Platform or Wholesale?
Different Implications for Retailers of Online Product Reviews

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Abstract

Online retailing is dominated by a channel structure in which a retailer either buys products from competing manufacturers and resells to consumers (wholesale scheme) or lets manufacturers directly sell to consumers on its platform for a commission (platform scheme), and is characterized by easy access to product reviews to facilitate consumers’ purchase decisions. We study how different types of information revealed by reviews affect the retailer under the wholesale scheme and platform scheme. We find that information provided by reviews on quality dimension homogenizes consumers’ perceived utility differences between products and increases upstream competition, which benefits the retailer under the wholesale scheme but hurts the retailer under the platform scheme. Information provided by reviews on fit dimension heterogenizes consumers’ estimated fits to products and softens upstream competition, which hurts the retailer under the wholesale scheme and benefits the retailer under the platform scheme. Together, we demonstrate that the quality information and fit information play very different roles in changing upstream competition, and whether the retailer benefits from reviews critically depends on its pricing scheme choice.

Keywords: online product reviews; pricing scheme; competition; game theory

1 Introduction

With the ubiquity of the Internet and the evident advantage with E-Commerce, online retailing has been continuously growing for the past few years. Forrest Research (2012) reported that online retail sales reached $200 billion in 2011 and accounted for 7 percent of overall retail sales in the U.S. Two unique features associated with online retailing platform have been well documented (Abhishek
et al., 2012; Chen and Xie, 2008). One, in addition to selling products using a wholesale scheme in which the retailer purchases products from manufacturer and then resells to consumers, an online retailer often lets others sell their products on its platform for a commission fee for each sale which we refer to as platform scheme. Platform-based retailing has become pervasive in online markets, Amazon being a dominant platform-based online retailer. Two, regardless of the pricing scheme, online retail platforms routinely provide features that enable consumers to post product reviews and read others’ reviews. Product reviews provide additional information to consumers about products’ qualities and their fits to consumers’ needs beyond standard product descriptions, and have become an important information source for consumers to mitigate the uncertainty about the quality of a product and about its fit to their needs (Chen and Xie, 2008).

Consumer surveys report that review strongly influence consumers’ purchase decisions (Deloitte and Touche, 2008; Cone, 2010). According to Deloitte and Touche (2008), 43% of surveyed consumers were reinforced of their original purchase intention by reviews and bought the product originally intended for, whereas 43% of consumers changed their opinions about which product to buy and 9% of consumers even abandoned their purchase after reading the product reviews. Recognizing the potentially strong impact of reviews, firms have begun to incorporate these as an important element of their advertising and promotion efforts (Harmon, 2004; Strauss, 2010). Academics have also examined the impact of product reviews. For instance, many empirical studies have investigated the effect of reviews on consumers’ purchase decisions and firm’s sales (e.g., Chevalier and Mayzlin, 2006; Liu, 2006; Zhu and Zhang, 2010). Analytical studies have examined the incentives for firms to supply reviews (Mayzlin, 2006), to manipulate reviews strategically (Dellarocas, 2006), and to adjust their pricing strategies (Sun, 2011b) and other marketing mix elements (Chen and Xie, 2008). Because product information affects consumers and firms, another strand of research has studied firms’ strategic decision on the product information provision (e.g., Lewis and Sappington, 1994; Gu and Xie, 2012; Johnson and Myatt, 2006; Sun, 2011a).

Clearly, in the presence of reviews that affect consumer decision making, sellers are forced to rethink about how they interact with consumers. Retailers that use their platforms to sell products from manufacturers may also have to carefully consider the impact of reviews on upstream firms and accordingly design their upstream strategies. It appears that one important decision related to a retailer’s upstream relationship with manufacturers is the pricing scheme adopted by the retailer.
As we stated earlier, both the wholesale scheme and platform scheme are being used in online retailing. For instance, Amazon uses wholesale scheme for only 7% of the more than two million products in the “Electronics” category and the remaining 93% are sold under the platform scheme. On the other hand, Amazon uses wholesale scheme for 64 of the top 100 bestsellers in the electronics category (Jiang et al., 2011). The relative fraction of items sold using these schemes appears to be dependent on product category too. For instance, while Amazon sells 16.7% of shoes directly, it sells only 3.1% of products in Sports & Outdoors category and 3.2% of products in the Jewelry category. Jiang et al. (2011) examine how such “cherry picking” of products to be sold directly by an online retailer affects the strategic interactions between an upstream manufacturer and the retailer.

The observations that product reviews affect consumers’ choice between competing products available in the platform and the retailer can choose to use wholesale or platform schemes give rise to a number of interesting questions. How do product reviews affect competition between upstream sellers? Does the pricing scheme affect the impact of reviews on upstream sellers? How does product category affect this impact? Finally, how can a retailer use the pricing scheme strategically to its advantage in the presence of reviews that it cannot influence or manipulate?

To address the above questions, we develop a game theoretic model in which a retailer either directly sells two substitutable products produced by different manufacturers or provides a platform for the manufacturers to sell their products. The selling party can exert effort to increase the traffic to the platform and boost the demands. The products differ in both their qualities and the fits to consumers’ needs. While all consumers value high quality rather than low quality, different consumers have different needs, with some consumers perceiving one product more suitable than the other product while others perceiving the opposite way. Each consumer has her own assessment of the quality of each product and its fit to her need. The online product reviews provide additional information in both the quality and fit dimensions. We distinguish the case in which the quality dimension plays a dominant role in determining consumers’ perceived utility differences between the two products, and the case in which the fit dimension plays an important role such that the fit is critical for some consumers. We call the former the quality-dominates-fit case and the latter the fit-dominates-quality case. We use the scenario without product reviews as the benchmark and study the effect of online product reviews on the competition between the two manufacturers and ultimately on the retailer under the two pricing schemes.
We find the information in the quality dimension and fit dimension conveyed by the online product reviews have very different effects on the competition. For the quality dimension, we show that reviews reduce the heterogeneity between consumers’ perceived quality differences by combining the review difference with consumers’ private assessment, which increases the competition between the two manufacturers. We call this reduced heterogeneity resulting from the reviews variance-reducing effect, which generally hurts manufacturers. In addition, reviews shift the mean perceived quality difference in favor of the product with favorable reviews. We call this mean-shifting effect, which generally benefits the manufacturer with favorable reviews and the retailer. In contrast, for the fit dimension, we demonstrate that consumers are differentiated further from each other in their perceived fits because of the reviews, since consumers have different needs and they learn better about the products’ fits to their needs with the additional product information conveyed by the reviews. We call this increased heterogeneity resulting from the reviews variance-increasing effect, which softens the competition between the two manufacturers and generally benefits the manufacturers.

Whether the retailer can benefit from the above effects of reviews depends critically on the pricing scheme used by the retailer, in both quality-dominates-fit and fit-dominates-quality cases. In the quality-dominates-fit case, under platform scheme, on the one hand, the intensified competition resulting from the variance-reducing effect tends to drive down the retail prices as well as manufacturers’ total revenue. On the other hand, the mean-shifting effect makes the two products’ market potentials asymmetric by shifting one product’s demand to the other’s. Because consumers that are shifted to the product with favorable reviews purchase at a higher price than the price they would have paid for the other product, the revenue increase in the product with favorable reviews outweighs the revenue decrease in the product with unfavorable reviews. As a result, the mean-shifting effect tends to increase the total revenue generated by the two products. Notice that the retailer takes a fraction of manufacturers’ revenues under platform scheme. Whether the retailer benefits from the reviews depends on the tradeoff between the variance-reducing effect and mean-shifting effect. If the reviews regarding the quality of the two products are not very different such that the variance-reducing effect dominates the mean-shifting effect, the retailer is hurt by the reviews; Otherwise, the retailer benefits. Under wholesale scheme, in a sharp contrast, the retailer always benefits from the reviews. This is because the variance-reducing effect increases the upstream
competition between the two manufacturers and drives the wholesale prices down, which gives the retailer more room to exploit the market because the retailer can lower the retail prices to increase demand while increasing the profit margins. In addition, the mean-shifting effect continues to play a positive role in increasing the total revenue from the two products. Furthermore, with the positive effect on its revenue from the variance-reducing and mean-shifting effects, the retailer has more incentive to boost its traffic, which amplifies the benefit from the increased upstream competition by the reviews. As a result, unlike under the platform scheme, the retailer always benefits from the reviews under the wholesale scheme.

In the fit-dominates-quality case, variance-increasing effect plays a main role in altering the upstream competition, while mean-shifting effect continues to exist if reviews regarding quality are different for the two products. Once again, the pricing scheme plays an important role in how the retailer is affected by reviews. Under platform scheme, the reduced competition from the variance-increasing effect tends to increase each manufacturer’s revenue. The mean-shifting effect similarly shifts the demand from one product to the other and increases the total revenue from the two products because of the price asymmetry. In addition, with the benefit from the variance-increasing effect and mean-shifting effect, the increase in the traffic-boosting effort from the manufacturer with favorable reviews offsets the possible decrease in the effort from the manufacturer with unfavorable reviews, and the total effort level is higher with reviews. As a result, the retailer always benefits from the reviews, because the retailer shares the manufacturers’ revenues. Under wholesale scheme, in contrast, the retailer may be hurt by the reviews, because the softened upstream competition resulting from the variance-increasing effect tends to drive up the wholesale prices which gives the retailer disadvantage in reselling. On the other hand, the mean-shifting effect continues to be beneficial to the retailer. Whether the retailer benefits from the reviews depends on the tradeoff between the variance-increasing effect and mean-shifting effect. If the reviews are mild such that the variance-increasing effect dominates the mean-shifting effect, the retailer hurts by the reviews; Otherwise, the retailer benefits.

Stated more generally, our main result is that when online product reviews mitigate consumers’ uncertainty about quality and fit dimensions of perceived utility difference between competing products, the retailer’s pricing schemes are critical in understanding the effect of reviews on retailers via the change in the upstream competition. We show that such effect of reviews varies depending
on which kinds of information from reviews play a dominant role. More importantly, the effect of reviews on the retailer varies depending on whether the retailer provides platform or sells directly. Consequently, retailers can use the upstream pricing scheme effectively to their advantage in the presence of product reviews.

Several recent studies have analyzed the effect of product reviews on firms. While some empirical studies find a significant positive association between rating valence and sales (Chevalier and Mayzlin, 2006; Clemons et al., 2006; Duan et al., 2008b; Chintagunta et al., 2010), others do not find a relationship between the two (Chen et al., 2004; Liu, 2006; Duan et al., 2008a). Meanwhile, researchers find that the variance of product ratings (Clemons et al., 2006), the volume of ratings (Liu, 2006; Duan et al., 2008a), the reviewer characteristics and product characteristics (Forman et al., 2008; Zhu and Zhang, 2010; Shen, 2008), and text reviews (Archak et al., 2011) have an impact on sales. These results suggest that sellers may have incentives to manipulate reviews of their products or adjust their marketing-mix strategies directed toward consumers to use reviews to their advantage. Dellarocas (2006) and Mayzlin (2006) analyze sellers’ incentives to manipulate the reviews and show that reviews are informative even under seller manipulation. Different from these studies, we investigate how the additional information of online product reviews affects online retailers via the impact of reviews on upstream players in a channel structure, considering different upstream pricing schemes.

Our study relates to the existing analytical work that models product reviews as information to enable consumers to identify products matching their needs (Chen and Xie, 2008) or estimate their true utilities more accurately (Li et al., 2011; Sun, 2011b). They typically consider the effect of reviews on sellers in a context of direct selling from sellers to consumers. For example, Chen and Xie (2008) study how a seller should adjust the amount of information provision to consumers in response to consumer reviews. They show that because of product reviews the seller should lower the product information provision and price when it has high product cost and sufficient “novice” consumers who have no knowledge of their fits to the product in the absence of product reviews. The seller can benefit from review supply only when it can ensure sufficiently large number of postings and small size of knowledgeable consumers. We differ in that we consider a channel structure with a retailer either providing platform for competing manufacturers to sell their products or selling directly to consumers and consumers facing two dimensional uncertainty about a product—both
the product quality and the fit to their needs. Shaffer and Zettelmeyer (2002) analyze the effects of information provision on the profits of channel members when the information is supplied by third parties. In their model, all consumers have the same product information, additional information has the same qualitative impact (positive or negative) on every consumer, and sellers have perfect knowledge of all product information. In our setting, however, consumers are uncertain about both product quality and the fit to consumers’ needs, and we study how online product reviews affect the product competition by changing consumers’ perceived utilities. In particular, we consider that consumers have private estimates of the qualities of products and fits to their needs, and online product reviews provide public and common additional information about quality and private and idiosyncratic additional information about fit to consumers. In addition, we focus on the effect of reviews on retailers under different pricing schemes.

Another related stream of research is the recent studies on platform provision in online retailing (Jiang et al., 2011; Abhishek et al., 2012). For instance, Jiang et al. (2011) study a retailer’s choice of products to be sold under platform scheme when product demand is uncertain. Abhishek et al. (2012) study online retailers’ pricing strategies in the context where they compete with a traditional brick-and-mortar channel. They found the online retailers would prefer the platform scheme when online channel cannibalizes the traditional channel whereas they prefer the wholesale scheme when online channel stimulates the demand in traditional channel. Differently, we focus on the effect of online product reviews on an online retailer that carries substitutable products, and show the effect varies depending on the retailer’s pricing scheme choice (wholesale scheme versus platform scheme) and the type of information conveyed by the reviews (in the quality dimension versus in the fit dimension).

The rest of this paper is organized as follows. In the next section, we lay out the model. In Section 3, we derive the main results of the effect of the reviews on the upstream competition and the retailer. Section 4 concludes the paper.

2 Model

We consider a retailer $R$ that carries two products, $A$ and $B$, produced by different manufacturers, and uses one of two pricing schemes—wholesale scheme and platform scheme. Under the wholesale
scheme, the manufacturers sell their products to the retailer and the retailer re-sells them to consumers. Under the platform scheme, the manufacturers sell their products directly to consumers on the retailer’s platform, and the retailer charges a commission fee for each sale.

Products A and B are imperfect substitutes. We call the manufacturer that produces product A (B) manufacturer A (B). The marginal production cost for each product is assumed to be zero. We use Hotelling’s horizontal differentiation model to capture consumer preference (Hotelling, 1929). We assume that products A and B are located at positions 0 and 1 of a line of length 1 (i.e., at the two ends of the line), respectively. A continuum of consumers with unit mass are uniformly distributed along the line, and each consumer has a unit demand. Consumer utility for a product is the quality of the product net the disutility from the mis-fit between the product and her need measured by the distance between the product’s and her locations on the line. We assume that the consumer’s disutility per unit distance is $t$, and thus for the consumer located at $\lambda$, her disutility for product A is $\lambda t$ and disutility for product B is $(1 - \lambda)t$. Therefore, if we denote the quality difference between products A and B as $\delta$, the utility difference between products A and B for a consumer located at position $\lambda$ is equal to

$$\delta + (1 - 2\lambda)t$$

Consumers are uncertain about both product quality and the misfit. The uncertainty in the misfit is modeled as that consumers know the locations of the two products but are not sure about their own locations. In the absence of online product reviews, based on the product description and other information sources, each consumer has her own assessments of the quality difference between the two products and of her location. We denote a consumer’s own assessment of the quality difference as $x_C$, and assume that $x_C$ satisfies uniform distribution over $[-\epsilon, \epsilon]$. For the fit dimension, a consumer’s assessment is that she is located at $y$, $y \in [0, 1]$, with probability $\beta_C$ and can be anywhere on the line equally with probability $(1 - \beta_C)$. Different consumers perceive different $y$, and we assume that consumers’ assessments are unbiased such that $y$ satisfies the uniform distribution as in the true distribution of consumers.

Online product reviews provide public information about the products, and consumers use this information in addition to their own assessments to evaluate the products. We denote as $x_R$ the
perceived quality difference in the two products revealed by the online product reviews. In the presence of online product reviews, consumers combine $x_C$ and $x_R$ to form their judgment of the quality difference between the two products. For the fit dimension, with the additional information provided by online product reviews, consumers know better about the fit. In particular, a consumer’s updated assessment is that she is at a specific location along the Hotelling line with probability $\beta_R$ and can be anywhere on the line equally with probability $(1 - \beta_R)$, where $\beta_R > \beta_C$. A consumer’s updated perceived location can be different from or the same as the perceived location in the absence of reviews.

Regardless of the review availability, we denote $\alpha (\alpha \in \mathbb{R}^+)$ as a consumer’s price sensitivity of each product that measures the marginal decrease in the demand of each product from a marginal increase in the prices of both products. By modeling the price sensitivity of $\alpha$, we implicitly model that consumers may have outside options and only consumers with perceived surplus above some threshold purchase such that the market is not always covered.$^1$

We call the demand for each product from the above described consumers with unit mass *base demand*, which is denoted as $D^0_i$, $i \in \{A, B\}$. We assume that the party that sells products to consumers can exert an effort to increase the traffic to the retailer’s site such that the demand for each product can be boosted. The selling party’s effort can be, for example, advertising on sponsored search. If we denote the effort as $\theta_i$, $i \in \{A, B\}$, we assume the demand for each product is

$$D_i = D^0_i [1 + \eta (\theta_A + \theta_B)]$$

(2)

where $\eta$ measures the effectiveness of the effort, and $\theta_i$ is the effort initiated by the retailer for product $i$ under the wholesale scheme or initiated by manufacturer $i$ under platform scheme. Notice that traffic-boosting effort initiated for one product also increases the demand for the other product, because under either scheme, the traffic goes to the retailer’s platform which lists both products. The cost of the effort $\theta_i$ is assumed to be $\theta_i^2$. To exclude trivial cases in which the manufacturers do not exert effort in equilibrium under the platform scheme, we assume that $\eta$ is not too small (see Footnote 4 for the specifics).

$^1$Technically, we assume $\alpha > 0$ to ensure an interior solution for retail prices in equilibrium. The main insights about how online reviews affect upstream competition retain the same without this assumption (such that the whole market is covered).
The sequence of events under each pricing scheme is as follows. Under the platform scheme, in stage 1, the retailer sets the commission rate \( k \) for each sale, which is the percentage of the revenue that the retailer charges for each sale. In stage 2, manufacturers determine their effort levels. In stage 3, the manufacturers set retail prices \( p_i \) simultaneously. In stage 4, consumers evaluate the difference of product utility, and make their purchase decisions. Under the wholesale scheme, in stage 1, the retailer determines the effort level for each product. In stage 2, manufacturers set wholesale prices \( w_i \) simultaneously. In stage 3, the retailer sets retail prices \( p_i \). In stage 4, consumers make their purchase decisions by evaluating difference of product utility. The assumed game sequences imply that effort levels are more strategic than prices under both pricing schemes, and in the case of platform scheme, the contract between the retailer and manufacturers that sets the commission rate is more strategic than both effort and prices.

We use the scenario without reviews as the benchmark to analyze the effect of reviews in section 3. In the scenario with reviews, reviews are observed by consumers, the manufacturers, and the retailer before they make their decisions. Therefore, consistent with many existing studies (e.g., Shaffer and Zettelmeyer, 2002; Sun, 2011b), we do not model the review generating process and instead we examine the effect of reviews when product reviews are in a steady state.\(^2\) Consumers’ own estimates about product quality difference and their locations are their private information. All players are risk neutral, and all other model parameters are common knowledge.

### 3 Effect of Online Product Reviews

In this section, we first derive each product’s demand as a function of the retail prices and online product review information. We then analyze the effects of online product reviews on the competition between the two manufacturers and the retailer under both the platform scheme and the wholesale scheme.

We start with consumer decision making in stage 4 of the game, which are the same under the two pricing schemes. For the quality dimension, without product reviews, a consumer’s perceived quality difference is \( x_C \); that is, \( \delta = x_C \) in Equation (1). With online product reviews, a consumer

\(^2\)The evidence is abundant that the reviews influence consumers' purchase decisions and product demands (e.g., Deloitte and Touche, 2008) and firms adjust product prices in response to the reviews (e.g., Jiang and Wang, 2008; Shin et al., 2011).
has two sources of information, $x_C$ and $x_R$, to estimate $\delta$. As shown by Bates and Granger (1969) using the minimum variance estimation, the consumer’s expected quality difference $\delta$ becomes

$$rx_C + (1 - r)x_R$$

(3)

where $r, r \in (0, 1)$, depends on the relative precisions of the two information sources,\(^a\) and the weight on the product reviews, $(1 - r)$, is high when the precision of the product review information is high. For the fit dimension, a consumer who perceives her locating at $y$ with probability $\beta$ has expected location $\beta y + \frac{1 - \beta}{2}$, and in the scenario without product reviews $\beta = \beta_C$ and in the scenario with reviews $\beta = \beta_R$. Altogether, we can formulate the perceived utility difference in Equation (1) for both scenarios uniformly as

$$V = [\gamma x_C + (1 - \gamma)x_R] + \left[1 - 2 \left(\beta y + \frac{1 - \beta}{2}\right)\right] t$$

(4)

in which $(\gamma, \beta) \in \{(1, \beta_C), (r, \beta_R)\}$ with $\gamma = 1$ and $\beta = \beta_C$ for the scenario without reviews and $\gamma = r$ and $\beta = \beta_R$ for the scenario with reviews.

Consumers’ net utility differences from the two products also depend on the prices. We thus formulate the net utility difference as

$$[\gamma x_C + (1 - \gamma)x_R] + (1 - 2y) \beta t - (p_A - p_B)$$

(5)

where $(1 - 2y)\beta t$ is from the simplification of the term on fit dimension in Equation (4). Clearly, besides consumers’ own assessments, the product reviews also affect consumers’ perceived net utility differences between the two products by changing $\gamma$ and $\beta$. We focus our analysis on the cases in which online product reviews play a mild or moderate role in changing the competition between the two manufacturers such that in equilibrium two manufacturers are comparably competitive. The extreme case in which the additional difference revealed by online product reviews is so dramatic such that one manufacturer has a dominant advantage in the market is not considered in this study.

We next distinguish two cases:

\(^a\)In particular, $r = \frac{s_{R}^2}{s_{C}^2 + s_{R}^2}$, where $s_{C}^2$ and $s_{R}^2$ are the variances of $x_C$ and $x_R$, respectively.
• Quality-dominates-fit case: in which the quality dimension dominates the fit dimension such that even the consumer who has the lowest fit with a product may derive higher net utility from the product than from the other when her perceived quality difference is strongly favorable toward the product.

• Fit-dominates-quality case: in which the fit dimension dominates the quality dimension for some consumers such that those consumers always derive higher net utility from the product that offers a better fit, regardless of the perceived quality difference.

In the quality-dominates-fit case, as illustrated in Figure 1a, for any consumer who perceives herself being located at $y$, by Equation (5), if her perceived quality difference is higher than $\frac{1}{\gamma} [p_A - p_B - (1 - \gamma)x_R - (1 - 2y)\beta]$, she derives higher net utility from product $A$; otherwise, she derives higher utility from product $B$. Therefore, we can formulate the demand for each product as

$$D_A^0 = \int_{-\epsilon}^\epsilon \frac{1}{2} dxdy - \alpha p_A = \frac{1}{2} - \frac{1}{2\gamma} [p_A - p_B - (1 - \gamma)x_R] - \alpha p_A$$
$$D_B^0 = \int_{-\epsilon}^\epsilon \frac{1}{2} dxdy - \alpha p_B = \frac{1}{2} + \frac{1}{2\gamma} [p_A - p_B - (1 - \gamma)x_R] - \alpha p_B$$

where the integral in product $i$'s demand measures the consumers who derive higher net utilities from product $i$ than from the other product, and $ap_i$ measures the consumers who leave for outside options, $i \in \{A, B\}$.

In the fit-dominates-quality case, consumers who perceive a strong fit with product $A$ always derive higher net utility from product $A$, and consumers who perceive a strong fit with product $B$ always derive higher net utility from product $B$, regardless of the perceived quality difference.
As illustrated in Figure 1b, we can denote the former consumer group as those located at $[0, y_A]$ and the latter as those located at $[y_B, 1]$ along the line. The consumers who perceive being located between $y_A$ and $y_B$ may derive higher net utility from product $A$ or from product $B$, depending on their perceived quality differences. The marginal consumer $y_A$ ($y_B$) is the one who derives the same net utility from the two products when perceiving the largest quality difference against product $A$ ($B$); that is, when $x_C = -\epsilon$ ($x_C = \epsilon$). By Equation (5), we have

$$y_A = \frac{1}{2\tau} \left[ -\gamma \epsilon + (1 - \gamma) x_R + \beta t - (p_A - p_B) \right] \quad (7)$$

$$y_B = \frac{1}{2\tau} \left[ \gamma \epsilon + (1 - \gamma) x_R + \beta t - (p_A - p_B) \right]$$

We then can formulate the base demand for each product in the fit-dominates-quality case as

$$D^0_A = \int_0^{y_A} dy + \int_{y_A}^{1} dy \int_{p_A - (1-\gamma)x_R - (1-2\tau)\beta t}^{p_B - (1-\gamma)x_R - (1-2\tau)\beta t} \frac{1}{2\tau} dxdy - \alpha p_A = \frac{1}{2} - \frac{1}{2\tau} \left[ p_A - p_B - (1 - \gamma) x_R \right] - \alpha p_A$$

$$D^0_B = \int_{y_A}^{1} dy \int_{-\epsilon}^{\gamma \epsilon} \frac{1}{2\tau} dxdy + \int_{y_B}^{1} dy \int_{p_A - (1-\gamma)x_R - (1-2\tau)\beta t}^{p_B - (1-\gamma)x_R - (1-2\tau)\beta t} \frac{1}{2\tau} dxdy - \alpha p_B = \frac{1}{2} + \frac{1}{2\tau} \left[ p_A - p_B - (1 - \gamma) x_R \right] - \alpha p_B \quad (8)$$

From Equations (6) and (8), we notice that firms’ demands are in a linear form in both cases. Moreover, each firm’s demands in both cases take the same structure except the coefficients of the terms in the brackets (i.e., $\frac{1}{2\gamma \epsilon}$ in the quality-dominates-fit case versus $\frac{1}{2\tau}$ in the fit-dominates-quality case). As a result, combining the relationship between the base demand and the efforts in Equation (2), we can uniformly characterize the demand as follows:

$$D_A = \left[ \left( \frac{1}{2} + \frac{1}{2\tau} (1 - \gamma) x_R \right) - \left( \frac{1}{2\tau} + \alpha \right) p_A + \frac{1}{2\tau} p_B \right] \left( 1 + \eta [\theta_A + \theta_B] \right)$$

$$D_B = \left[ \left( \frac{1}{2} - \frac{1}{2\tau} (1 - \gamma) x_R \right) - \left( \frac{1}{2\tau} + \alpha \right) p_B + \frac{1}{2\tau} p_A \right] \left( 1 + \eta [\theta_A + \theta_B] \right) \quad (9)$$

where $\tau \in \{\gamma \epsilon, \beta t\}$ with $\tau = \gamma \epsilon$ for the quality-dominates-fit case and $\tau = \beta t$ for case the fit-dominates-quality case. This expression evidently demonstrates that the assumptions that we impose on consumers’ true and perceived preferences and distribution of consumers’ perceived quality difference are equivalent to the assumptions on linear demand functions, which have been commonly used in the literature (e.g., Choi, 1991). The online product reviews affect the competition between the two manufacturers via changing the parameters of the above demand functions.

We next analyze the effects of online product reviews under the platform scheme and then analyze the effects under the wholesale scheme.
3.1 Effect of Online Product Reviews under Platform Scheme

Using backward induction, we next derive the equilibrium retail prices, the commission rate charged by the retailer, and the retailer’s profit.

In stage 3 of the game, the manufacturers maximize their profits by choosing the optimal prices given their effort levels and the commission rate \( k \) pre-announced by the retailer; that is,

\[
\max_{p_i} \pi_i = (1 - k) p_i D_i - \theta_i^2
\]

where \( i \in \{A, B\} \) and \( D_i \) is specified in Equation (9). By the first-order conditions, we can derive the manufacturers’ optimal prices. In stage 2 of the game, the manufacturers maximize their profit by choosing their effort levels; that is,

\[
\max_{\theta_i} \pi_i = (1 - k) p_i D_i - \theta_i^2 = (1 - k) p_i D_i^0 [1 + \eta (\theta_A + \theta_B)] - \theta_i^2
\]

where the second equality is by Equation (2). Solving the first-order conditions, we can derive the manufacturers’ optimal effort levels as functions of the commission rate. In stage 1 of the game, anticipating the retail prices set by manufacturers and their effort levels, the retailer maximizes its profit by choosing the commission rate \( k \); that is,

\[
\max_k \pi_R = k (p_A D_A + p_B D_B) = k (p_A D_A^0 + p_B D_B^0) [1 + \eta (\theta_A + \theta_B)]
\]

Based on the first-order conditions, we can derive the retailer’s optimal commission rate. The retailer’s equilibrium profit can be derived by substituting the optimal commission rate, optimal effort levels, and optimal retail prices to the retailer’s profit function. We summarize the equilibrium outcome in the following lemma.

Lemma 1. Under the platform scheme, the equilibrium retail prices, commission rate, and the retailer’s profit are as follows.

(a) Retail prices:

\[
p_A = \frac{\tau}{1 + 4 \alpha \tau} + \frac{(1 - \gamma) x_R}{3 + 4 \alpha \tau}
\]

\[
p_B = \frac{\tau}{1 + 4 \alpha \tau} - \frac{(1 - \gamma) x_R}{3 + 4 \alpha \tau}
\]
(b) Commission rate:

\[
k = \frac{2r(1+4\alpha r)^2(3+4\alpha r)^2 + (1+2\alpha r)(1+4\alpha r)^2 x_B^2}{2(1+2\alpha r)[2\tau(3+4\alpha r)^2 + (1-\gamma)^2(1+4\alpha r)^2 x_B^2]} \eta^2
\]  

(15)

(c) The retailer’s Profit:

\[
\pi_R = \frac{[2r(1+4\alpha r)^2(3+4\alpha r)^2 + (1+2\alpha r)(1+4\alpha r)^2 x_B^2]}{8\tau(1+4\alpha r)^4(3+4\alpha r)^4}\eta^2
\]

(16)

where \( \tau \in \{\beta t, \gamma \epsilon\} \) and \((\gamma, \beta) \in \{(1, \beta_c), (r, \beta_R)\}\), with \( \tau = \gamma \epsilon \) for the quality-dominates-fit case and \( \tau = \beta t \) for the fit-dominates-quality case and in each case with \((\gamma, \beta) = (1, \beta_c)\) for the scenario without reviews and \((\gamma, \beta) = (r, \beta_R)\) for the scenario with reviews.

Proof. All proofs are in the appendix unless indicated otherwise.

3.1.1 Quality-Dominates-Fit Case under Platform Scheme

Without loss of generality, we consider \( x_R \geq 0 \); that is, the online product reviews favor manufacturer \( A \) on the quality dimension. The following proposition summarizes the effects of product reviews, in which we use the notations (e.g., \( \tilde{\pi}_R \)) with tildes for the scenario without reviews and use the notations with hats (e.g., \( \hat{\pi}_R \)) for the scenario with reviews. We follow the same convention when coming to similar comparison later.

**Proposition 1.** Under the platform scheme, in the quality-dominates-fit case, in the presence of online product reviews with \( x_R \geq 0 \):

(a) Product B’s retail price is lower (i.e., \( \tilde{p}_B > \hat{p}_B \)); Product A’s retail price is lower (i.e., \( \tilde{p}_A > \hat{p}_A \)) if and only if

\[
x_R < \frac{\epsilon(3+4\alpha r)}{(1+4\alpha r)(1+4\alpha \epsilon)}
\]  

(17)

(b) The retailer’s commission rate \( k \) is higher (i.e., \( \tilde{k} < \hat{k} \)) and retailer’s profit is lower (i.e., \( \tilde{\pi}_R > \hat{\pi}_R \)) if

\[
x_R^2 < \frac{r\epsilon^2(3+4\alpha r)^2[1+2\alpha(1+r)\epsilon]}{(1-r)(1+4\alpha r)^2(1+2\alpha r)(1+4\alpha \epsilon)^2} \eta^2
\]  

(18)

\footnote{Notice that \( k \) is the commission rate and should be no greater than 1. By the optimization problem in Equation (11), the equilibrium effort is positive if and only if \( k < 1 \). Therefore, the condition for positive equilibrium effort is equivalent to the condition required for \( k < 1 \), which is \( \eta^2 > \frac{2(1+4\alpha \tau)^2}{\tau(1+2\alpha \tau)} \) by Equation (15).}
Otherwise, the retailer’s commission rate is lower and retailer’s profit is higher.

In the symmetric case with $x_R = 0$, the conditions in Inequalities (17) and (18) are apparently satisfied and thus we have the following corollary.

**Corollary 1.** Under platform scheme, in the presence of the symmetric product reviews (i.e., $x_R = 0$), the retail prices are lower; the retailer’s commission rate is higher and retailer’s profit is lower.

The intuition for the symmetric case is as follows. In the scenario without product reviews, each consumer’s perceived quality difference is from her own private assessment. In the scenario with reviews, the consumer combines her own assessment with the quality difference assessment revealed by the online product reviews. Because the quality difference revealed by the reviews is public and common to all consumers, the presence of product reviews reduces the heterogeneity of consumers’ perceived quality difference and thus reduces the heterogeneity of their perceived utility differences. Figure 2a illustrates the effect of the online product reviews on the perceived utility differences in this symmetric case. Because consumers put some weight on the common component—perceived quality difference revealed by the reviews—in evaluating the utility differences, the span of their evaluations is reduced and thus the variance of their evaluations is reduced because of the product reviews. We call this effect *variance-reducing effect*.

The reduced heterogeneity in consumers’ perceived utility differences between the two products makes the two products more substitutable overall and makes consumers more price sensitive to a specific product, and thus it increases the competition between the two manufacturers. The increased substitutability and competition can also be seen from the demand functions. In this...
symmetric case, the base demand functions in Equation (6) can be rewritten as

$$D^0_i = \frac{1}{2} - \alpha p_i - \frac{1}{2\gamma \epsilon} (p_i - p)$$  \hspace{1cm} (19)$$

where \(\{i, \bar{i}\} = \{A, B\}\). Note that the coefficient of the price difference term (i.e., \(\frac{1}{2\gamma \epsilon}\) in this case) measures the substitutability between the two products: the larger the coefficient is, the more substitutable the two products are. The effect of online product reviews on the demand function is that it reduces \(\gamma\) from 1 to \(r\) (where \(r < 1\)), and thus it increases the substitutability between the two products. The increased competition between the two manufacturers drives their retail prices down as well as the revenues from the base demands.

Notice that from Equation (11), the marginal benefit of the traffic-boosting effort for manufacturer \(i\) is \((1 - k) p_i D^0_i \eta\). (The marginal cost is \(2\theta_i\) and thus the optimal effort level is \((1 - k) p_i D^0_i \eta / 2\).) Clearly, both the commission rate and the revenue from the base demand affect manufacturers’ incentives to exert effort. Given any commission rate \(k\), the decreased revenue from the base demand resulting from the increased competition because of the product reviews reduces manufacturers’ incentive to exert effort to boost traffic. As a result of the decreased efforts and the decreased revenue from the base demand, the retailer’s revenue is always lower under the same commission rate. Therefore, in equilibrium, the retailer’s optimal revenue (by choosing its optimal commission rate) is reduced by online product reviews. The retailer charges a higher commission rate in the presence of the reviews, because, with the decreased revenue from the base demand, the marginal benefit of the traffic-boosting effort is less sensitive to the commission rate.

In the general case in Proposition 1, the online product reviews have asymmetric effect on each manufacturer. The favorable quality information toward product \(A\) revealed by the reviews (i.e., \(x_R > 0\)) uniformly changes each consumer’s perceived quality difference between the two products favorably toward product \(A\). As a result, in the presence of the favorable reviews toward product \(A\), on average consumers’ perceived quality differences and thus their perceived utility differences between the two products are favorable for product \(A\). We call this effect mean-shifting effect. Figure 2b illustrates such an effect. With favorable reviews for product \(A\), consumers are more likely to have a higher utility from product \(A\). As a result, the mean of their perceived utility differences is shifted toward the right-hand side and is changed favorably for product \(A\). In the demand functions
outlined in Equation (6), the mean shifting is reflected in the shifting from product B’s potential market size to product A’s such that, compared to the symmetric case, manufacturer A’s potential market size increases (from $\frac{1}{2}$ to $\left[\frac{1}{2} + \frac{1}{2r}\left(1-r\right)x_R\right]$) and manufacturer B’s decreases (from $\frac{1}{2}$ to $\left[\frac{1}{2} - \frac{1}{2r}\left(1-r\right)x_R\right]$). Manufacturer B may suffer from the reduced potential market size resulting from unfavorable reviews, in addition to increased competition resulting from the variance-reducing effect as in the symmetric case. As a result, the retail price for product B is reduced. For manufacturer A, the favorable reviews have positive effect on its retail price because of the enhanced market potential, whereas the increased competition resulting from the variance-reducing effect has a negative effect. Whether the manufacturer increases its retail price depends on the relative strength of the two effects, and more favorable reviews make the positive effect more significant, which in turn induces manufacturer A to charge a higher retail price. Inequality (17) pinpoints the condition showing that only if the reviews are favorable enough, the retail price for product A become higher in the presence of reviews.

Although it hurts by the variance-reducing effect, the retailer may benefit from the mean-shifting effect. The benefit from the mean-shifting effect might come from two sources. First, the mean-shifting effect makes the base demands asymmetric in terms of their potential market sizes, which may increase the total revenue generated from the base demands, compared to the symmetric case. For example, shifting the potential demand from product B to product A, per se, allows manufacturer A to charge a higher retail price for product A and receive a higher realized demand, at the cost of a lower retail price with a lower realized demand for product B. Notice the gain from the increased price and increased demand for product A outweighs the loss from the decreased price and decreased demand for product B, because the changes in both the price and demand are more significant for product A than product B due to A’s dominance in the market potential. Therefore, the increase of the degree of the asymmetry in the market potentials can increase the total revenue generated from the base demand. When the reviews are highly favorable to one product, the total revenue generated from the base demand could be higher that the total revenue in the benchmark case, despite the variance-decreasing effect. Second, when the reviews are highly favorable to manufacturer A, the increased profit from the base demand for product A can induce manufacturer A to exert considerably high effort, which can compensate the effort reduction of manufacturer B. As a result, the total effort level is even higher, compared to the
no-review scenario. Inequality (18) essentially shows that the retailer can benefit from the reviews if the reviews are highly favorable to one product over the other.

The change in the equilibrium commission rate is in the opposite direction of the change in the retailer’s revenue because of the similar reason as in the symmetric case. When the total revenue from the base demands is lower, the total effort level (which is \((1 - k) \left( p_AD_A^0 + p_BD_B^0 \right) \eta / 2\), by the first-order condition of Equation (11)) is less sensitive to the commission rate, and, as a result, the retailer tends to charge a higher commission rate in equilibrium.

3.1.2 Fit-Dominates-Quality Case under Platform Scheme

We again assume \(x_R \geq 0\), without loss of generality.

**Proposition 2.** Under platform scheme, in the fit-dominates-quality case, in the presence of online product reviews with \(x_R \geq 0\):

(a) Product A’s retail price is higher (i.e., \(\hat{\bar{p}}_A < \bar{p}_A\)); Product B’s retail price is higher (i.e., \(\hat{\bar{p}}_B < \bar{p}_B\)) if and only if

\[
x_R < \frac{t(\beta_R - \beta_C)(3 + 4\alpha_\beta_R t)}{(1 - r)(1 + 4\alpha_\beta_C t)(1 + 4\alpha_\beta_C t)}
\]  

(b) The retailer’s commission rate \(k\) is lower (i.e., \(\hat{k} > \bar{k}\)) and retailer’s profit is higher (i.e., \(\hat{\bar{\pi}}_R < \bar{\bar{\pi}}_R\)).

In the symmetric case with \(x_R = 0\), the condition in Inequality (20) is satisfied because \(\beta_R > \beta_C\) and we thus have the following corollary.

**Corollary 2.** Under platform scheme, in the presence of the symmetric product reviews (i.e., \(x_R = 0\)), the retail prices are higher; the retailer’s commission rate is lower and retailer’s profit is higher.

The intuition for the symmetric case is as follows. Different from the quality dimension in which the true quality difference is the same for all consumers and the product reviews add a common component in evaluating the quality difference across all consumers, in the fit dimension consumers have different preferences and online product reviews provide more information for them to further calibrate their own fits. With the additional information, consumers become less uncertain about the products’ fits to their needs (from with probability \(\beta_C\) to with probability \(\beta_R\) for some degree of the fit). The reduced uncertainty thus makes consumers more heterogeneous in terms of their
perceived fits, which tends to increase the heterogeneity in consumers’ perceived utility differences. Figure 3a illustrates the effect of the online product reviews on the perceived utility differences in this symmetric case. Contrary to the effect in the quality dimension, the information provided by the product reviews in the fit dimension tends to increase the variance of consumers’ perceived utility differences. We call this effect variance-increasing effect.

The increased heterogeneity in consumers’ perceived utility differences between the two products makes the two products less substitutable overall and makes consumers less price sensitive to a specific product, and thus it softens the competition between the two manufacturers. The decreased substitutability and competition can also be seen from the demand functions. In this symmetric case, the demand functions in Equation (8) can be rewritten as

\[ D_i^0 = \frac{1}{2} - \alpha p_i - \frac{1}{2\beta_{t}} (p_i - \bar{p}_i) \]

where \( \{i, \bar{i}\} = \{A, B\} \). As discussed previously, the larger the coefficient of the price difference term (i.e., \( \frac{1}{2\beta_{t}} \) in this case) is, the more substitutable the two products are. The effect of online product reviews on the demand function is that it increases \( \beta \) from \( \beta_C \) to \( \beta_R \) and thus it decreases the substitutability between the two products. The softened competition between the two manufacturers increases their retail prices as well as the revenues from the base demands.

As in the quality-dominates-fit case, by Equations (11), the marginal benefit of the traffic-boosting effort is \( (1 - k) p_i D_i^0 \eta \), the proportion of the revenue from the base demand that each manufacturer receives. With the increased revenue from the base demand resulting from the increased competition by the product reviews, manufacturers have more incentive to exert effort to
boost traffic. As a result of the increased revenue from the base demand and the manufacturers’ increased incentives to exert effort, the total revenue generated in the market is higher, and so is the retailer’s revenue because the retailer takes an (optimal) proportion of the total revenue. Notice that the retailer’s proportion, or the commission rate, in this case is lower, because, with the increased revenue from the base demand, the marginal benefit of the traffic-boosting effort is more sensitive to the commission rate. In other words, being aware of the increased revenue from the base demand, the retailer stimulates manufacturers’ traffic-boosting efforts by reducing the commission rate that the retailer charges. As a result, the total revenue from the market is boosted, and the retailer’s revenue is also boosted although it charges a lower commission rate.

In the general case as prescribed in Proposition 2, the online product reviews have asymmetric effect on each manufacturer. As in the quality-dominates-fit case and as illustrated in Figure 3b, the mean-shifting effect continues to exist (i.e., $x_R > 0$). In the demand functions outlined in Equation (9), the mean shifting is reflected in an increase in manufacturer A’s potential market size (from $\frac{1}{2}$ to $\left[\frac{1}{2} + \frac{1}{2\beta_R t} (1-r)x_R \right]$) and a decrease in manufacturer B’s (from $\frac{1}{2}$ to $\left[\frac{1}{2} - \frac{1}{2\beta_R t} (1-r)x_R \right]$). Manufacturer A benefits from the favorable reviews and the resulting increased potential market size, in addition to softened competition resulting from the variance-increasing effect as in the symmetric case. As a result, because of the reviews, the manufacturer increases its retail price, its revenue from the base demand is higher, and it has more incentive to boost the traffic. For manufacturer B, the unfavorable reviews have a negative effect on its retail price because of the reduced appeal in the market, whereas the softened competition resulting from the variance-increasing effect, as in the symmetric case, has a positive effect. Whether the manufacturer increases its retail price depends on the relative strength of the two effects. When unfavorable reviews are mild, the softened competition effect dominates, which induces manufacturer B to charge a higher price compared to the no-reviews scenario. Inequality (20) describes such a condition.

In the presence of asymmetric reviews, as in the quality-dominates-fit case, retailer benefits from the mean-shifting effect. As before, the mean-shifting effect makes the potential market sizes asymmetric in the base demand, which increases the total revenue from the base demands. With the increased total revenue from the base demand, the total effort level is also enhanced. In addition to the mean-shifting effect, the retailer also benefits from the variance-increasing effect, as discussed in the symmetric case. As a result of the both positive effects, the retailer’s profit is higher in the
presence of the reviews, although it charges a lower commission rate.

3.2 Effect of Online Product Reviews under Wholesale Scheme

Using backward induction, we derive the equilibrium wholesale price, retail prices, and the retailer’s profit under the wholesale scheme.

In stage 3 of the game, the retailer maximizes its profit given its effort level and wholesale prices by choosing the optimal retail price for each product; that is,

$$\max_{p_A, p_B} \pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B - \theta_A^2 - \theta_B^2$$  \hspace{2cm} (22)

By the first-order conditions, we can derive the retailer’s optimal prices, which are functions of wholesale prices. In stage 2 of the game, anticipating the retailer’s reaction in response to the wholesale prices, the manufacturers maximize their profits by choosing their optimal prices; that is,

$$\max_{w_i} \pi_i = w_iD_i, \ i \in \{A, B\}$$  \hspace{2cm} (23)

Based on the first-order conditions, we can obtain the optimal wholesale price for each manufacturer. In stage 1 of the game, the retailer maximizes its profit by choosing the effort level; that is,

$$\max_{\theta_A, \theta_B} \pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B - \theta_A^2 - \theta_B^2$$  \hspace{2cm} (24)

$$= [ (p_A - w_A)D_A^0 + (p_B - w_B)D_B^0 ] [ 1 + \eta (\theta_A + \theta_B) ] - \theta_A^2 - \theta_B^2$$  \hspace{2cm} (25)

where the second equality is because of Equation (2). Solving (24) based on the first-order conditions, we can derive the retailer’s optimal effort levels. Substituting the optimal effort levels to the retailer’s objective function, we can derive the retailer’s equilibrium profit. We summarize the equilibrium outcome in the following lemma.

**Lemma 2.** Under the wholesale scheme, the equilibrium wholesale prices, retail prices, and the retailer’s profit are as follows.

(a) Wholesale prices:

$$w_A = \frac{\tau}{1 + 4\alpha \tau} + \frac{(1 - \gamma)x_R}{3 + 4\alpha \tau}$$  \hspace{2cm} (26)
\[ w_B = \frac{\tau (1-\gamma)x_R}{3+4\alpha\tau} \]  

(b) Retail prices:

\[ p_A = \frac{1+6\alpha\tau}{4\alpha(1+4\alpha\tau)} + \frac{5+6\alpha\tau(1-\gamma)x_R}{4(1+\alpha\tau)(3+4\alpha\tau)} \]  

\[ p_B = \frac{1+6\alpha\tau}{4\alpha(1+4\alpha\tau)} - \frac{5+6\alpha\tau(1-\gamma)x_R}{4(1+\alpha\tau)(3+4\alpha\tau)} \]  

(c) The retailer’s profit:

\[ \pi_R = \left[ \left( \frac{(1+2\alpha\tau)^2}{8\alpha(1+4\alpha\tau)^2} + \frac{(1+2\alpha\tau)^2(1-\gamma)x_R^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2} \right) + \left[ \frac{(1+2\alpha\tau)^2}{8\alpha(1+4\alpha\tau)^2} + \frac{(1+2\alpha\tau)^2(1-\gamma)x_R^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2} \right]^2 \right] \eta^2 \]  

where \( \tau \in \{ \beta t, \gamma \epsilon \} \) and \( (\gamma, \beta) \in \{(1, \beta_c), (r, \beta_R)\} \), with \( \tau = \gamma \epsilon \) for the quality-dominates-fit case and \( \tau = \beta t \) for the fit-dominates-quality case and in each case with \( (\gamma, \beta) = (1, \beta_c) \) for the scenario without reviews and \( (\gamma, \beta) = (r, \beta_R) \) for the scenario with reviews.

### 3.2.1 Quality-Dominates-Fit Case under Wholesale Scheme

**Proposition 3.** Under wholesale scheme, in the quality-dominates-fit case, in the presence of online product reviews with \( x_R \geq 0 \):

(a) Product B’s wholesale price is lower (i.e., \( \hat{w}_B > \bar{w}_B \)); Product A’s wholesale price is lower (i.e., \( \hat{w}_A > \bar{w}_A \)) if and only if

\[ x_R < \frac{\epsilon}{(1+4\alpha\tau)(1+4\alpha\epsilon)} \]  

(b) Product B’s retail price is lower and the retailer’s profit is higher (i.e., \( \hat{p}_B > \bar{p}_B \) and \( \hat{\pi}_R < \bar{\pi}_R \)); Product A’s retail price is lower (i.e., \( \hat{p}_A > \bar{p}_A \)) if and only if

\[ x_R < \frac{2(1+\alpha\epsilon)(3+4\alpha\tau)}{(1+4\alpha\tau)(1+4\alpha\epsilon)(5+6\alpha\epsilon)} \]  

In the symmetric case with \( x_R = 0 \), the conditions in Inequalities (31) and (32) are apparently satisfied and thus we have the following corollary.

**Corollary 3.** Under wholesale scheme, in the presence of the symmetric product reviews (i.e., \( x_R = 0 \)), both the wholesale prices and retail prices are lower, and retailer’s profit is higher.

The intuition of the effect of reviews on upstream competition for the symmetric case is similar
to the case under platform scheme. As illustrated in section 3.1.1, reviews have a variance-reducing effect, which makes the two products more substitutable overall and increases the competition between the two manufacturers. The increased competition drives their wholesale prices down. As a result, the retailer benefits from the increased competition between the two manufacturers, and the benefits are two-fold. First, with the lower wholesale prices, the retailer can lower its retail prices to increase the base demand for each product while increasing its profit margin from each sale at the same time. Therefore, the profit derived from the base demands increases because of the increased competition between the upstream manufacturers. Second, with the increased profit from the base demand, the retailer has more incentive to boost the traffic, which further benefits the retailer’s total profit.

In the general case (i.e., $x_R > 0$) as prescribed in Proposition 3, the mean-shifting effect, as illustrated in section 3.1.1, also plays a role in the equilibrium outcome. Manufacturer $B$ hurts by the unfavorable reviews, in addition to the increased competition because of the variance-reducing effect. Therefore, manufacturer $B$ charges a lower wholesale price, which in turn induces the retailer to charge a lower retail price for product $B$. For manufacturer $A$, the favorable reviews have positive effect on its wholesale price, whereas the increased competition resulting from the variance-reducing effect has a negative effect. Inequality (31) shows that when the positive reviews are mild, the variance-reducing effect dominates the mean-shifting effect, and thus the wholesale price for product $A$ is lower because of the reviews. The change in the wholesale price, together with the change in the base demand for product $A$ because of the favorable reviews, changes the retail price of product $A$ in a similar fashion, as characterized by Inequality (32).

The retailer benefits from the reviews from three sources. First, as in symmetric case, the variance-reducing effect intensifies the upstream competition, which, per se, reduces the wholesale prices and thus increases the retailer’s revenue from base demands. Second, as illustrated in section 3.1.1, the mean-shifting effect makes the downstream demand asymmetric in terms of their potential market sizes, which engenders more room for the retailer to exploit its market and benefits the retailer. Both the variance-reducing effect and mean-shifting effect increase the retailer’s profit from the base demand. Third, as in the symmetric case, with the increased profit from the base demand, the retailer has more incentive to boost the traffic and boost the demand, which benefits the retailer. In other words, the possibility of boosting the traffic enables the retailer to benefit
from the reviews even more.

3.2.2 Fit-Dominates-Quality Case under Wholesale Scheme

**Proposition 4.** Under wholesale scheme, in the fit-dominates-quality case, in the presence of online product reviews with $x_R \geq 0$:

(a) Product A’s wholesale price is higher (i.e., $\hat{w}_A < \tilde{w}_A$); Product B’s wholesale price is higher (i.e., $\hat{w}_B < \tilde{w}_B$) if and only if

$$x_R < \frac{t(\beta_R - \beta_C)(3+4t\beta_R)}{(1-r)(1+4t\beta_C t)} \quad (33)$$

(b) Product A’s retail price is higher (i.e., $\hat{p}_A < \tilde{p}_A$); Product B’s retail price is higher (i.e., $\hat{p}_B < \tilde{p}_B$) if and only if

$$x_R < \frac{2t(\beta_R - \beta_C)(1+4t\beta_R)(3+4t^2\beta_R)}{(1-r)(1+4t\beta_C t)(1+4t^2\beta_R)(5+6t\beta_R)} \quad (34)$$

The retailer’s profit is lower (i.e., $\tilde{\pi}_R > \hat{\pi}_R$) if and only if

$$x_R^2 < \frac{4^2(\beta_R - \beta_C)\beta_R(1+t\alpha\beta_R)(3+4t\beta_R)^2[1+3t\alpha\beta_R+ta\beta_C(3+8t\alpha\beta_R)]}{(1-\gamma)^2(1+4t\alpha\beta_C^2t^2(1+4t\alpha\beta_R)^2)} \quad (35)$$

In the symmetric case with $x_R = 0$, the conditions in Inequalities (33), (34), and (35) are all satisfied because $\beta_R > \beta_C$ and we thus have the following corollary.

**Corollary 4.** Under wholesale scheme, in the presence of the symmetric product reviews (i.e., $x_R = 0$), both the wholesale prices and retail prices are higher, and the retailer’s profit is lower.

The intuition of the effect of reviews on upstream competition for the symmetric case is similar to the case under the platform scheme. As illustrated in section 3.1.2, the online product reviews have a variance-increasing effect, which makes the two products less substitutable overall and makes consumers less price sensitive to a specific product, and thus it softens the competition between the two manufacturers. The softened competition between the two manufacturers increases their wholesale prices. The increased wholesale prices induce the retailer to increase its retail prices, which decreases the base demand for each product. Meanwhile, the increased wholesale prices leave the retailer with lower profit margin from each sale. As a result, the retailer earns less profit from
the base demands because of online product reviews. Furthermore, with the decreased profit from the base demands, the retailer has less incentive to boost its traffic, which, in addition to direct loss in the profit from base demands, hurts the retailer’s profit.

In the general case (i.e., $x_R > 0$) as prescribed in Proposition 4, the online product reviews have asymmetric effect on each manufacturer, and the mean-shifting effect, as illustrated in section 3.1.2, also plays a role in the equilibrium outcome. Manufacturer $A$ benefits from the favorable reviews, in addition to the softened competition because of the variance-increasing effect. Therefore, manufacturer $A$ charges a higher wholesale price, which in turn induces the retailer to charge a higher retail price for product $A$. For manufacturer $B$, the unfavorable reviews have negative effect on its wholesale price, whereas the softened competition resulting from the variance-increasing effect has a positive effect. Inequality (33) shows that when the positive reviews are mild, the variance-increasing effect dominates the mean-shifting effect, and thus the wholesale price for product $B$ is higher because of the reviews. The change in the wholesale price, together with the change in the base demand for product $B$ because of the unfavorable reviews, changes the retail price of product $B$ in a similar fashion, as characterized by Inequality (34).

The retailer is affected by the online product reviews through both the variance-increasing effect and mean-shifting effect. First, as in symmetric case, the variance-increasing effect softens the upstream competition, which, per se, increases the wholesale prices and thus decreases the retailer’s revenue from base demands. Second, as illustrated in section 3.1.1, the mean-shifting effect makes the downstream demand asymmetric in terms of their potential market sizes, which engenders more room for the retailer to exploit its market and benefits the retailer. Whether the retailer can benefit from the reviews depends on the balance between the variance-increasing effect and the mean-shifting effect. When the reviews are mild and the mean-shifting effect is limited, the variance-increasing effect dominates the mean-shifting effect, which hurts the retailer’s profit from the base demands. Furthermore, when the retailer’s profit from the base demands is lower, the retailer has less incentive to invest on the traffic-boosting effect, which amplifies the decrease in the revenue. Inequality (35) characterizes such a case.
4 Conclusion

We examine the effect of online product reviews in a channel structure with a retailer carrying two substitutable products. The retailer may use wholesale scheme and sell products by itself, or use platform scheme and let manufacturers sell directly to consumers. We consider that consumers face uncertainty in both the product quality and fit to their needs, and product reviews provide additional information and reduce their uncertainties. Consumers agree on the preference order of the attributes in the quality dimension and have idiosyncratic preferences for the same attribute in the fit dimension. We identify the quality-dominates-fit case in which the quality dimension plays a dominant role in determining consumers’ perceived utility differences of the competing products and the fit-dominates-quality case in which the fit dimension plays a more important role. We show the effect of the reviews on an online retailer can be opposite depending on which dimension plays a dominant role and which pricing scheme the retailer uses.

We find, in quality-dominates-fit case, online product reviews homogenize consumers’ perceived utility differences between the two products and increase the competition between the manufacturers. Under the platform scheme, the retailer hurts by reviews unless the the reviews are strongly favorable to one product against the other, whereas, under the wholesale scheme, the retailer always benefits from reviews. In contrast, in the fit-dominates-quality case, online product reviews heterogenize consumers’ estimated fits to the products and soften the competition between the manufacturers. Under platform scheme, the retailer always benefits from the reviews, whereas, under wholesale scheme, the retailer hurts by the reviews unless the reviews are strongly favorable to one product against the other. Together, we show that the effect of online product reviews on the retailer is very different under different pricing schemes via changing the upstream competition. The effect of reviews also varies depending on the different kinds of information conveyed by reviews.

Online retailers have been deploying a variety of technologies to mitigate consumer uncertainty and match consumers with their preferred products. Online review platforms and recommendation systems are a few examples of these. Our results suggest that the effect of the same reviews on a retailer can be the opposite depending on the pricing scheme used. For example, some reviews may add the information in the quality dimension. The retailer should always welcome such reviews if the retailer uses wholesale scheme, but might want to avoid such reviews under platform scheme.
The reviews that provide fit information about products always benefit the retailer under platform scheme but may hurt the retailer under wholesale scheme. The rule of thumb of fostering reviews is that intended information to be revealed by reviews should be tailored according to the pricing scheme used by a retailer. Under such circumstances where retailer can benefit from reviews, retailers should encourage and even induce consumers and/or third parties to generate relevant reviews. For instance, retailers should make the review platform easy to use to facilitate the review generating process, and, in particular, they may provide some review templates to direct users toward generating information about product qualities or fits.

While controlling review generation and access process, review manipulation, and adjusting marketing-mix elements including strategic pricing are options that a retailer may have at its disposal to benefit from reviews, these options may also suffer from issues such as a potential decrease in consumers’ trust of the retailer. Alternatively, the contract that the retailer enters into with upstream sellers can be a strategic tool that a retailer can use to its advantage. The upstream contract is less likely to suffer from issues related to consumer mistrust. Finally, product category in the sense of whether fit or quality plays a dominant role in consumer decision making significantly affects a retailer’s optimal choice. As we noted in the introduction section, anecdotal observations suggest that the relative fraction of products for which Amazon uses the wholesale scheme or platform scheme varies across product categories; however, whether the relative importance of quality and fit varies across these categories and whether this relative importance influences Amazon’s choice of pricing scheme are open empirical questions.

References


Strauss, Samantha. 2010. Retailers can easily leverage amazon customer reviews (and others) with kaggle. Tech. rep.


Appendix

A.1 Proof of Lemma 1

Proof. We denote $a_A \equiv \frac{1}{2} + \frac{1}{2\tau}(1 - \gamma)x_R$, $a_B \equiv \frac{1}{2} - \frac{1}{2\tau}(1 - \gamma)x_R$, $b \equiv \frac{1}{2} + \alpha$, and $c \equiv \frac{1}{2\tau}$. The demand functions in Equation (9) then can be rewritten as

$$D_A = D^0_A [1 + \eta(\theta_A + \theta_B)] = (a_A - bp_A + cp_B) [1 + \eta(\theta_A + \theta_B)]$$
$$D_B = D^0_B [1 + \eta(\theta_A + \theta_B)] = (a_B - bp_B + cp_A) [1 + \eta(\theta_A + \theta_B)]$$

(36)

The manufacturers’ optimization problems in stage 3 are characterized by the first-order conditions of Equation (10):

$$\frac{\partial \pi_A}{\partial p_A} = (1 - k) (a_A - 2bp_A + cp_B) [1 + \eta(\theta_A + \theta_B)] = 0$$
$$\frac{\partial \pi_B}{\partial p_B} = (1 - k) (a_B - 2bp_B + cp_A) [1 + \eta(\theta_A + \theta_B)] = 0$$

from which we can derive the manufacturers’ optimal retail prices:

$$p_A = \frac{2a_A b + a_B c}{4b^2 - c^2}$$
$$p_B = \frac{2a_B b + a_A c}{4b^2 - c^2}$$

(37)

The manufacturers’ optimization problems in stage 2 are characterized by the first-order conditions of Equation (11):

$$\frac{\partial \pi_A}{\partial \theta_A} = (1 - k)p_A D^0_A \eta - 2\theta_A = 0$$
$$\frac{\partial \pi_B}{\partial \theta_B} = (1 - k)p_B D^0_B \eta - 2\theta_B = 0$$

from which we can derive the manufacturers’ optimal effort levels $\theta_i = (1 - k)p_i D^0_i \eta/2, i \in \{A, B\}$.

We denote $\pi^*_R \equiv p_A D^0_A + p_B D^0_B$. By substituting the optimal effort levels into Equation (12), the retailer’s objective function becomes

$$\pi_R = k \left( p_A D^0_A + p_B D^0_B \right) [1 + \eta(\theta_A + \theta_B)] = k \pi^*_R \left( 1 + \frac{(1-k)\eta^2 \pi^*_R}{2} \right)$$

(38)

Then, the retailer’s optimization problem in stage 1 is characterized by the first-order condition of
the above objective function:

\[
\frac{\partial \pi_R}{\partial k} = \pi_R^0 \left( 1 + \frac{(1-k)\eta^2 \pi_R^0}{2} \right) - k \pi_R^0 \frac{\eta^2 \pi_R^0}{2} = 0
\]

from which we can derive the retailer’s optimal \( k \):

\[ k = \frac{2+\eta^2 \pi_R^0}{2\eta^2 \pi_R^0} \quad (39) \]

Substituting \( k \) back to the objective function in Equation (38), the retailer’s optimal profit is

\[
\pi_R = \frac{2+\eta^2 \pi_R^0}{2\eta^2 \pi_R^0} \pi_R^0 \left( 1 + \frac{\eta^2 \pi_R^0 - 2}{4} \right) = \frac{1}{8\eta^2} \left( 2 + \eta^2 \pi_R^0 \right)^2 = \frac{\pi_R^0}{2} + \left[ \frac{1}{2} \left( \frac{\pi_R^0}{\eta} \right)^2 + \frac{1}{2\eta^2} \right] \quad (40)
\]

The equilibrium \( p \) follows by substituting \( a_A, a_B, b, \) and \( c \) into Equations (37). We can also derive the equilibrium \( \pi_R^0 \):

\[
\pi_R^0 = p_A D_A^0 + p_B D_B^0 = \frac{\tau(1+2\alpha r)}{(1+4\alpha r)^2} + \frac{(1+2\alpha r)(1-\gamma)^2 x^2}{\tau(3+4\alpha r)^2} \quad (41)
\]

The equilibrium \( k \) and \( \pi_R \) follow by substituting \( \pi_R^0 \) into Equations (39) and (40).

A.2 Proof of Proposition 1

Proof. By Lemma 1, with \( \gamma = 1 \) and \( \tau = \epsilon \) we have the equilibrium outcome for the scenario without reviews and with \( \gamma = r \) and \( \tau = r \epsilon \) we have the equilibrium outcome for the scenario with reviews.

(a) \( \tilde{p}_B > \hat{p}_B \) because

\[
\tilde{p}_B = \frac{\epsilon}{1+4\alpha \epsilon} > \frac{\epsilon}{1/r + 4\alpha \epsilon} \geq \frac{r \epsilon}{1+4\alpha \epsilon} - \frac{(1-r)r x_R}{3+4\alpha r} = \hat{p}_B
\]

\( \tilde{p}_A > \hat{p}_A \) if and only if

\[
\tilde{p}_A - \hat{p}_A = \frac{\epsilon}{1+4\alpha \epsilon} - \frac{r \epsilon}{1+4\alpha \epsilon} - \frac{(1-r)r x_R}{3+4\alpha r} > 0
\]

which leads to the condition in Inequality (17).
(b) \( \tilde{k} < \check{k} \) if and only if
\[
\tilde{k} - \check{k} = \frac{2(1+4\alpha)^2 + \epsilon(1+2\alpha)\eta^2}{2\epsilon(1+2\alpha)\eta^2} - \frac{2r(1+4\alpha)^2 + (1+2\alpha)[r^2\epsilon^2(3+4\alpha)^2 + (1-r)^2(1+4\alpha)^2x_R^2] \eta^2}{2(1+2\alpha)[r^2\epsilon^2(3+4\alpha)^2 + (1-r)^2(1+4\alpha)^2x_R^2] \eta^2} < 0
\]
and \( \tilde{\pi}_R > \check{\pi}_R \) if and only if
\[
\tilde{\pi}_R - \check{\pi}_R = \frac{[2(1+4\alpha)^2 + \epsilon(1+2\alpha)\eta^2]^2}{8(1+4\alpha)^4 \eta^2} - \frac{[2r(1+4\alpha)^2 + (1+2\alpha)[r^2\epsilon^2(3+4\alpha)^2 + (1-r)^2(1+4\alpha)^2x_R^2] \eta^2]^2}{8r^2\epsilon^2(1+4\alpha)^4(3+4\alpha)^4 \eta^2} > 0
\]
Each of the above two conditions can be simplified to the following condition:
\[
re^2(3+4\alpha)^2[1 + 2\alpha(1+r)\epsilon] - (1-r)(1+4\alpha)^2(1 + 2\alpha\epsilon)(1+4\alpha)^2x_R^2 > 0
\]
which leads to the condition in Inequality (18).

A.3 Proof of Proposition 2

Proof. By Lemma 1, with \( \gamma = 1 \) and \( \tau = \beta_C t \) we have the equilibrium outcome for the scenario without reviews and with \( \gamma = r \) and \( \tau = \beta_R t \) we have the equilibrium outcome for the scenario with reviews.

(a) \( \tilde{p}_A < \hat{p}_A \) because
\[
\tilde{p}_A = \frac{\beta_C t}{1+4\alpha\beta_C t} < \frac{\beta_R t}{1+4\alpha\beta_R t} \leq \frac{\beta_R t}{1+4\alpha\beta_R t} + \frac{(1-r)x_R}{3+4\alpha\beta_R t} = \hat{p}_A
\]

\( \tilde{p}_B < \hat{p}_B \) if and only if
\[
\tilde{p}_B - \hat{p}_B = \frac{\beta_C t}{1+4\alpha\beta_C t} - \frac{\beta_R t}{1+4\alpha\beta_R t} + \frac{(1-r)x_R}{3+4\alpha\beta_R t} < 0
\]
which leads to the condition in Inequality (20).

(b) From Equations (39) and (40), it is easy to see that \( \check{k} \) decreases in \( \pi^0_R \) and \( \pi_R \) increases in \( \pi^0_R \). Therefore, to conclude Part (b), we need to show \( \tilde{\pi}^0_R < \hat{\pi}^0_R \). In Equation (41), if we view \( \tilde{\pi}^0_R \) as a
function of \(x_R^2\), we have \(d\tilde{\pi}_R^0(x_R^2)/dx_R^2 > 0\), and thus \(\tilde{\pi}_R^0(x_R^2) \geq \tilde{\pi}_R^0(0)\). Meanwhile, we have

\[
\frac{\tilde{\pi}_R^0(0) - \tilde{\pi}_R^0}{\beta_R(1 + 2\alpha\beta_R t)} - \frac{\beta_C t(1 + 2\alpha\beta_C t)}{(1 + 4\alpha\beta_C t)^2} = \frac{(\beta_R - \beta_C)(1 + 2\alpha(\beta_R + \beta_C) t^2)}{(1 + 4\alpha\beta_R t)^2(1 + 4\alpha\beta_C t)^2} > 0
\]

Therefore, \(\tilde{\pi}_R^0(x_R^2) > \tilde{\pi}_R^0\) and Part (b) follows.

### A.4 Proof of Lemma 2

**Proof.** We denote \(a_A \equiv \frac{1}{2} + \frac{1}{2\gamma}(1 - \gamma)x_R\), \(a_B \equiv \frac{1}{2} - \frac{1}{2\gamma}(1 - \gamma)x_R\), \(b \equiv \frac{1}{2\gamma} + \alpha\), and \(c \equiv \frac{1}{2\gamma}\). The demand functions in Equation (9) then can be rewritten as

\[
D_A = (a_A - bp_A + cp_B) [1 + \eta (\theta_A + \theta_B)]
\]

\[
D_B = (a_B - bp_B + cp_A) [1 + \eta (\theta_A + \theta_B)]
\]

(42)

The retailer’s optimization problem in stage 3 is characterized by the first-order conditions of Equation (22):

\[
\frac{\partial \pi_R}{\partial p_A} = a_A - bp_A + cp_B + c(p_B - w_B) - b(p_A - w_A) [1 + \eta (\theta_A + \theta_B)] = 0
\]

\[
\frac{\partial \pi_R}{\partial p_B} = a_B - bp_B + cp_A + c(p_A - w_A) - b(p_B - w_B) [1 + \eta (\theta_A + \theta_B)] = 0
\]

from which we can derive the retailer’s optimal retail prices as functions of the wholesale prices:

\[
p_A = \frac{w_A}{2} + \frac{a_A b + a_B c}{2(b^2 - c^2)}
\]

\[
p_B = \frac{w_B}{2} + \frac{a_B b + a_A c}{2(b^2 - c^2)}
\]

(43)

The manufacturers’ optimization problems in stage 2 are characterized by the first-order conditions of Equation (23) (noticing that \(p_i\) in \(D_i\) is a function of \(w_i\) as in Equation (43)):

\[
\frac{\partial \pi_A}{\partial w_A} = \frac{1}{2} (a_A - 2bw_A + cw_B) [1 + \eta (\theta_A + \theta_B)] = 0
\]

\[
\frac{\partial \pi_B}{\partial w_B} = \frac{1}{2} (a_B - 2bw_B + cw_A) [1 + \eta (\theta_A + \theta_B)] = 0
\]

from which we can derive the optimal wholesale prices:

\[
w_A = \frac{2a_A b + a_B c}{4b^2 - c^2}
\]

\[
w_B = \frac{2a_B b + a_A c}{4b^2 - c^2}
\]

(44)
Substituting the above optimal wholesale prices into Equation (43), we derive the optimal retail prices:

\[
P_A = \frac{2a_A b + a_A c}{2(c^2 - b^2)} + \frac{a_A b + a_A c}{2(c^2 - b^2)} \\
P_B = \frac{2a_B b + a_B c}{2(c^2 - b^2)} + \frac{a_B b + a_B c}{2(c^2 - b^2)}
\] (45)

We denote \( \pi^0_R \equiv (p_A - w_A)D_A^0 + (p_B - w_B)D_B^0 \). Then, the retailer’s optimization problem in stage 1 is characterized by the first-order conditions of Equation (25):

\[
\frac{\partial \pi_R}{\partial \theta_i} = \pi^0_R \theta - 2\theta_i = 0
\]

from which we can derive the optimal effort levels \( \theta_i = \pi^0_R \theta / 2 \). Substituting \( \theta_i \) back to the objective function in Equation (25), the retailer’s optimal profit is

\[
\pi_R = \pi^0_R + \frac{(\pi^0_R \theta)^2}{2}
\] (46)

The equilibrium \( w_i \) and \( p_i \) follow by substituting \( a_A, a_B, b, \) and \( c \) into Equations (44) and (45). We can also derive

\[
\pi^0_R = (p_A - w_A)D_A^0 + (p_B - w_B)D_B^0 = \frac{(1 + 2\alpha \tau)^2}{8\alpha(1 + 4\alpha \tau)^2} + \frac{(1 + 2\alpha \tau)^2(1 - \gamma)^2 x_R^2}{87(1 + \alpha \tau)(3 + 4\alpha \tau)^2}
\] (47)

The retailer’s equilibrium profit follows by substituting the above \( \pi^0_R \) into Equation (46).

A.5 Proof of Proposition 3

Proof. By Lemma 2, with \( \gamma = 1 \) and \( \tau = \epsilon \) we have the equilibrium outcome for the scenario without reviews and with \( \gamma = r \) and \( \tau = r \epsilon \) we have the equilibrium outcome for the scenario with reviews.

(a) \( \tilde{w}_B > \hat{w}_B \) because

\[
\tilde{w}_B = \frac{\epsilon}{1 + 4\alpha \epsilon} > \frac{\epsilon}{1 + r + 4\alpha \epsilon} \geq \frac{r \epsilon}{1 + 4\alpha \epsilon} \geq \frac{(1 - r) x_R}{3 + 4\alpha \epsilon} = \hat{w}_B
\]

\( \tilde{w}_A > \hat{w}_A \) if and only if

\[
\tilde{w}_A - \hat{w}_A = \frac{\epsilon}{1 + 4\alpha \epsilon} - \frac{r \epsilon}{1 + 4\alpha \epsilon} - \frac{(1 - r) x_R}{3 + 4\alpha \epsilon} > 0
\]
which leads to the condition in Inequality (31).

(b) \( \tilde{p}_B > \hat{p}_B \) because

\[
\tilde{p}_B = \frac{1 + 6\alpha}{4\alpha(1 + 4\alpha)} > \frac{1 + 6\alpha r}{4\alpha(1 + 4\alpha)} \geq \frac{1 + 6\alpha r}{4\alpha(1 + 4\alpha)} - \frac{(5 + 6\alpha r)(1-r)x_R}{4(1 + 4\alpha r)(3 + 4\alpha r)} = \hat{p}_B
\]

\( \tilde{p}_A > \hat{p}_A \) if and only if

\[
\tilde{p}_A - \hat{p}_A = \frac{1 + 6\alpha}{4\alpha(1 + 4\alpha)} - \frac{1 + 6\alpha r}{4\alpha(1 + 4\alpha)} - \frac{(5 + 6\alpha r)(1-r)x_R}{4(1 + 4\alpha r)(3 + 4\alpha r)} > 0
\]

which leads to the condition in Inequality (32).

From Equation (46), it is easy to see that \( \pi_R \) increases in \( \pi_0^R \). Therefore, showing \( \hat{\pi}_R < \tilde{\pi}_R \) is equivalent to showing \( \hat{\pi}_R^0 < \tilde{\pi}_R^0 \). In Equation (47), if we view \( \hat{\pi}_R^0 \) as a function of \( x_R^2 \), we have \( d\hat{\pi}_R^0(x_R^2)/dx_R^2 > 0 \), and thus \( \hat{\pi}_R^0(x_R^2) \geq \hat{\pi}_R^0(0) \). Meanwhile, we have

\[
\hat{\pi}_R^0(0) - \tilde{\pi}_R^0 = \frac{(1 + 2\alpha r)^2}{8\alpha(1 + 4\alpha r)^2} - \frac{(1 + 2\alpha r)^2}{8\alpha(1 + 4\alpha r)^2} = \frac{4\alpha(1-r)(1 + 3\alpha r) + 8\alpha^2 r^2}{8\alpha(1 + 4\alpha r)^2(1 + 4\alpha r)^2} > 0
\]

Therefore, \( \hat{\pi}_R^0 < \tilde{\pi}_R^0(x_R^2) \) and thus \( \hat{\pi}_R < \tilde{\pi}_R \).

A.6 Proof of Proposition 4

Proof. By Lemma 2, with \( \gamma = 1 \) and \( \tau = \beta_{Ct} \) we have the equilibrium outcome for the scenario without reviews and with \( \gamma = r \) and \( \tau = \beta_{Rt} \) we have the equilibrium outcome for the scenario with reviews.

(a) \( \dot{w}_A < \dot{w}_A \) because

\[
\dot{w}_A = \frac{\beta_{Ct}}{1 + 4\alpha \beta_{Ct}} < \frac{\beta_{Rt}}{1 + 4\alpha \beta_{Rt}} \leq \frac{\beta_{Rt}}{1 + 4\alpha \beta_{Rt}} + \frac{(1-r)x_R}{3 + 4\alpha \beta_{Rt}} = \dot{w}_A
\]

\( \dot{w}_B < \dot{w}_B \) if and only if

\[
\dot{w}_B - \dot{w}_B = \frac{\beta_{Ct}}{1 + 4\alpha \beta_{Ct}} - \frac{\beta_{Rt}}{1 + 4\alpha \beta_{Rt}} + \frac{(1-r)x_R}{3 + 4\alpha \beta_{Rt}} < 0
\]

which leads to the condition in Inequality (33).
(b) \( \hat{p}_A < \hat{p}_B \) because

\[
\hat{p}_A = \frac{1+6\alpha \beta c t}{4\alpha(1+4\alpha \beta c t)} < \frac{1+6\alpha \beta R t}{4\alpha(1+4\alpha \beta R t)} \leq \frac{1+6\alpha \beta R t}{4\alpha(1+4\alpha \beta R t)} + \frac{(5+6\alpha \beta R t)(1-r)x_R}{4(1+\alpha \beta R t)(3+4\alpha \beta R t)} = \hat{p}_A
\]

\[
\hat{p}_B < \hat{p}_B \text{ if and only if }
\]

\[
\hat{p}_B - \hat{p}_B = \frac{1+6\alpha \beta c t}{4\alpha(1+4\alpha \beta c t)} - \frac{1+6\alpha \beta R t}{4\alpha(1+4\alpha \beta R t)} + \frac{(5+6\alpha \beta R t)(1-r)x_R}{4(1+\alpha \beta R t)(3+4\alpha \beta R t)} < 0
\]

which leads to the condition in Inequality (34).

From Equation (46), it is easy to see that \( \pi_R \) increases in \( \pi^0_R \), and therefore \( \hat{\pi}_R > \hat{\pi}_R \) if and only if \( \hat{\pi}_R > \hat{\pi}_R \). By Equation (47), \( \hat{\pi}_R > \hat{\pi}_R \) if and only if

\[
\hat{\pi}_R - \hat{\pi}_R = \frac{(1+2\alpha \beta c t)^2}{8\alpha(1+4\alpha \beta c t)^2} - \frac{(1+2\alpha \beta R t)^2}{8\alpha(1+4\alpha \beta R t)^2} - \frac{(1+2\alpha \beta c t)^2(1-\gamma)^2x^2_R}{8\alpha(1+4\alpha \beta c t)(3+4\alpha \beta c t)^2} > 0
\]

which leads to the condition in Inequality (35).