Our results suggest different pricing strategies during the initial sale period for different product types. When the firm offers a niche product, it should set a lower price for a better-quality product to take advantage of the impact of positive word of mouth. When the offered product is popular, the firm is able to charge a higher price for a better-quality product to enjoy the direct profit from the initial sale, even after taking into account the negative impact of high price on consumer reviews. In addition, when the firm adopts the special appeal disclosure strategy, the size of the targeted consumer segment is higher for high quality or high popularity products. The firm is more likely to adopt the upper-bound pricing strategy and the general appeal disclosure strategy as product popularity increases.
This paper studies the optimal design of seller-managed consumer review systems. There are several directions for future research. It would be interesting to investigate the optimal design problem if the consumer review system is managed by a third party. There might be other relevant contextual factors such as consumers’ online experience and reviewer identity. Experienced consumers may learn more from product reviews than novice consumers. Reviews written by true customers may be perceived differently from those written by anonymous users.
Figures and Tables

Figure 1: Conceptual Model of Consumer Review Systems as an Information Sharing Mechanism to Reduce Product Uncertainty
Figure 2: Examples of Consumer Taste (Misfit) Distribution
Figure 3: Mapping from Consumers’ Post-Purchase Utility to Product Rating
Figure 4: Firm’s optimal strategy for neutral products ($\theta = 1$) corresponding to parameter space.

Notes:

- Figures 4-6 are based on parameter values $\varepsilon = 0.03$, $v = 5$, and $V = 100$. Other parameter values generate qualitatively the same results.
- We focus on the shaded area where all ratings exist. The parameter space can be partitioned into three regions: in region A, $p_1^* = 0$, $k^* = 1$, and $s^* \in \{2,3,\ldots,\bar{s}\}$; in region B, $p_1^* = 0$, $k^* < 1$, and $s^* \in \{2,3,\ldots,\bar{s}\}$; in region C, $p_1^* = \bar{p}_1$, $k^* < 1$, and $s^* \in \{2,3,\ldots,\bar{s}\}$. 

Figure 5: Firm’s Optimal Strategy when $\theta = 0.5$

Notes:

- We focus on the shaded area where all ratings exist. The parameter space can be partitioned into three regions: in region A, $p_i^* = 0$, $k^* = 1$, and $s^* = 2$; in region B, $p_i^* = 0$, $k^* < 1$, and $s^* = 2$; in region C, $p_i^* = \bar{p}_1$, $k^* < 1$, and $s^* = 2$.
- The double arrows indicate the changing directions of the separating lines when $\theta$ increases.
Figure 6: Firm’s Optimal Strategy when $\theta = 1.5$

Notes:

- We focus on the shaded area where all ratings exist. The parameter space can be partitioned into six regions: in region A, $p_i^* = 0$, $k^* = 1$, and $s^* = \bar{s} = 10$; in region B, $p_i^* = 0$, $k^* < 1$, and $s^* = \bar{s} = 10$; in region C, $p_i^* = \bar{p}_i$, $k^* < 1$, and $s^* = \bar{s} = 10$; in region D, $p_i^* = \bar{p}_i$, $k^* = 1$, and $s^* = \bar{s} = 10$; in region E, $0 < p_i^* < \bar{p}_i$, $k^* = 1$, and $s^* = \bar{s} = 10$; in region F, $0 < p_i^* < \bar{p}_i$, $k^* < 1$, and $s^* = \bar{s} = 10$.
- The double arrows indicate the changing directions of the separating lines when $\theta$ increases.
Table 1: Literature Review of Consumer Review Systems

<table>
<thead>
<tr>
<th>Paper</th>
<th>Methodology</th>
<th>Design of review systems</th>
<th>Review outcomes</th>
<th>Contextual characteristics</th>
<th>Main findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archak et al. 2011</td>
<td>Empirical</td>
<td>● 1-5 star scale</td>
<td>● Mean</td>
<td>● Product publicity measured by Google</td>
<td>● Textual review content can be used to learn consumers’ preferences for different product features and to help predict product sales.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Volume</td>
<td>search volume</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Length</td>
<td>● Product age</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Textual content</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chen and Xie 2008</td>
<td>Analytical</td>
<td>● Binary decision</td>
<td>● Informativeness</td>
<td>● Product cost</td>
<td>● When the product cost is high (low) and there are sufficient expert (novice) consumers, the firm should increase (decrease) the amount of available product information.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>whether or not to supply consumer reviews</td>
<td></td>
<td>● Consumer expertise</td>
<td>Product/market conditions are identified for firms to facilitate consumer reviews as well as to strategically delay the availability of consumer reviews.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Variable timing of</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>offering consumer reviews</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chevalier and Mayzlin 2006</td>
<td>Empirical</td>
<td>● 1-5 star scale</td>
<td>● Mean</td>
<td>● N/A</td>
<td>● Mean rating is positively related to sales.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Volume</td>
<td></td>
<td>● The impact of one-star reviews is greater than the impact of five-star reviews.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Percentage of one-star and five-star ratings</td>
<td></td>
<td>● The relationship between review length and sales is positive and statistically significant at Amazon.com. However, the relationship is negative and insignificant at bn.com.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clemons et al. 2006</td>
<td>Empirical</td>
<td>● 0-5 scale</td>
<td>● Mean</td>
<td>● Firm age</td>
<td>● Mean, standard deviation, and mean of top quartile of the ratings are positively associated with sales growth.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Standard deviation</td>
<td></td>
<td>Volume and mean of bottom quartile of the ratings do not have a significant impact on sales growth.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Volume</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Mean of top quartile</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Mean of bottom quartile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dellarocas et al. 2007</td>
<td>Empirical</td>
<td>● 1-5 scale</td>
<td>● Mean</td>
<td>● Professional critics reviews</td>
<td>● Consumer reviews increase the accuracy of forecasting box office revenue.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>● Volume</td>
<td>● Reviewer age and gender distributions</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>● Marketing cost</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>● Movie genre</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>● MPAA rating</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>● Theaters</td>
<td></td>
</tr>
<tr>
<td>Paper</td>
<td>Methodology</td>
<td>Design of review systems</td>
<td>Review outcomes</td>
<td>Contextual characteristics</td>
<td>Main findings</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-------------</td>
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<td>-----------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Dellarocas et al. 2010</td>
<td>Empirical</td>
<td>1-5 scale</td>
<td>Mean, Volume</td>
<td>Movie genre, MPAA rating, Theaters, Sequel, Competition, Holiday</td>
<td>Consumers are more likely to contribute reviews for both hit and niche products.</td>
</tr>
<tr>
<td>Duan et al. 2008</td>
<td>Empirical</td>
<td>3-13 scale</td>
<td>Mean, Volume</td>
<td>Professional critics reviews, Movie production budget, Marketing cost, Number of screens</td>
<td>A movie’s box office revenue and mean consumer rating significantly influence the volume of consumer reviews. The volume of consumer reviews, in turn, significantly influences the box office revenue.</td>
</tr>
<tr>
<td>Forman et al. 2008</td>
<td>Empirical</td>
<td>1-5 star scale</td>
<td>Mean, Volume, Percentage of equivocal reviews</td>
<td>Reviewer identity disclosure, Shared geographical location</td>
<td>Reviews with reviewer identity information are perceived as more helpful by online community members. The prevalence of reviewer identity disclosure is positively associated with sales.</td>
</tr>
<tr>
<td>Gao et al. 2011</td>
<td>Empirical</td>
<td>1-5 scale</td>
<td>Mean, Volume</td>
<td>Peer rating, Offline rating, Physician gender, certification, experience, Practice location</td>
<td>Patients are less likely to review physicians with low perceived quality online. Opinions expressed in online reviews are exaggerated compared to offline opinions.</td>
</tr>
<tr>
<td>Hu et al. 2009</td>
<td>Empirical</td>
<td>1-5 star scale</td>
<td>Mean, Percentage of one-star and five-star ratings</td>
<td>N/A</td>
<td>Purchasing bias and under-reporting bias exist in online consumer reviews, which result in a J-shaped distribution of product reviews. Measures such as standard deviation and two modes of the product ratings should be used to better predict sales.</td>
</tr>
<tr>
<td>Kuksov and Xie 2010</td>
<td>Analytical</td>
<td>Binary scale {0,1}</td>
<td>Mean</td>
<td>Market growth rate</td>
<td>Depending on the market growth rate, there are three possible optimal pricing and frills decisions for the firm – lowering price, lowering price and offering frills, and raising price and offering frills.</td>
</tr>
<tr>
<td>Paper</td>
<td>Methodology</td>
<td>Design of review systems</td>
<td>Review outcomes</td>
<td>Contextual characteristics</td>
<td>Main findings</td>
</tr>
<tr>
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<td>------------------------</td>
<td>------------------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Liu 2006</td>
<td>Empirical</td>
<td>• Textual user comments</td>
<td>• Volume</td>
<td>• Professional critics reviews</td>
<td>• Consumer reviews offer significant explanatory power for box office revenue.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Percentages of positive and negative messages</td>
<td>• Movie production budget</td>
<td>• Most of the explanatory power comes from the volume and not from the valence (percentages of positive and negative messages) of consumer reviews.</td>
</tr>
<tr>
<td>Sun 2012</td>
<td>Analytical and Empirical</td>
<td>• Uniformly distributed rating based on consumption utility for the analytical part</td>
<td>• Mean</td>
<td>• Product popularity</td>
<td>• When the mean rating is low, a higher variance corresponds to a higher demand.</td>
</tr>
<tr>
<td>Zhu and Zhang 2010</td>
<td>Empirical</td>
<td>• 1-10 scale</td>
<td>• Mean</td>
<td>• Misfit cost</td>
<td>• All three review outcomes (mean, coefficient of variation, and volume) are more influential for less popular games and games targeted at consumers with greater internet experience.</td>
</tr>
<tr>
<td>This paper</td>
<td>Analytical</td>
<td>• Variable rating scale</td>
<td>• Mean</td>
<td>• Product popularity</td>
<td>• A low rating scale is suggested for niche products and a high rating scale is suggested for popular products.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Variable product attribute disclosure</td>
<td>• Volume</td>
<td>• Misfit cost</td>
<td>• Firms should disclose specific product attribute information to attract the desired consumer segment when product quality is low relative to misfit cost, and the resulting optimal size of the targeted consumer market increases in product popularity and product quality.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Product attribute summary</td>
<td>• Product quality</td>
<td>• For niche products, firms are advised to adopt lower-bound pricing for high-quality products to take advantage of the positive word of mouth.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• For popular products, firms are advised to adopt upper-bound pricing for high-quality products to enjoy the direct profit from the initial sale without damaging the product review outcomes.</td>
</tr>
</tbody>
</table>
Table 2: Model Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu )</td>
<td>True quality of the product</td>
</tr>
<tr>
<td>( \underline{\nu} ) and ( \overline{\nu} )</td>
<td>Lower bound and upper bound of consumers’ pre-purchase perceived product quality. Consumers share a common pre-purchase belief that the product quality ( \nu ) is uniformly distributed between ( \underline{\nu} ) and ( \overline{\nu} ).</td>
</tr>
<tr>
<td>( \hat{\nu}_i )</td>
<td>Consumers’ pre-purchase expected product quality in period ( i )</td>
</tr>
<tr>
<td>( t )</td>
<td>Consumers’ unit misfit cost</td>
</tr>
<tr>
<td>( x )</td>
<td>A particular customer’s taste for the product ( x \in [0,1] ). Since the product is located at 0, ( x ) also represents the product misfit for the customer.</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>PDF of consumer taste (misfit) distribution</td>
</tr>
<tr>
<td>( F(x) )</td>
<td>CDF of consumer taste (misfit) distribution</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Product popularity parameter ( \theta \in [0,2] ). ( \theta \in [1,2] ) correspond to popular products, ( \theta \in [0,1] ) correspond to niche products, and ( \theta = 1 ) corresponds to neutral products.</td>
</tr>
<tr>
<td>( \hat{x}_i )</td>
<td>Consumers’ pre-purchase expected product misfit in period ( i )</td>
</tr>
<tr>
<td>( \hat{u}_i )</td>
<td>Consumers’ pre-purchase expected utility in period ( i )</td>
</tr>
<tr>
<td>( u(x) )</td>
<td>First-period consumer ( x )’s post-purchase utility</td>
</tr>
<tr>
<td>( w(x) )</td>
<td>Utility score of a consumer based on the consumer’s post-purchase utility ( u(x) )</td>
</tr>
<tr>
<td>( R(\cdot) )</td>
<td>Rating function defined in formula (1)</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Consumers’ propensity to review the product</td>
</tr>
<tr>
<td>( p_i )</td>
<td>Price of the product in period ( i )</td>
</tr>
<tr>
<td>( \bar{p}_i )</td>
<td>Maximum first-period price and ( \bar{p}_i = \frac{(\overline{\nu} + \nu)}{2} - \left( \frac{2 - \theta}{3 - 6} \right) t )</td>
</tr>
<tr>
<td>( s )</td>
<td>Rating scale of the consumer review system and ( s = {2,3,...,\bar{s}} )</td>
</tr>
<tr>
<td>( \bar{s} )</td>
<td>Maximum rating scale</td>
</tr>
<tr>
<td>( k )</td>
<td>Product attribute disclosure</td>
</tr>
<tr>
<td>( \mu(\cdot) )</td>
<td>Mean rating of the product</td>
</tr>
<tr>
<td>( m(\cdot) )</td>
<td>Numerator of consumer mean rating</td>
</tr>
<tr>
<td>( n(\cdot) )</td>
<td>Rating volume</td>
</tr>
<tr>
<td>( \pi_i )</td>
<td>Firm’s profit in period ( i )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Firm’s total profit of two periods</td>
</tr>
</tbody>
</table>
Appendix A: Discussion of parameter conditions such that all rating levels are rated

Given the firm’s choice of first-period price $p_1$, if $v - p_1 - \ln\left(\frac{1 - \varepsilon}{\varepsilon}\right) \geq 0$ is not satisfied, then no first-period consumers will give the highest rating 1 and the overall ratings will scale down.

Similarly, if $t \geq v - p_1 + \ln\left(\frac{1 - \varepsilon}{\varepsilon}\right)$ is not satisfied, then no first-period consumers will give the lowest rating 0 and the overall ratings will scale up. For example, if the quality is high such that $v > t + p_1 - \ln\left(\frac{1 - \varepsilon}{\varepsilon}\right)$, $w^{-1}\left(\frac{i}{s-1} - \varepsilon\right) \geq 1$ and $w^{-1}\left(\frac{i}{s-1} + \varepsilon\right) < 1$ with $1 < i < s - 1$, then the lowest consumer rating will be $\frac{i}{s-1}$. The review volume and mean rating can be updated accordingly.

A more general setting is that all rating levels exist in the consumer rating results. We therefore solve the model for this general setting and assume parameter conditions $v - p_1 - \ln\left(\frac{1 - \varepsilon}{\varepsilon}\right) \geq 0$ and $t \geq v - p_1 + \ln\left(\frac{1 - \varepsilon}{\varepsilon}\right)$ are satisfied for $0 \leq p_1 \leq \bar{p}_1$, where $\bar{p}_1 = \left(\frac{v + v}{2}\right) - \left(\frac{2}{3} - \frac{\theta}{6}\right)t$.

Appendix B: Derivations of Important Values

Derivation of the review volume $n(p_1, s)$

When there are ratings for each of the rating levels, the total review volume is given by

$$n(p_1, s) = \int_0^{w^{-1}(1-\varepsilon)} f(x) dx + \sum_{i=1}^{s-2} \left[ \int_{w^{-1}(\frac{i}{s-1} - \varepsilon)}^{w^{-1}(\frac{i}{s-1} + \varepsilon)} f(x) dx \right] + \int_{w^{-1}(s)}^{s} f(x) dx,$$

where

$$w^{-1}(1-\varepsilon) = \frac{1}{i} \left[ v - p_1 - \ln\left(\frac{1 - \varepsilon}{\varepsilon}\right) \right], \quad w^{-1}\left(\frac{i}{s-1} + \varepsilon\right) = \frac{1}{i} \left[ \ln\left(\frac{(1-\varepsilon)(s-1) - i}{i + \varepsilon(s-1)}\right) + v - p_1 \right].$$
\[ w^{-1}(\frac{i}{s-1} - \epsilon) = \frac{1}{t} \left[ \ln \frac{(1+\epsilon)(s-1) - i}{i - \epsilon(s-1)} + v - p_i \right], \quad w^{-1}(\epsilon) = \frac{1}{t} \left[ v - p_i + \ln \frac{1-\epsilon}{\epsilon} \right], \] and

\[ f(x) = \theta + 2x(1-\theta). \] Substituting the corresponding terms into the expression of the review volume yields

\[ n(p_i, s) = 1 - \frac{2(1-\theta)(v-p_i) + t\theta}{t^2} \ln \left( \frac{1-\epsilon}{\epsilon} D_1 \right), \] where

\[ D_1 = \frac{\left[ 1-(s-1)\epsilon \right] \left[ 2-(s-1)\epsilon \right] \ldots \left[ (s-2)-(s-1)\epsilon \right]}{\left[ 1+(s-1)\epsilon \right] \left[ 2+(s-1)\epsilon \right] \ldots \left[ (s-2)+(s-1)\epsilon \right]} \text{ for } s \geq 3. \] When \( s = 2 \), the review volume can be simplified as

\[ n(p_i, s) = 1 - \frac{2(1-\theta)(v-p_i) + t\theta}{t^2} \ln \left( \frac{1-\epsilon}{\epsilon} \right). \]

**Derivation of the average rating \( \mu(p_i, s) \)**

Mean rating is given by

\[ \mu(p_i, s) = \frac{1}{n(p_i, s)} \left[ (1) \int_0^{w^{-1}(1-\epsilon)} f(x) \, dx + \sum_{i=1}^{s-2} \int_{w^{-1}(\frac{i}{s-1}+\epsilon)}^{w^{-1}(1-\epsilon)} f(x) \, dx \right]. \]

The denominator of \( \mu(p_i, s) \) is just the review volume \( n(p_i, s) \). We only need to derive the numerator of \( \mu(p_i, s) \), denoted by \( m(p_i, s) \).

The first term of \( m(p_i, s) \) can be simplified as

\[ \int_0^{w^{-1}(1-\epsilon)} f(x) \, dx = \frac{1-\theta}{t^2} \left[ v - p_i - \ln \left( \frac{1-\epsilon}{\epsilon} \right) \right] + \frac{\theta}{t} \left[ v - p_i - \ln \left( \frac{1-\epsilon}{\epsilon} \right) \right]. \] The second term can be rewritten as

\[ \sum_{i=1}^{s-2} \int_{w^{-1}(\frac{i}{s-1}+\epsilon)}^{w^{-1}(1-\epsilon)} f(x) \, dx = -\frac{2(1-\theta)(v-p_i) + t\theta}{t^2} \ln D_1 \]

\[ -\left( \frac{1-\theta}{t^2} \right) \sum_{i=1}^{s-2} \left( \frac{i}{s-1} \right) \left[ \ln \left( \frac{(s-1-i)-(s-1)\epsilon}{i+(s-1)\epsilon} \right)^2 - \ln \left( \frac{(s-1-i)+(s-1)\epsilon}{i-(s-1)\epsilon} \right)^2 \right]. \]

Substituting these terms back to \( m(p_i, s) \) yields

\[ m(p_i, s) = \left( \frac{1-\theta}{t^2} \right) \left[ v - p_i - \ln \left( \frac{1-\epsilon}{\epsilon} \right) \right]^2 \]

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\[\left(\frac{\theta}{t}\right)\left[v-p_1-\ln\left(\frac{1-\epsilon}{\epsilon}\right)\right]-\frac{2(1-\theta)(v-p_1)+t\theta}{t^2}\ln D_1-\left(1-\theta\right)\frac{\int(s-2)/2}{t^2}\sum_{i=1}^{\int(s-2)/2} \frac{s-1-2i}{s-1}\ln D_2\ln D_3,\]

where \(\int(s-2)/2\) takes the integer part of \((s-2)/2\),

\[D_2 = \left\langle \frac{s-1-i-(s-1)\epsilon}{i+(s-1)\epsilon} \right\rangle \left\langle \frac{(s-1-i)+(s-1)\epsilon}{i-(s-1)\epsilon} \right\rangle,\]

and

\[D_3 = \left\langle \frac{i+(s-1)\epsilon}{(s-1-i)-(s-1)\epsilon} \right\rangle \left\langle \frac{(s-1-i)+(s-1)\epsilon}{i-(s-1)\epsilon} \right\rangle\]

for \(s \geq 3\). When \(s = 2\), we can further simplify

\[m(p_1,s) = m(p_1,s) = \frac{1-\theta}{t^2}\left[v-p_1-\ln\left(\frac{1-\epsilon}{\epsilon}\right)\right]^2 + \frac{\theta}{t}\left[v-p_1-\ln\left(\frac{1-\epsilon}{\epsilon}\right)\right].\]

Appendix C: Proofs of Propositions and Lemmas

**Proof of Lemma 1**

We first analyze the properties of rating volume \(n(p_1,s)\). Since \(\partial\left(\frac{1-\epsilon}{\epsilon}\right)/\partial \epsilon = -\frac{1}{\epsilon^2} < 0\) and

\[\partial \left[\frac{i-(s-1)\epsilon}{i+(s-1)\epsilon}\right]/\partial \epsilon = \frac{-2i(s-1)}{[i+(s-1)\epsilon]^2} < 0,\] for a given \(s\), the natural logarithm term in \(n(p_1,s)\) decreases in \(\epsilon\) and thus rating volume increases in \(\epsilon\).

At \(\epsilon = \frac{1}{2(s-1)}\), we can rewrite the natural logarithm term as

\[\ln\frac{(1-\epsilon)[1-2(s-2)\epsilon-\epsilon][1-2(s-2)\epsilon+\epsilon][1-2(s-3)\epsilon+\epsilon]...(1-5\epsilon)(1-3\epsilon)}{\epsilon[1-2(s-2)\epsilon+\epsilon][1-2(s-3)\epsilon+\epsilon]...(1-3\epsilon)(1-\epsilon)} = \ln 1 = 0.\] Thus all customers rate when \(\epsilon \geq \frac{1}{2(s-1)}\). When \(\epsilon < \frac{1}{2(s-1)}\), the natural logarithm term is positive and not every
customer will rate the product. Since the total market size is 1, this implies that
\[ 2(1-\theta)(v - p_1) + t\theta > 0. \]

Since \( \frac{\partial n(p_1,s)}{\partial p_1} = \frac{4(1-\theta)}{t^2} \ln \left( \frac{1-\varepsilon}{\varepsilon} D_1 \right) \) and \( \frac{\partial n(p_1,s)}{\partial v} = \frac{4(\theta-1)}{t^2} \ln \left( \frac{1-\varepsilon}{\varepsilon} D_1 \right) \), for popular products the rating volume decreases in the first-period price but increases in the product quality, whereas for niche products the rating volume increases in the first-period price but decreases in the product quality. For neutral products, the rating volume is independent of the first-period price and product quality.

Next, we analyze the properties of mean rating \( \mu(p_1,s) \). Mean rating \( \mu(p_1,s) \) is the weighted average of the reviews where each rating level is weighted by the corresponding number of ratings and the rating volume is the total weight. Recall that we rewrite \( \mu(p_1,s) \) as \( \mu(p_1,s) = m(p_1,s)/n(p_1,s) \), the relationship between mean rating and the first-period price can be represented as
\[ \frac{\partial \mu(p_1,s)}{\partial p_1} = \frac{n(p_1,s) \partial m(p_1,s)/\partial p_1 - m(p_1,s) \partial n(p_1,s)/\partial p_1}{\left[ n(p_1,s) \right]^2} \text{, where } \frac{\partial n(p_1,s)}{\partial p_1} \]
is given earlier and \( \frac{\partial m(p_1,s)}{\partial p_1} = \frac{1}{t^2} \left[ -2(1-\theta) \left( v - p_1 - \ln \frac{1-\varepsilon}{\varepsilon} D_1 \right) - \theta t \right] < 0 \). For niche and neutral products (\( 0 \geq \theta \leq 1 \)), \( \partial n(p_1,s)/\partial p_1 \geq 0 \) and thus the mean rating decreases in price. For popular products (\( 1 < \theta \leq 2 \)), the numerator of \( \partial \mu(p_1,s)/\partial p_1 \) satisfies
\[ n(p_1,s) \partial m(p_1,s)/\partial p_1 - m(p_1,s) \partial n(p_1,s)/\partial p_1 < n(p_1,s) \left[ \partial m(p_1,s)/\partial p_1 - \partial n(p_1,s)/\partial p_1 \right] < 0 \]
since \( m(p_1,s) < n(p_1,s) \), \( \partial n(p_1,s)/\partial p_1 < 0 \), and \( t \geq v - p_1 + \ln \left( \frac{1-\varepsilon}{\varepsilon} \right) \). Thus the mean rating
decreases in the first-period price for all product types. Since \( \partial \mu(p_1, s)/\partial p_1 = -\partial \mu(p_1, s)/\partial v \), the mean rating increases in product quality.

**Proof of Lemma 2**

The second-period consumers’ expected product quality is given by

\[
\hat{v}_2 = n(p_1, s)\left[ v + \mu(p_1, s)(\bar{v} - v) \right] + \left[ 1 - n(p_1, s) \right] \left( \frac{\bar{v} + v}{2} \right).
\]

Substituting the \( n(p_1, s) \) and \( \mu(p_1, s) \) terms derived in Appendix B back to \( \hat{v}_2 \) yields

\[
\hat{v}_2 = v + (\bar{v} - v) \left( 1 - \theta \right) \left[ (v - p_1) + D_4 - D_5 \right] + \theta t (v - p_1),
\]

where

\[
D_4 = \left[ \ln \left( \frac{1 - \varepsilon}{\varepsilon} \right) \right]^2
\]

and

\[
D_5 = \sum_{i=1}^{\text{int}(s-2)/2} \left( \frac{s-1-2i}{s-1} \right) \ln D_2 \ln D_3 > 0.
\]

The second-period expected quality decreases in the first-period price, since

\[
\frac{\partial \hat{v}_2}{\partial p_1} = -\frac{(\bar{v} - v) \left[ 2(1 - \theta)(v - p_1) + \theta t \right]}{t^2} < 0.
\]

The second-period expected quality increases in the true quality \( v \), since

\[
\frac{\partial \hat{v}_2}{\partial v} = \frac{\bar{v} - v}{t^2} \left[ 2(1 - \theta)(v - p_1) + \theta t \right] > 0,
\]

which is inferred from \( n(p_1, s) \leq 1 \).

To check the impact of the rating scale on the second-period expected quality, we need to compare \( \hat{v}_2(p_1, s+1) - \hat{v}_2(p_1, s) \), which can be simplified as

\[
\frac{(\theta - 1)(\bar{v} - v)}{t^2} \left[ \sum_{i=1}^{\text{int}(s-1)/2} \left( \frac{s-2i}{s} \right) \ln D_6 \ln D_7 - \sum_{i=1}^{\text{int}(s-2)/2} \left( \frac{s-1-2i}{s-1} \right) \ln D_2 \ln D_3 \right],
\]

where \( D_6 \) and \( D_7 \) are in the same form as \( D_2 \) and \( D_3 \) but replacing \( s \) with \( s+1 \). Because \( i \) is up to \( \text{int}(s-1)/2 \) or \( \text{int}(s-2)/2 \), the values of \( D_2, D_3, D_6, D_7 \) are all greater than 1. For a given \( i \), the values \( \frac{s-2i}{s} \),...
\( D_6 \) and \( D_2 \) increase in \( s \), which implies that \( \left( \frac{s-2i}{s} \right) \ln D_6 \ln D_2 > \left( \frac{s-1-2i}{s-1} \right) \ln D_2 \ln D_3 \). Hence the sign of the second-period expected quality difference depends on \( \theta \). Specifically,

\[
\hat{v}_2(p_1, s+1) > \hat{v}_2(p_1, s) \text{ for } \theta > 1; \quad \hat{v}_2(p_1, s+1) < \hat{v}_2(p_1, s) \text{ for } \theta < 1; \text{ and } s \text{ has no impact on } \hat{v}_2 \text{ for } \theta = 1.
\]

When \( p_1 + \ln \left( \frac{1-e}{e} \right) < v < t + p_1 - \ln \left( \frac{1-e}{e} \right) \), all rating levels exist. This implies that

\[
\frac{\partial \hat{v}_2}{\partial \theta} = \frac{v - y}{t^2} \left[ (v - p_1)(t - v + p_1) + D_3 - D_4 \right] > 0.
\]

Thus the second-period expected quality increases in the product popularity parameter \( \theta \).

We next evaluate the impact of the unit misfit cost on the second-period expected quality.

Since \( \frac{\partial \hat{v}_2}{\partial t} = - (\bar{v} - y) \frac{2(1-\theta) [ (v - p_1)^2 + D_4 - D_3 ] + \theta t (v - p_1) }{t^3} \), we know that \( \frac{\partial \hat{v}_2}{\partial t} < 0 \) for niche and neutral products (\( \theta \leq 1 \)). At \( \theta = 2 \), we have \( \frac{\partial \hat{v}_2}{\partial t} \bigg|_{\theta = 2} < 0 \). Since the numerator of \( \frac{\partial \hat{v}_2}{\partial t} \) is monotone in \( \theta \), \( \frac{\partial \hat{v}_2}{\partial t} < 0 \) for all popular products.

**Proof of Proposition 1**

To determine the firm’s optimal choice on the rating scale \( s \), we need to compare its profit level at the rating scale \( s \) with that at \( s+1 \) for any given first-period price level \( p_1 \). The firm’s profit function can be simplified as \( \pi(p_1, s) = p_1 + \hat{v}_2(p_1, s) - t\bar{x}_i \), and the profit difference is then given by \( \pi(p_1, s+1) - \pi(p_1, s) = \hat{v}_2(p_1, s+1) - \hat{v}_2(p_1, s) \). As shown in Lemma 2, the sign of the profit difference depends on \( \theta \). Specifically, \( \pi(p_1, s+1) > \pi(p_1, s) \) for \( \theta > 1 \);
\( \pi(p_i, s+1) = \pi(p_i, s) \) for \( \theta < 1 \); and \( s \) has no impact on profit for \( \theta = 1 \). As a result, the firm selects the maximum rating scale \( s^* = \bar{s} \) for popular products, the minimum rating scale \( s^* = 2 \) for niche products, and any integer between 2 and \( \bar{s} \) for neutral products.

**Proof of Proposition 2**

The first derivative of profit over \( p_i \) is given by

\[
\frac{\partial \pi(p_i, s)}{\partial p_i} = 1 + (\bar{v} - v) \frac{\partial m(p_i, s)}{\partial p_i} - \left( \frac{\bar{v} - v}{2} \right) \frac{\partial n(p_i, s)}{\partial p_i},
\]

where

\[
\frac{\partial m(p_i, s)}{\partial p_i} = \frac{2(1 - \theta)}{t^2} \ln D_i - \frac{2(1 - \theta)}{t^2} \left[ v - p_i - \ln \left( \frac{1 - \epsilon}{\epsilon} \right) \right] - \frac{\theta}{t}
\]

and

\[
\frac{\partial n(p_i, s)}{\partial p_i} = \frac{4(1 - \theta)}{t^2} \left[ \ln \left( \frac{1 - \epsilon}{\epsilon} \right) + \ln D_i \right].
\]

Thus the first and second derivatives of profit with respect to \( p_i \) can be written as

\[
\frac{\partial \pi}{\partial p_i} = 1 - \left( \frac{\bar{v} - v}{t^2} \right) \left[ 2(1 - \theta)(v - p_i) + t\theta \right]
\]

and

\[
\frac{\partial^2 \pi}{\partial p_i^2} = \frac{2(1 - \theta)(\bar{v} - v)}{t^2}.
\]

Since the shape of the profit function depend on product popularity, we analyze three cases – popular, neutral, and niche products, separately.

For popular products \( (\theta > 1) \), \( \frac{\partial^2 \pi}{\partial p_i^2} < 0 \). Thus the profit function is concave in \( p_i \). Solving first order condition yields

\[
p_i^* = v - \frac{t\theta(\bar{v} - v) - t^2}{2(\theta - 1)(\bar{v} - v)}.
\]

This interior solution is feasible, if

\[
0 \leq v - \frac{t^2 - t\theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \leq p_i^*. \text{ Therefore, if the true valuation is high, i.e.,}
\]

\[
v > p_i^* + \frac{t^2 - t\theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)},\text{ then we will have one boundary solution } p_i^* = \bar{p}_i; \text{ if the true valuation}
\]
is medium, i.e., \( t^2 - t \theta (\bar{v} - v) \leq v \leq \bar{p}_1 + \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), we will then have the interior solution \( p_1^* = v - \frac{t \theta (\bar{v} - v) - t^2}{2(\theta - 1)(\bar{v} - v)} \); if the true valuation is low, i.e., \( v < \frac{t^2 - t \theta (\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then we will have the other boundary solution \( p_1^* = 0 \).

For neutral products (\( \theta = 1 \)), \( \frac{\partial \pi}{\partial p_1} = 1 - \frac{\bar{v} - v}{t} \) which means the profit function is linear in \( p_1 \). If customers’ quality uncertainty is high relative to the misfit cost, i.e., \( \bar{v} - v > t \), then the profit decreases in \( p_1 \) and thus \( p_1^* = 0 \). If customers’ quality uncertainty is low relative to the misfit cost, i.e., \( \bar{v} - v < t \), then the profit increases in \( p_1 \) and thus the optimal price is the maximum value \( p_1^* = \bar{p}_1 \).

For niche products (\( \theta < 1 \)), \( \frac{\partial^2 \pi}{\partial p_1^2} > 0 \). Thus the optimal \( p_1 \) will take a boundary solution and we need to compare \( \pi(0,s) \) and \( \pi(\bar{p}_1,s) \), to determine the optimal price \( p_1 \). The profit difference can be written as

\[
\pi(\bar{p}_1,s) - \pi(0,s) = \bar{p}_1 + \left[ m(\bar{p}_1,s) - m(0,s) \right] (\bar{v} - v) - \left[ n(\bar{p}_1,s) - n(0,s) \right] \left( \frac{\bar{v} - v}{2} \right),
\]

where

\[
m(\bar{p}_1,s) - m(0,s) = \frac{1 - \theta}{t} \left[ \bar{p}_1^2 - 2\bar{p}_1 \left( v - \ln \frac{1 - \varepsilon}{\varepsilon} \right) \right] - \frac{\theta \bar{p}_1}{t} + \frac{2(1 - \theta) \bar{p}_1}{t^2} \ln D_1 \quad \text{and}
\]

\[
n(\bar{p}_1,s) - n(0,s) = \frac{4(1 - \theta) \bar{p}_1}{t^2} \left[ \ln \left( \frac{1 - \varepsilon}{\varepsilon} \right) + \ln D_1 \right].
\]

Combining terms together, we can simplify
the profit difference as 
\[ \pi(\bar{p}_i, s) - \pi(0, s) = \bar{p}_i \left\{ \frac{(\bar{v} - v)(1 - \theta)}{t^2} \bar{p}_i + 1 - \frac{(\bar{v} - v)[2(1 - \theta)v + t\theta]}{t^2} \right\}. \]

Therefore, if \( v \leq \frac{\bar{p}_i}{2} + \frac{t^2 - t\theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i^* = \bar{p}_i \); if \( v > \frac{\bar{p}_i}{2} + \frac{t^2 - t\theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \), then \( p_i^* = 0 \).

**Proof of Lemma 3**

In order to determine the firm’s optimal choice of rating scale \( s \), we need to compare its profit level at the rating scale \( s \) with that at \( s + 1 \) for any given pair of \( k \) and \( p_i \). The firm’s profit function can be simplified as 
\[ \pi(p_i, k, s) = p_i + F(k)\hat{v}_2(p_i, s) - \int_0^k xf(x)dx, \]
and the profit difference is then given by 
\[ \pi(p_i, k, s + 1) - \pi(p_i, k, s) = F(k)\left[\hat{v}_2(p_i, s + 1) - \hat{v}_2(p_i, s)\right]. \]

As shown in Lemma 2, the sign of the profit difference depends on \( \theta \). Specifically, 
\[ \pi(p_i, k, s + 1) > \pi(p_i, k, s) \] for \( \theta > 1 \) and the firm selects the maximum rating scale \( s^* = \bar{s} \) for popular products; 
\[ \pi(p_i, k, s + 1) < \pi(p_i, k, s) \] for \( \theta < 1 \) and the firm selects the minimum rating scale \( s^* = 2 \) for niche products; and \( s \) has no impact on profit for neutral products (\( \theta = 1 \)).

We next derive the solution structure of the design choice \( k \). The firm’s profit function is a cubic function of \( k \). For a given pair of \( p_i \) and \( s \), the first derivative of profit with respect to \( k \) is given by 
\[ \frac{\partial \pi}{\partial k} = (\hat{v}_2 - kt)[\theta + 2k(1 - \theta)]. \]

We are interested in finding the optimal value of \( k \) for \( 0 \leq k \leq 1 \) with \( \theta \in [0, 2] \). Solving first order condition leads to two roots \( k_1 = \hat{v}_2/t \) and \( k_2 = \theta/2(\theta - 1) \), with one root corresponding to the local maximum and the other corresponding to the local minimum. The coefficient of the \( k^3 \) term is \( 2(\theta - 1)/3 \), which is negative for niche products (\( 0 \leq \theta < 1 \)) and positive for popular products (\( 1 < \theta \leq 2 \)). Thus for niche products
(0 ≤ θ < 1), the second root \( k_2 = \theta/2(\theta - 1) \) is negative and corresponds to the local minimum, and the first root \( k_1 = \hat{v}_2/t \) corresponds to the local maximum. If \( \hat{v}_2 < t \), then \( k^* = \hat{v}_2/t < 1 \) is the optimal design choice, which corresponds to the special appeal disclosure strategy; if \( \hat{v}_2 > t \), then \( k^* = 1 \) is the optimal design choice, which corresponds to the general appeal disclosure strategy.

For popular products (1 < θ ≤ 2), the second root \( k_2 = \theta/2(\theta - 1) \geq 1 \). Thus if \( \hat{v}_2 < t \), then \( k^* = \hat{v}_2/t < 1 \) (the local maximum) is the optimal design choice, which corresponds to the special appeal disclosure strategy; if \( \hat{v}_2 > t \), then the firm’s profit increases in \( k \) for 0 ≤ \( k \) ≤ 1 and \( k^* = 1 \) is the optimal design choice, which corresponds to the general appeal disclosure strategy.

For niche products, the parameter condition corresponding to \( \hat{v}_2 < t \) is given by \( v < C_0 + p_1 \); for neutral products, the parameter condition is given by \( v < C_1 + p_1 \); for popular products, the parameter condition is given by \( v < C_2 + p_1 \), where

\[
C_0 = \frac{-\theta t + \sqrt{\theta^2 t^2 - 4(D_4 - D_3)(1-\theta)^2 + 4(1-\theta)t^2(t-v)/(\overline{v} - v)}}{2(1-\theta)}, \quad C_1 = \frac{t(t-v)}{\overline{v} - v}, \quad C_2 = \frac{\theta t - \sqrt{\theta^2 t^2 - 4(D_4 - D_3)(\theta - 1)^2 - 4(\theta - 1)t^2(t-v)/(\overline{v} - v)}}{2(\theta - 1)}.
\]

**Proof of Proposition 3**

For neutral products with \( \theta = 1 \), the firm’s profit is \( \pi(k, p_1, s) = p_1 + \hat{v}_2 k - tk^2/2 \), where second-period consumers’ perceived quality \( \hat{v}_2(p_1) = \overline{v} + \left(\frac{v-p_1}{t}\right) \). The firm’s profit is determined by its product attribute disclosure (\( k \)) and pricing (\( p_1 \)) choices and does not depend
on rating scale (s). As shown in Lemma 3, there are two possible optimal product attribute disclosure levels, \( k = 1 \) or \( \hat{v}_2/t \). If \( k = 1 \), then \( \pi(k, p_s) \) is a linear function in \( p_1 \). If \( k = \hat{v}_2/t \), then \( \pi(k, p_s) \) is a convex quadratic function in \( p_1 \). In either case, there are only two possible optimal price levels, \( p_1 = 0 \) or \( \bar{p}_1 \), where the maximum price can be simplified to \( \bar{p}_1 = \frac{\overline{v} + v - t}{2} \) for neutral products. The firm’s optimal solutions for \( k \) and \( p_1 \) interact with each other and have to be solved simultaneously.

Therefore, there are four possible solutions for the firm’s profit maximization problem:

- \( p_1^* = 0 \) and \( k^* = \hat{v}_2/t < 1 \) (solution A);
- \( p_1^* = 0 \) and \( k^* = 1 \) (solution B);
- \( p_1^* = \bar{p}_1 \) and \( k^* = \hat{v}_2/t < 1 \) (solution C);
- \( p_1^* = \bar{p}_1 \) and \( k^* = 1 \) (solution D).

Based on Lemma 3, we know that for neutral products, \( k^* = \hat{v}_2/t < 1 \) is the optimal solution if \( v < C_1 + p_1 \). Since there are two possible optimal price levels (0 and \( \bar{p}_1 \)), we need to consider three cases.

Case 1: If \( v \geq C_1 + \bar{p}_1 \), then solution B dominates solution A and solution D dominates solution C. Thus we need to compare the profits of solutions B and D. Comparison of the two profits show that if \( t \leq \overline{v} - \nu \), then solution B is optimal; otherwise, solution D is optimal.

Case 2: If \( C_1 \leq v < C_1 + \bar{p}_1 \), then solution B dominates solution A and solution C dominates solution D. Thus we need to compare the profits of solutions B and C. Comparison of the two profits show that if \( v \geq C_4 \), where \( C_4 = \frac{t \left[2t - \overline{v} - \nu - 2\sqrt{(t - \overline{v})^2 - \nu^2}\right] + \overline{v}^2 - \nu^2}{2(\overline{v} - \nu)} \), then solution B is optimal; otherwise, solution C is optimal.
Case 3: If $v < C_1$, then solution A dominates solution B and solution C dominates solution D. Thus we need to compare the profits of solutions A and C. Comparison of the two profits show that if $v \geq C_3$, where $C_3 = \frac{4t^3 - t(3v + \overline{v})(\overline{v} - v) + (\overline{v} + v)(\overline{v} - v)^2}{4(\overline{v} - v)^2}$, then solution B is optimal; otherwise, solution C is optimal.

We summarize the above comparisons and simplify the separating conditions to derive Proposition 3.

References


