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PRIVATE CONTRACTS IN TWO-SIDED MARKETS*

GASTÓN LLANES[†] AND FRANCISCO RUIZ-ALISEDA[‡]

ABSTRACT. We study a two-sided market in which a platform connects consumers and sellers, and signs private contracts with sellers. We compare this situation with a two-sided market with public contracts. We find that the platform provider sets positive (negative) royalties to sellers and earns a negative (positive) markup on consumers when contracts are private (public). Thus, private contracting has a significant effect on the price structure. Private contracting leads to lower platform profits, consumer surplus, and social welfare. We study the welfare effects of most-favored-nation clauses, price-forcing contracts, and integration with sellers; and relate our results with the agency model of sales. Our results indicate that enhancing the market power of a dominant platform over sellers may increase welfare because it acts as a commitment device for inducing lower seller prices, mitigating the hold-up problem borne by consumers when they cannot observe sellers' contracts.

KEYWORDS: Two-Sided Markets, Platforms, Vertical Relations, Most-Favored Nation, Price-Forcing Contracts, Resale Price Maintenance, Integration, Agency Model of Sales (JEL: L12, L14, L42).

1. INTRODUCTION

Private contracts are common in two-sided markets. For example, Amazon signs private contracts with publishers, Netflix with movie studios, Sony and Nvidia with game developers, Spotify with record companies, HMOs with health-care providers, Google with phone manufacturers, Apple with cellphone carriers, and Intel and Microsoft with computer manufacturers. In this paper, we show that private contracting has a critical impact on the platform's price structure, industry profitability, and social welfare, and that it helps explain many commonly observed features of two-sided markets.

We study a two-stage model of a platform that connects buyers and sellers of platform-based products. Such products may be substitutes or complements. In the first stage, the platform provider chooses the membership or access fees to be paid by

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buyers and sellers, and sets the royalty fees that sellers have to pay for each unit of the good they sell to consumers; then sellers decide whether to accept the two-part-tariff contract offered by the platform provider, and finally, consumers decide whether to join the platform. In the second stage, sellers post prices, and consumers who have purchased access to the platform choose how much to buy from each seller.

Our aim is to compare different information structures and shed light on how each of them affects equilibrium royalty fees, access prices, platform profits, and welfare. In particular, we compare the *public contracts* case, in which the platform provider’s pricing scheme is publicly observable –the standard assumption in the two-sided markets literature (see Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006; Hagiu, 2006; Weyl, 2010, for example)– with the *private contracts* case, in which the platform’s offer to each seller is observed only by that seller –an assumption that is commonly used in the vertical relations literature (see Hart and Tirole, 1990; O’Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004, for example).

When the contract offered to a seller is private, equilibrium behavior depends on how sellers and buyers form beliefs about other players’ private information when they observe out-of-equilibrium play. In line with the literature, we assume that consumers form “passive beliefs” (Hart and Tirole, 1990; O’Brien and Shaffer, 1992; Hagiu and Halaburda, 2014) and sellers form “wary beliefs” (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when observing unexpected behavior by the platform. Thus, a consumer who observes a price for the platform that differs from the one expected in equilibrium believes that sellers’ behavior is unaffected. In turn, a seller observing an unexpected two-part tariff believes that the platform provider is acting opportunistically when pursuing such a deviation from expected play. In particular, the seller (rightly) conjectures that the platform provider has deviated in a profit-maximizing manner with the other seller.¹

We find that the conclusions drawn from a model of a two-sided market with private contracts stand in stark contrast with those of a model with public contracts. When contracts are public, equilibrium royalty fees are *negative* and the platform provider’s markup on consumers is *positive*. When contracts are private, on the other hand, royalty fees are *positive* and the platform provider’s markup on consumers is *negative*. Thus, the price structure is completely overturned when contracts are private instead of public. Our results explain the price patterns observed in many industries in which

¹Wary beliefs mitigate the opportunistic behavior of the platform provider relative to having sellers form passive beliefs. It is well-known from Rey and Vergé (2004) that there might exist no equilibria if sellers form passive beliefs in a setting like the one we shall consider.

contracts are private (for example, video game consoles or ebook readers are generally sold at a price below marginal cost and games and ebooks are sold at a positive markup).

We also find that private contracting results in lower profit for the platform provider, as well as lower consumer and social surplus. This result contrasts with those of papers studying private contracts in one-sided markets (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004), in which private contracting lowers industry profits but increases consumer surplus and welfare.

To understand our results, start by considering the public contracts case. On the one hand, the platform captures sellers' profits through the fixed fee and (indirectly) controls their pricing through royalty fees. Sellers' prices have a dual impact on consumer demands: they affect per-capita demand for sellers' products, and consumers' demand for platform access. On the other hand, the platform has direct control on the consumers' access prices. Such prices affect the demand for platform access, but not per-capita demands for sellers' products, so they are a more efficient instrument for extracting consumers' rents. As a consequence, the platform provider chooses royalties to induce a zero price by sellers and charges positive access prices to buyers.

Consider now an intermediate case in which sellers observe all contractual offers, but sellers' contracts are unobserved by consumers. Consumers then anticipate that the platform will behave opportunistically, choosing the unobservable royalty fees to induce collusive pricing by sellers. This hold-up problem anticipated by consumers lowers their demand for platform access. The platform lowers consumer access prices for two reasons: to compensate such a demand decrease, and because seller revenue per consumer increases (given that seller prices are set at the collusive level, each consumer who joins the platform becomes more valuable in terms of the revenues she generates when consuming sellers' products). Using a simple revealed-preference argument, it is straightforward to show that the effect of the lower demand dominates the effect of higher prices, and platform's profits decrease as a result. Consumer surplus also decreases, since fewer consumers join the platform and the surplus each one of them obtains is lower.

Finally, consider the private contracts case, and assume for the time being that sellers' goods are substitutes. In this case, one may be tempted to extrapolate Rey and Vergé's (2004) finding that the platform provider must be worse off (relative to the intermediate case mentioned above), for it loses part of its market power vis-à-vis sellers. Such an extrapolation would be incorrect because it would miss the feedback loops that arise in a two-sided market.

In particular, we find that in a two-sided market framework, decreasing the market power on one side may enhance market power on the other side.² Sellers fear that the platform will behave opportunistically, offering lower royalties to other sellers when they accept their contract. Thus, the royalties that sellers are willing to accept are lower, which implies that royalties and sellers' prices decrease relative to the intermediate case. This decrease in seller prices, in turn, encourages consumers to join the platform. Therefore, the lack of commitment when setting sellers' royalties acts as a commitment device for inducing lower seller prices, and it mitigates the hold-up problem borne by consumers when they cannot observe sellers' contracts. As a result, the platform provider can charge higher access prices to consumers and still increase the number of consumers joining the platform. These effects dominate the smaller profit per consumer that can be extracted from sellers, so platform profits increase relative to the intermediate case in which only buyers are uninformed.

In contrast with the substitutes case, when sellers' products are complementary, the platform earns less from sellers (for a given number of consumers), but also attracts fewer consumers, relative to the intermediate case.³ The loss of market power by the platform provider makes it less capable of internalizing the double marginalization problem faced by sellers (Cournot, 1838), so consumers expect sellers to charge higher prices. The platform becomes less valuable for buyers, and the hold-up problem becomes more severe. Even though the platform provider charges lower prices to attract consumers, platform sales decrease and the platform provider is harmed by the lower usage of the platform by consumers and the smaller profit appropriated from sellers.

Comparing now the public contracts and private contracts cases, it holds when contracts are private rather than public that consumers fear being taken advantage of by the platform because they cannot observe the actual royalties that the platform will receive from sellers. When sellers' goods are substitutes, the hold-up problem that consumers face is *mitigated* by the loss of market power that the platform provider bears when it secretly contracts with each seller. Our contribution in this case is to show that the consumers' initial concern is not mitigated enough by this loss in control. The platform provider's profits are therefore smaller when contracts are private rather than public. Consumer surplus and social welfare decrease as well. When sellers' offer complements instead of substitutes, our contribution is to show that the

²As is standard, we define market power as a firm's ability to charge prices above marginal cost.

³To the best of our knowledge, the complements case has not been analyzed by the vertical relations literature dealing with secret contracts.

consumers' concern about the platform provider's opportunistic behavior is *accentuated* because it has less control over the double marginalization problem faced by the sellers. Relative to public contracting, private contracts again result in higher royalties, higher prices charged by sellers, lower prices for the platform, lower profitability for the platform provider, and lower consumer and social welfare.

We also study the welfare effects of Most-Favored Nation (MFN) contractual clauses, price-forcing contracts (i.e., allowing the platform to contract on sellers' prices), and integration between the platform and sellers. We find that MFN clauses increase welfare when seller's products are complements, and reduce welfare when seller's products are substitutes, whereas contracting on sellers' prices increases welfare in both cases. The difference is that MFN clauses solve the commitment problem with sellers, but not with consumers, whereas *price-forcing contracts solve the commitment problem* with both sellers and consumers. Integration with all sellers can also help prevent the welfare losses from private contracting, but *incomplete* integration (when the platform integrates with some, but not all sellers) is socially desirable only when seller's products are complements.

Overall, our findings suggest that enhancing the platform's market power may be beneficial because it prevents consumers from facing a hold-up problem that would harm adoption and overall platform profitability. For instance, contracts in which sellers' prices are set by the platform may be good for the provider of the platform and for consumers as well. Such price-forcing contracts are so beneficial because they fully alleviate consumers' fears of being held up after joining to the platform. This result explains the pervasive use of price-forcing contracts in two-sided markets (as in the cases of iTunes, Amazon Prime, and Netflix).

Our paper contributes to the literature on two-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006). To the best of our knowledge, the entire literature assumes that contracts are publicly observable to all parties. The only exception in which one of the two sides does not observe the price charged to the other side is the paper by Hagiu and Halaburda (2014), which examines how price transparency affects market outcomes. Our result that contractual transparency is beneficial because it allows the platform to commit not to trick consumers into purchasing a platform that will have expensive goods sold by sellers is different from Hagiu and Halaburda's (2014) insight because buyers and sellers do not interact in their setting. In fact, if sellers' prices were contractible in our setting, then the platform would commit not to hold up consumers and consumers would benefit from it (think of iTunes, for example). In contrast with the two-sided markets literature, we

also allow sellers to enjoy market power, so the platform provider shapes their competitive interaction through its choice of royalty fees.

Our paper also builds on the literature on vertical relations regulated by secret contracts, with important contributions by Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and Rey and Vergé (2004). Using their terminology, the upstream supplier in our setting has another type of customer with whom downstream firms interact, and such interaction is shaped by the upstream supplier's decisions. This two-sidedness of the problem implies that there are cross-group network effects, so the issues and results are very different from this literature.

2. THE MODEL

We study a two-sided market composed of a platform provider, $n \geq 2$ sellers, and a continuum of consumers. The platform provider produces a platform good (such as a video console) at a normalized marginal cost of zero. Sellers sell platform-specific products (such as video games) to consumers (e.g., gamers) who buy the platform good. Sellers produce at zero marginal cost (again a normalization).

Consumers are uniformly spread on the positive real line and firm 0 is located at the left end. The utility of a consumer located at distance $x \in [0, \infty)$ from the platform provider if she purchases the platform good at price p_0 , and $q_i \geq 0$ units of the product of seller $i \in \{1, \dots, n\}$ at price p_i per unit is (see Vives, 2001, for example)

$$U_x(p_0, p_1, q_1, \dots, p_n, q_n) = u(p_1, q_1, \dots, p_n, q_n) - x - p_0,$$

where

$$u(p_1, q_1, \dots, p_n, q_n) = \sum_{i=1}^n q_i - \frac{1}{2} \left(\sum_{i=1}^n q_i^2 + \theta \sum_{i=1}^n \sum_{j=1; j \neq i}^n q_i q_j \right) - \sum_{i=1}^n p_i q_i.$$

Parameter $\theta \in (-1, 1)$ captures the degree of complementarity/substitution between sellers' goods. If $\theta < 0$, goods are complements, with their degree of complementarity decreasing with θ . If $\theta = 0$, goods are independent. If $\theta > 0$, goods are substitutes, with their degree of substitutability increasing with θ .

We consider the following two-stage model. In the first stage, the platform provider offers contracts to sellers and sets a price p_0 for consumers. Sellers decide whether to accept the contract, and then consumers observe both p_0 and how many sellers have accepted the contract before having to decide whether to buy the platform good. In the second stage, sellers set prices for their products, and consumers decide how many products to buy. Our timing reflects the fact that consumers use the platform for many periods, during which platform-specific products are continuously being

launched. For instance, buyers of a video console often buy it without observing the prices charged for the games they will consume during the lifetime of the console.

A contract between seller $i \in \{1, \dots, n\}$ and the platform provider consists of a fixed fee f_i and a per-unit royalty fee w_i .⁴ If seller i accepts the contract and then sells Q_i units to consumers, its total payment to the platform provider is $f_i + w_i Q_i$. We will take n as given, and for the most part of the paper we will let $n = 2$. We discuss what happens as n increases in Section 6.

In Section 3, we study a one-sided market with public contracts. That is, we assume that $p_0 = 0$, and that consumers and sellers observe all contracts before making their decisions. In Section 4, we study a two-sided market with public contracts. This game is analogous to the previous one, except that we allow for $p_0 \neq 0$. In Section 5, we study a two-sided market with private contracts. We first examine an intermediate situation in which consumers do not observe the contracts offered to sellers, but sellers observe all contracts. We then examine the private-contract case, in which consumers do not observe the contracts offered to sellers, and each seller only observes the contract she is offered. We assume throughout that p_0 is contractible and is written in the contract offered to any seller.⁵

Even though we assume that the number of sellers is fixed, the market is two-sided because the number of consumers who join the platform affects the profitability of sellers. Thus, there are indirect network effects between buyers and sellers. In Section 4, we also show that the platform's price structure has a non-trivial effect on membership decisions and the level of transactions.

In Sections 3 and 4, we seek for symmetric subgame perfect equilibria (SPE). In Section 5, we seek for symmetric perfect Bayesian equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed.

3. PUBLIC CONTRACTS IN A ONE-SIDED MARKET

In order to understand the basic mechanisms at play, in this section we consider a one-sided market in which p_0 is constrained to be zero. Comparing this model with

⁴Our main results do not depend on fixed fees being available. The proof is available on request.

⁵Even if p_0 is not contractible, reputational concerns may prevent the platform provider from behaving opportunistically with sellers. That p_0 is known by sellers when they have to decide whether to accept contracts is standard in some industries such as video games (see Hagiu, 2006, for example). If p_0 was chosen after sellers have decided to accept the platform's offers, sellers would anticipate a hold-up problem that would harm the platform. Note also that it is in principle easier to contract upon p_0 than upon other seller's fees because sellers will eventually observe p_0 , but they may never be able to observe the royalty fees paid by other sellers.

that of Section 4 also allows us to show how the two-sidedness of the market affects the results.

We start by studying the second stage. After observing p_i ($i \in \{1, 2\}$), consumers who have purchased the platform good decide how many units of sellers' products to consume. Looking at interior solutions of a consumer's utility maximization problem yields the following *per-capita* demand for the product of seller i :

$$q_i(p_i, p_j) = \frac{1 - \theta - p_i + \theta p_j}{1 - \theta^2}. \quad (1)$$

Per-capita consumption does not depend on the distance between the consumer and the platform. Thus, the overall demand for seller i 's product is $Q_i(p_i, p_j) = x_0 q_i(p_i, p_j)$, where x_0 is the number of consumers who choose to buy the platform good in the first stage. Seller $i \in \{1, 2\}$ solves the following problem given a price p_j by the other seller:

$$\max_{p_i} (p_i - w_i) Q_i(p_i, p_j) - f_i,$$

where f_i is a cost already sunk and the total number of consumers, x_0 , is given from the first stage. Seller i 's first-order condition is

$$x_0 (1 - \theta - 2p_i + w_i + \theta p_j) = 0,$$

so its equilibrium price can be easily shown to be

$$p_i(w_i, w_j) = \frac{(2 + \theta)(1 - \theta) + 2w_i + \theta w_j}{(2 + \theta)(2 - \theta)}. \quad (2)$$

It readily follows from (1) that each consumer buys

$$q_i(w_i, w_j) = \frac{(1 - \theta)(2 + \theta) - w_i(2 - \theta^2) + \theta w_j}{(1 - \theta^2)(4 - \theta^2)} \quad (3)$$

units of product i .

We now turn to the analysis of the first stage. By symmetry, optimal royalties are such that $w_1 = w_2 = w$. Recall that in the one-sided market case, $p_0 = 0$. In the first stage, given w , the utility of consumer x is

$$U_x^o(w) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2} - x,$$

where the superscript o refers to the one-sided, public-contracts case. This results in a demand for the platform good equal to

$$x_0^o(w) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2}.$$

Anticipating how play will evolve in the second stage, seller i will accept the contract offered by the platform if and only if $f_i \leq x_0(p_i - w_i) q_i$. The platform provider sets $f_i = x_0(p_i - w_i) q_i$, and solves

$$\max_w x_0^t(w) [p_1(w, w) q_1(w, w) + p_2(w, w) q_2(w, w)]. \quad (4)$$

It is then easy to prove the following result.

Proposition 1. *If firm 0 provides a one-sided platform and contracts are publicly observed by all parties, equilibrium royalties are*

$$w^o = \frac{3\theta - 2}{4},$$

the equilibrium price charged by seller $i \in \{1, 2\}$ is

$$p_i^o = \frac{1}{4},$$

per-capita consumption of each product is

$$q_i^o = \frac{3}{4(1 + \theta)},$$

the number of consumers that joins the platform is

$$x_0^o = \frac{9}{16(1 + \theta)},$$

platform profits are

$$\pi_0^o = \frac{27}{128(1 + \theta)^2},$$

and consumer surplus is

$$cs^o = \frac{81}{512(1 + \theta)^2}.$$

Note that $w^o < 0$ for $\theta < 2/3$ and $w^o > 0$ for $\theta > 2/3$. To understand this result, note that, given that the platform perfectly predicts second stage prices as a function of royalties, it can solve the problem in expression (4) as if it was choosing prices p_i instead of royalties w_i . The first-order condition with respect to price p_1 is

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) + \frac{\partial x_0}{\partial p_1} (p_1 q_1 + p_2 q_2) = 0.$$

Seller 1, on the other hand, chooses price p_1 according to the following first-order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0.$$

When choosing a price, seller 1 ignores two effects: the effect that a change in p_1 has on the per-capita demand of seller 2, and the effect it has on the number of consumers

who join the platform. The first effect is positive or negative depending on whether $\partial q_2 / \partial p_1$ is positive or negative, and the second effect is always negative.

It is straightforward to see that the platform can make seller 1 internalize these two effects by choosing an appropriate royalty fee. In particular, it should choose a royalty fee so that

$$-w_1 \frac{\partial q_1}{\partial p_1} = p_2 \frac{\partial q_2}{\partial p_1} + \frac{\partial x_0}{\partial p_1} \frac{p_1 q_1 + p_2 q_2}{x_0}.$$

When $\theta \leq 0$, the two terms on the right hand side are negative. Thus, the optimal royalty fee is negative. The royalty fee will be positive only if θ is positive and sufficiently large to overcome the negative effect of the change in the number of consumers joining the platform. This is precisely the result in Proposition 1.

4. PUBLIC CONTRACTS IN A TWO-SIDED MARKET

We now allow the platform good to be priced at $p_0 \neq 0$. Second-stage decisions (for a given number of consumers and pair of royalty fees) are equivalent to those of the previous section (see expressions (2) and (3)). In the first stage, given $w_1 = w_2 = w$ and p_0 , the utility of consumer x is

$$U_x^t(w, p_0) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2} - x - p_0,$$

where the superscript t refers to the two-sided, public-contracts case. It follows that the demand for the platform good is

$$x_0^t(w, p_0) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2} - p_0.$$

As in the previous section, the platform provider sets $f_i = x_0(p_i - w_i)q_i$, but now solves

$$\max_{w, p_0} x_0^t(w, p_0) [p_0 + p_1(w, w)q_1(w, w) + p_2(w, w)q_2(w, w)],$$

which leads to the following result.

Proposition 2. *If firm 0 provides a two-sided platform and contracts are publicly observed by all parties, equilibrium royalties are*

$$w^t = -(1 - \theta) < 0,$$

the equilibrium price charged by seller $i \in \{1, 2\}$ is

$$p_i^t = 0,$$

the equilibrium price for the platform good is

$$p_0^t = \frac{1}{2(1+\theta)} > 0,$$

per-capita consumption of each product is

$$q_i^t = \frac{1}{1+\theta},$$

the number of consumers who join the platform is

$$x_0^t = \frac{1}{2(1+\theta)},$$

platform profits are

$$\pi_0^t = \frac{1}{4(1+\theta)^2},$$

and consumer surplus is

$$cs^t = \frac{1}{8(1+\theta)^2}.$$

Note that, in contrast with the previous case, in this case the optimal royalty fee is always negative, and goes to zero as θ goes to one. To understand this result, we start by noting that the first-order condition of the platform with respect to price p_0 is

$$x_0 + \frac{\partial x_0}{\partial p_0} (p_0 + p_1 q_1 + p_2 q_2) = 0.$$

Given that the platform absorbs sellers' profits through the fixed fee, it can act as if it was choosing price p_1 instead of royalty fee w_1 . Then, it would choose price p_1 according to the following first-order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) + \frac{\partial x_0}{\partial p_1} (p_0 + p_1 q_1 + p_2 q_2) = 0.$$

The first-order condition with respect to p_0 implies that

$$p_0 + p_1 q_1 + p_2 q_2 = x_0,$$

given that $\partial x_0 / \partial p_0 = -1$. Since Roy's identity implies that $\partial x_0 / \partial p_1 = -q_1$, the first-order condition becomes:

$$x_0 \left(p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) = 0.$$

In a symmetric equilibrium:

$$x_0 p_1 \left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) = 0,$$

so it is optimal to induce sellers to sell their products at a price of zero. Given that seller 1 chooses price p_1 so that

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0,$$

the royalty fee must be negative so that sellers choose prices equal to zero. Note that prices go to marginal cost as $\theta \rightarrow 1$ due to pure Bertrand competition, so the royalty converges to zero when products become perfect substitutes. Hence the results in Proposition 2.

Finally, note that the results in this section stand in stark contrast with those of the vertical relations literature. For instance, it is well known when $\theta = 0$ that an upstream supplier would avoid double marginalization and hence would not distort pricing by a downstream retailer, which is not the case in our setting because of the presence of indirect network effects.

5. PRIVATE CONTRACTS

In this section, we assume that the contracts between the platform provider and sellers are private. Thus, consumers cannot observe any of the contracts offered to sellers, and a seller can only observe the contract it is offered. We will seek for symmetric Perfect Bayesian Equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed.⁶ In what follows, let p_0^* denote the price charged to consumers by the platform provider in a symmetric PBE. Also, let w^* denote the royalty fee that is offered to seller $i \in \{1, 2\}$ in a symmetric PBE, and f^* the associated fixed fee.

Regarding the formation of out-of-equilibrium beliefs, note that, upon observing any $p_0 \neq p_0^*$, rational consumers would realize that such a deviation affects sellers' profits and potentially their incentives to enter the market (this happens when $p_0 > p_0^*$). They should therefore conclude that a price deviation must be accompanied by a change in the fixed fee and/or a change in the royalty fee offered to each seller. We will look at equilibria in which consumers rationalize any price deviation by conjecturing that there was no deviation in the royalty fee offered to each seller; hence, consumers believe upon observing $p_0 \neq p_0^*$ that the platform is simply adjusting the fixed fee offered to each seller just to make it break-even given w^* . These beliefs are

⁶No asymmetric equilibrium exists, so the symmetry requirement is without loss of generality, at least if one restricts attention to equilibria in which the pricing strategy and beliefs held by a seller are polynomial functions of the royalties it observes.

in the spirit of “passive beliefs” (Hart and Tirole, 1990), but they require some rationality by consumers. In particular, when consumers observe a price deviation, they acknowledge that this should have had an impact on the sellers’ willingness to accept the contract, and they reason that the absence of such an impact must be due to a change in the fixed fee offered to each seller. We refer to this weak form of passive beliefs held by consumers as “weakly passive beliefs,” and note that the main implication of such belief formation is that consumers always expect the interaction of sellers in the product market to be unaffected by the choice of p_0 .⁷

Because a seller anticipates such unsophisticated behavior by consumers when $p_0 \neq p_0^*$, it believes that $p_0 \neq p_0^*$ conveys no information about contract offers. Thus, sellers therefore form passive beliefs with respect to deviations in p_0 . However, seller $i \in \{1, 2\}$ is assumed to form “wary beliefs” (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when it observes an unexpected contract offer. In such case, it believes that the platform provider must have made an offer to j that maximizes the platform’s total profit given the price that it charges to consumers and the contract offered to seller i . Of course, in equilibrium, a seller anticipates perfectly the offer made by the platform to the other seller, but the formation of wary beliefs by sellers implies that, if the platform deviates from equilibrium play, then sellers will correctly infer how it is deviating. We also assume that a seller that forms wary beliefs conjectures that the other seller also does, and also conjectures that the platform provider does not want to drive any seller out of the market.

5.1. Contracts observable to sellers, but unobservable to consumers. Before examining equilibrium play when the contract offer received by a seller is solely observed by such a seller, it is useful to examine an intermediate case in which sellers observe each other’s contract, but consumers do not. As we show next, such unobservability gives rise to a hold-up problem that consumers anticipate: consumers will (correctly) believe that the platform provider will induce sellers to charge high (collusive) prices. Sellers will earn more for each consumer who joins the platform, but the platform’s value for consumers will be harmed by such beliefs. Both these forces induce the platform provider to lower access prices for buyers, thereby setting a negative markup on them.

⁷The outcome would be the same under the standard strong form of passive beliefs (corresponding to situations in which consumers do not change their equilibrium beliefs when observing out-of-equilibrium behavior). However, it would be harder to interpret some situations. For example, upon observing $p_0 > p_0^*$, a consumer who kept her beliefs about f^* and w^* should conclude that the sellers are accepting a contract that gives them negative profits, for consumer demand is smaller than it should be in equilibrium (since we shall show later on that consumer demand for the platform does not affect competition between sellers, which solely depends on royalty fees).

To see these issues formally, let us denote the contract offered to each seller in equilibrium by $(\widehat{f}, \widehat{w})$. Because consumers cannot observe deviations from this contract and form weakly passive beliefs when observing any p_0 , their demand for the platform when observing price p_0 equals

$$x_0(p_0, \widehat{w}) = \frac{(1 - \widehat{w})^2}{(1 + \theta)(2 - \theta)^2} - p_0.$$

The platform provider extracts all the surplus from sellers, and chooses p_0 , w_1 and w_2 to maximize

$$x_0(p_0, \widehat{w}) [p_0 + p_1(w_1, w_2)q_1(w_1, w_2) + p_2(w_2, w_1)q_2(w_2, w_1)].$$

The first-order condition corresponding to w_i is

$$\frac{\theta(1 - \theta)(2 + \theta)^2 - (8 - 6\theta^2)w_i + 2\theta^3 w_j}{(1 - \theta^2)(4 - \theta^2)^2} = 0.$$

Rearranging this equation allows us to give it an interpretation that will be useful later on. When seller j receives an offer involving royalty fee w_j , it infers that the platform provider finds it optimal to charge seller i with a royalty fee w_i that solves the above first-order condition. Thus, given a royalty offer of w , a seller believes that the other seller is being offered a royalty equal to

$$\widehat{w}^*(w) = \frac{\theta(1 - \theta)(2 + \theta)^2 + 2\theta^3 w}{2(4 - 3\theta^2)}, \quad (5)$$

Therefore, $\widehat{w}^*(\cdot)$ can be interpreted as a seller's belief about the royalty fee offered to the other seller. Such a belief is correct both on and off the equilibrium path because the platform anticipates that sellers will have complete information when pricing, so there is no way to fool them. The function $\widehat{w}^*(\cdot)$ will serve as a useful benchmark when we further assume in the next subsection that sellers cannot observe each other's contract offers.

We now complete our solution of the model. It is straightforward to show that the equilibrium royalty fee is

$$\widehat{w} = \frac{\theta}{2}.$$

Thus, the royalty fee is positive if $\theta > 0$ and negative if $\theta < 0$. The first-order condition corresponding to p_0 can be written as

$$\frac{(2 - \theta)^2}{4(1 + \theta)(2 - \theta)^2} - \frac{1}{2(1 + \theta)} - 2p_0 = 0,$$

so

$$\widehat{p}_0 = -\frac{1}{8(1 + \theta)} < 0.$$

In equilibrium, the platform induces seller i to charge price

$$\widehat{p}_i = \frac{1}{2} > 0,$$

and gains

$$\widehat{\pi}_0 = \frac{9}{64(1+\theta)^2}.$$

To understand these results, we can proceed as in the previous sections. If the platform acts as if it was choosing price p_1 instead of royalty fee w_1 , it would choose price p_1 according to the following first order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) = 0.$$

Note that this first-order condition differs from the one in Section 4 because consumers do not observe changes in royalty fees, so their decision to buy the platform good depends only on their *beliefs* about the equilibrium royalty. In a symmetric equilibrium, it holds that

$$-\left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) p_i = q_i.$$

Thus, the optimal implied price for sellers is positive. This contrasts with the result in the public-contracts case, in which the optimal price was zero.

It is easy to see that the optimal price p_0 solves

$$p_0 = \frac{U(p_1, p_2) - p_1 q_1 - p_2 q_2}{2}.$$

This equation shows that the platform has incentives to lower p_0 , in comparison with the public-contracts case, for two reasons: to compensate the decrease in consumer surplus from consumption of seller goods ($U(p_1, p_2) < U(0, 0)$), and because seller surplus per consumer increases ($p_1 q_1 + p_2 q_2 > 0$). In the case at hand, it turns out that the platform lowers p_0 so much that it ends up setting a negative access fee for consumers.

Finally, note that seller i chooses price p_i so that

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0.$$

Thus, the royalty needs to be positive if the cross-price effect $\partial q_2 / \partial p_1$ is positive (substitutes), and negative if the cross-price effect is negative (complements).

Summarizing, we find that when consumers do not observe royalty fees, they are less reactive to changes in the intensity of competition between sellers, since they cannot observe deviations from the royalty fees they expect in equilibrium. As a consequence, the platform has incentives to behave opportunistically, and choose royalties

to induce collusive pricing by sellers. Consumers correctly foresee the hold-up problem when they decide whether to join the platform, so their utility from joining the platform decreases. The platform has incentives to lower access prices for consumers for two reasons: to compensate the lower demand for platform access, and because seller revenue per consumer increases.

5.2. Contracts unobservable to sellers and consumers. We now turn to the analysis of the private-contracts case, in which the contract offer received by a seller is solely observed by this seller.

At the beginning of the second stage, seller $i \in \{1, 2\}$ knows p_0 , x_0 , f_i and w_i , and chooses a price for its product based on this information. Taking into account that seller i 's overall demand product equals $Q_i(p_i, p_j) = x_0 q_i(p_i, p_j)$, we can solve for the second-stage subgames.

Let $B(\widehat{w})$ denote the belief formed by seller i about the royalty fee paid by seller j to the platform provider.⁸ We follow Rey and Vergé (2004), and restrict attention to equilibria in which seller i 's belief about the royalty fee paid by the other seller does not depend on the fixed fee it observes. Not only is the pricing strategy of seller i independent from the fixed fee it already paid, but it is also independent from p_0 (and hence from x_0). Such a price has no signaling role and it does not affect belief formation, which seems a reasonable assumption given that x_0 is simply a scaling factor in seller i 's second-stage profit.⁹

In what follows, let $p_i(w_i)$ denote the strategy of seller $i \in \{1, 2\}$ in the second-stage subgame if it has observed an offer of (w_i, f_i) and price p_0 . Seller $i \in \{1, 2\}$ chooses p_i to maximize

$$(p_i - w_i)Q_i(p_i, p_j(B(w_i))) - f_i$$

with f_i already sunk. The first-order condition is

$$1 - \theta + w_i - 2p_i(w_i) + \theta p_j(B(w_i)) = 0. \quad (6)$$

We now turn to analyzing the first stage of play. Regardless of the price p_0 that consumers observe, they believe that both sellers face a royalty fee w^* , so they expect

⁸Because we are looking at symmetric equilibria, the belief function $B(\cdot)$ does not depend on the label of the seller receiving the unexpected offer. Note that, in general, $B(\cdot)$ is an unrestricted function except for the constraint that $B(w^*) = w^*$ (i.e., conjectured beliefs are fulfilled along the equilibrium path). In our case, we restrict the function so that beliefs are wary.

⁹Therefore, it does not affect equilibrium pricing in the second-second if sellers believe that it does not convey some information, making it self-fulfilling that it is pointless for the platform provider to use it for signaling purposes.

a price

$$p_i^* = \frac{1 - \theta + w^*}{2 - \theta}$$

for each unit they purchase from seller $i \in \{1, 2\}$ in the second stage. Given price p_0 , the overall utility expected by consumer x is

$$U_x(w^*, p_0) = \frac{(1 - w^*)^2}{(1 + \theta)(2 - \theta)^2} - x - p_0,$$

so the demand for the platform good is

$$x(w^*, p_0) = \frac{(1 - w^*)^2}{(1 + \theta)(2 - \theta)^2} - p_0.$$

The platform provider's total profit if it charges p_0 and makes a private offer of (w_1, f_1) and (w_2, f_2) to sellers 1 and 2 is

$$\begin{aligned} \pi_0(w_1, f_1, w_2, f_2, p_0) &= x(p_0) [p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) \\ &\quad + w_2 q_2(p_1(w_1), p_2(w_2))] + f_1 + f_2, \end{aligned}$$

since the platform provider can perfectly forecast actual sales made by sellers 1 and 2. In order for seller 2 (say) to form wary beliefs, her inference about seller 1's contract upon observing a price of p_0 and an offer of (w_2, f_2) must be such that $B(w_2)$ maximizes $\pi_0(w, f, w_2, f_2, p_0)$ with respect to w and f subject to the constraint that

$$f \leq (p_1(w) - w)x(p_0)q_1(p_1(w), p_2(B(w))).$$

Taking into account that the constraint must bind at the optimum, and that (6) implies

$$q_1(p_1(w), p_2(B(w))) = \frac{p_1(w) - w}{1 - \theta^2},$$

we have

$$B(w_2) \in \operatorname{argmax}_w \pi_0(w, w_2, f_2, p_0), \quad (7)$$

where

$$\begin{aligned} \pi_0(w, w_2, f_2, p_0) &= x(p_0) \left\{ p_0 + w q_1(p_1(w), p_2(w_2)) \right. \\ &\quad \left. + w_2 q_2(p_2(w_2), p_1(w)) + \frac{[p_1(w) - w]^2}{1 - \theta^2} \right\} + f_2. \end{aligned}$$

The first-order condition is

$$q_1(p_1(w), p_2(w_2)) + \frac{2[p_1(w) - w]}{1 - \theta^2} \left(\frac{dp_1(w)}{dw} - 1 \right) + \left[w \frac{\partial q_1(p_1(w), p_2(w_2))}{\partial p_1} + w_2 \frac{\partial q_2(p_2(w_2), p_1(w))}{\partial p_1} \right] \frac{dp_1(w)}{dw} = 0. \quad (8)$$

Since our purpose at this stage is to build some intuition, let us assume for now that a unique solution to equation (8) exists for any w_2 . Let $w_1^*(w_2)$ denote that solution, and note that it must coincide with $B(w_2)$ even if $w_2 \neq w^*$ because sellers form wary beliefs even when off the equilibrium path. Using the implicit function theorem, we obtain the following result:

$$\frac{dB(w_2)}{dw_2} = \frac{dw_1^*(w_2)}{dw_2} = - \frac{\frac{\partial}{\partial w_2} \left(\frac{dp_1(w)}{dw} + \frac{dp_2(w_2)}{dw_2} \right)}{\partial^2 \pi_0(w, w_2, f_2, p_0) / \partial w^2}.$$

If $\pi_0(w, w_2, f_2, p_0)$ is strictly concave with respect to w (as we shall later show), symmetry yields that

$$\text{sign} \left(\frac{dB(w)}{dw} \right) = \text{sign} \left(\theta \frac{dp(w)}{dw} \right).$$

Whenever it holds that $dp(w)/dw > 0$, which is an intuitive property that equilibrium prices should satisfy,¹⁰ we have that $dB(w)/dw \gtrless 0$ if and only if $\theta \gtrless 0$, according well with what one may have expected: sellers' prices are *strategic complements* if $\theta > 0$ and *strategic substitutes* if $\theta < 0$, and the platform provider aims at softening competition between sellers under strategic complementarity and at toughening such competition under strategic substitutability.

Having shed some light on some of the properties that the equilibrium satisfies, we proceed to showing existence and characterizing it. To this end, evaluating the first-order condition at $w = B(w_2)$, and letting $p_i(w) = p(w)$ because of symmetry, we obtain

$$0 = 1 - \theta - p(B(w_2)) + \theta p(w_2) + (\theta w_2 - B(w_2)) \frac{dp(B(w_2))}{dw} + 2[p(B(w_2)) - B(w_2)] \left[\frac{dp(B(w_2))}{dw} - 1 \right]. \quad (9)$$

If one focuses on PBE such that $p(\cdot)$ and $B(\cdot)$ are polynomial functions, then Rey and Vergé (2004) show that there is no loss of generality in restricting attention to affine functions, so one can readily solve the system of differential equations given by (9) and (6) (after dropping subscripts) to obtain the following result:

¹⁰Note that we shall restrict attention to polynomial pricing strategies, and that in such cases there is no loss in further restricting them to be affine.

Proposition 3. *The unique symmetric PBE in which $p(w)$ and $B(w)$ are polynomial functions is such that $p(w) = \Theta_\theta + \Sigma_\theta w$ and $B(w) = \Gamma_\theta + \Phi_\theta w$ for some constants $\Theta_\theta \in [0, 1]$, $\Sigma_\theta \in [\frac{1}{2}, 1] > 0$, $\Gamma_\theta \in [0, 1]$, and $\Phi_\theta \in [-1, 1]$. In such an equilibrium, it always holds that*

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \geq 0$$

for $i \in \{1, 2\}$ and $w^* \geq 0$ for any $\theta \in (-1, 1)$, with $w^* = 0$ if and only if $\theta = 0$. Platform profits are

$$\pi_0^* = \left(\frac{1 - (p_i^*)^2}{2(1 + \theta)} \right)^2,$$

and consumer surplus is

$$cs^* = \frac{1}{2} \left(\frac{1 - (p_i^*)^2}{2(1 + \theta)} \right)^2.$$

Proof. See Appendix A. ■

Contrary to the case in which sellers can observe each other's royalties (subsection 5.1), royalty fees are never negative under private contracting, regardless of whether price competition between sellers displays strategic complementarity ($\theta > 0$) or strategic substitutability ($\theta < 0$).

When sellers can observe each other's royalties, $\theta > 0$ implies that $d\hat{w}^*(w)/dw > 0$ (see expression (5)), so an increase in the royalty fee a seller observed would (correctly) make it believe that the other seller's royalty offer must have increased, since the platform aims at softening competition, and hence in equilibrium $\hat{w} = \theta/2 > 0$. The converse happens if $\theta < 0$ (so that $d\hat{w}^*(w)/dw < 0$), with $\hat{w} = \theta/2 < 0$ in these cases because the platform wishes to toughen competition.

When sellers *cannot* observe each other's offers, their beliefs become more sensitive to observed royalties. This overreaction to changes in the royalty fee observed is a straightforward effect of wary beliefs.

Figure 1 plots $d\hat{w}^*(w)/dw$ (see solid curve) relative to $dB(w)/dw$ (see dashed curve) as parameter θ varies, and shows that beliefs become more sensitive to changes in the offer received from the platform when sellers cannot observe each other's contracts.

The determinants of how the equilibrium royalty fee relates to θ are different when sellers can observe each other's royalty offers and when they cannot. When they can observe them as in subsection 5.1, the platform's incentives to deviate have to do with making competition between sellers softer (if $\theta > 0$) or tougher (if $\theta < 0$), as we just mentioned. When sellers cannot observe each other's royalty offers, the platform's incentives to deviate greatly depend on how a seller that receives an unexpected offer updates her beliefs about the royalties of the other seller. In particular, such a seller

FIGURE 1. Comparison of beliefs

(correctly) infers that the platform must be simultaneously deviating with the other seller in a way that the opportunistic platform does not care about seller 2's profitability. Indeed, taking into account that the platform extracts all the surplus that seller i expects to make when observing royalty fee w_i , it holds that the payoff to the platform if it chooses w_1 , w_2 and p_0 equals

$$\begin{aligned}
 \widehat{\pi}_0(w_1, w_2, p_0) &= x(p_0) \left\{ p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \right. \\
 &\quad + [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(B(w_1))) \\
 &\quad \left. + [p_2(w_2) - w_2] q_2(p_2(w_2), p_1(B(w_2))) \right\} \\
 &= x(p_0) \left\{ p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \right. \\
 &\quad + [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(w_2)) \\
 &\quad + [p_1(w_1) - w_1] [q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))] \\
 &\quad \left. + [p_2(w_2) - w_2] q_2(p_2(w_2), p_1(B(w_2))) \right\}.
 \end{aligned}$$

Clearly, maximizing this payoff with respect to w_1 is equivalent to maximizing

$$\begin{aligned}
 \widehat{\pi}'_0(w_1, w_2) &= [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(w_2)) \\
 &\quad + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \\
 &\quad + [p_1(w_1) - w_1] [q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))],
 \end{aligned}$$

so the platform cares about seller 1's actual profit, the actual royalty revenue generated by each seller and the change in seller 1's profit because of the formation of wary beliefs. By the envelope theorem, seller 1's actual profit when w_1 changes by a small

amount is $-q_1(p_1(w_1), p_2(w_2))$, so $w^* = B(w^*)$ implies that

$$\left. \frac{\partial \widehat{\pi}'_0(w_1, w_2)}{\partial w_1} \right|_{w_1=w_2=w^*} = 0$$

is equivalent to

$$\left\{ [p(w^*) - w^*] \theta \frac{dB(w^*)}{dw} - (1 - \theta) w^* \right\} \frac{dp(w^*)}{dw} = 0.$$

The fact that $p(w^*) > w^*$ then implies that

$$w^* = \frac{\theta}{1 - \theta} \frac{dB(w^*)}{dw} [p(w^*) - w^*]$$

must be nonnegative because we showed earlier that $\theta(dB(w^*)/dw) \geq 0$.

As we have shown, the sign of w^* depends on how the second argument of $q_1(p_1(w_1), p_2(B(w_1)))$ varies with w_1 , that is, on whether an increase in w_1 will stimulate seller 1's sales via the conjectured price change performed by seller 2. Because seller 1 always believes that this is indeed the case, w^* is always nonnegative.

When sellers can observe each other's offer, we showed in Section 5.1 that the equilibrium royalty fee is positive if and only if competition between sellers displays strategic complementarity. Figure 2 compares royalty fees in the two models (the dashed curve corresponds to the case of private contracting).

FIGURE 2. Comparison of royalty fees

Because $w^* < \widehat{w}$ if and only if $\theta > 0$, it should come as no surprise that the comparison of sellers' prices in both situations is as illustrated in Figure 3 (the dashed curve corresponds to the case of private contracting).

Relative to when sellers can observe each other's contract, it holds when they cannot that the platform provider loses part of its market power vis-à-vis sellers because of its opportunistic behavior when dealing with each on a one-on-one basis (as in Rey

FIGURE 3. Comparison of seller prices

and Vergé, 2004). This smaller market power implies that the platform provider cannot sufficiently raise sellers' prices through the royalty fees when goods are substitutes; when goods are complements, the smaller market power of the platform provider implies that it cannot sufficiently lower prices charged by sellers so as to mitigate the double marginalization problem first pointed out by Cournot (1838) for the case of perfect complements.

The difference in pricing by sellers illustrated by the previous figure has key implications for platform pricing, since one of the two determinants of platform demand is how much utility consumers expect to attain given the anticipated pricing by sellers. When $\theta < 0$, consumers correctly anticipate that sellers will charge higher prices when they cannot observe each other's offer than when they can, so the platform provider has an incentive to lower the platform's price relative to when sellers can observe each other's offer. When $\theta > 0$, the sellers charge lower prices when they cannot observe each other's offer than when they can, so the platform provider has an incentive to raise the platform's price relative to when sellers can observe each other's offer.

The other determinant of platform pricing is how much overall profit is generated per consumer through the two sellers. Figure 4 shows how total profit generated by sellers per customer varies with θ (the dashed curve represents the situation when seller cannot observe each other's offer).

Because sellers are induced to price collusively when they can observe each other's offer, it holds that per-consumer profitability is at least as large as when they cannot observe each other's offer. This implies that, regardless of the value of θ , the platform provider has an incentive to set a higher price for the platform when sellers cannot observe each other's offer than when they can. Interestingly, note that the incentive is

FIGURE 4. Comparison of seller profits per customer

very small when $\theta > 0$: in such cases, the platform provider's opportunistic behavior is hardly costly in terms of generating sellers' profits. The effect highlighted by Rey and Vergé (2004) is present, but it is not very strong.

Overall, we find that pricing by the platform is mostly driven by the anticipated effect of sellers' prices on consumer utility. On the one hand, when $\theta > 0$, the platform provider prices higher when sellers cannot observe each other's offer than when they can: the effect on consumer demand of having lower prices dominates the effect of appropriating less profit through sellers. On the other hand, when $\theta < 0$, the effect of having lower consumer utility when sellers cannot observe each other's offer always dominates the lower per-consumer profitability that arises when sellers cannot observe each other's offer. This is illustrated by Figure 5 (the dashed curve corresponds to the case of private contracting).

FIGURE 5. Comparison of consumer access prices

It should then not be very surprising that platform profits are greater when sellers cannot observe each other's offer than when they can if and only if $\theta > 0$, as Figure

6 shows (the dashed curve represents the situation when seller cannot observe each other's offer). A similar result holds for consumer and total welfare, since they are proportional to platform profits both when sellers cannot observe each other's offer and when they can.

FIGURE 6. Comparison of platform profit

We now turn to our main result. The following proposition shows the effects of private contracts in a two-sided market by comparing the equilibrium of this section with that of the previous two sections.

Proposition 4. *Equilibrium royalties can be positive or negative in a one-sided market with public contracts, are negative in a two-sided market with public contracts, and are positive in a two-sided market with private contracts. The price of the platform good for consumers is positive in a two-sided market with public contracts, and is negative in a two-sided market with private contracts. Comparing two-sided market models, private contracts lead to lower profit, consumer surplus, and welfare.*

The first two claims in the proposition follow from comparing the equilibria of the models in Sections 3, 4, and 5. The proof for the last claim is included in the proof of Proposition 3.

6. PRIVATE CONTRACTS WITH A LARGE NUMBER OF SELLERS

An interesting case to examine is that in which n becomes very large and strategic interaction among sellers becomes very weak. If $\theta > 0$, sellers compete monopolistically and the platform ends up having an absolute control over sellers' prices through royalty fees. As $n \rightarrow \infty$ in these cases, both $p(w)$ and $B(w)$ converge pointwise to w ,

whereas w^* and p^* both converge to $1/2$ for $\theta > 0$.¹¹ The platform provider therefore has no incentive to act opportunistically with respect to any seller and it induces collusive pricing by sellers in equilibrium, thus depressing consumers' incentives to join the platform as in subsection 5.1. This gives a rationale for the platform provider to limit entry by sellers into the platform: when the platform provider can act opportunistically, it prefers that there be strategic interaction among sellers so as to commit to control less their pricing and thus alleviate the consumers' hold-up problem. Restricting massive market participation by sellers would not only benefit the platform but also consumers, so it would be welfare enhancing.¹²

7. POLICY IMPLICATIONS

7.1. Most-favored nation and price-forcing clauses. Contracts between platform and sellers in two-sided markets often use Most-Favored Nation (MFN) and price-forcing clauses. For example, the private contract between Spotify (an online music platform) and Sony Music (a record company) included MFN clauses,¹³ and seller prices are regulated by the platform in the case of iTunes. We next study the welfare implications of each instrument.

Suppose first that it is commonly known that the platform provider includes MFN clauses in their contracts with sellers. In this way, the platform provider commits not to price discriminate between sellers, so we model this situation by assuming that sellers have “symmetry beliefs” (McAfee and Schwartz, 1994). That is, sellers believe that $B(w) = w$ for all w . It is straightforward to see that with this type of beliefs, results are identical to those of subsection 5.1. Therefore, the introduction of MFN clauses makes the platform and consumers worse off when goods are substitutes, but it makes all of them better off when goods are complements. The reason is that MFN clauses solve sellers' information problems, but not consumers', which intensifies the hold-up problem when sellers' products are substitutes, and alleviates it when they are complements.

¹¹All these arguments can be shown formally, and the proof is available upon request. We note that $p(w) = (1 + w)/2$ and $B(w) = 0$ when $\theta = 0$, so $w^* = 0$ and $p^* = 1/2$ when sellers' products are independent. Therefore, there is a discontinuity at $\theta = 0$ in terms of pricing strategies, belief functions and royalties, but not in terms of the price that sellers are induced to charge.

¹²Note, however, that the platform's incentive to limit entry has a negative effect on consumers' utility because it reduces product variety.

¹³Contracts such as this one are generally private. However, in the case at hand, the contract was made public by hackers that entered into Sony's servers. See <http://www.theverge.com/2015/5/19/8621581/sony-music-spotify-contract>, accessed October 1, 2015, for more details.

Proposition 5. *Relative to when MFN clauses are not used, the introduction of such clauses increases total welfare if and only if $\theta < 0$.*

Suppose now that the platform includes price-forcing clauses in its contract with sellers, so that the platform can commit to sellers' prices vis-à-vis consumers. The best the platform can do is to replicate the results in Section 4 in order to make the consumers' hold-up problem disappear and thus alleviate consumers' fears when adopting the platform, so it is dominant for the platform to induce prices equal to 0. Doing so makes not only the platform better off, but also consumers, which leads to our next result.

Proposition 6. *Relative to when price-forcing clauses are not used, the introduction of such clauses always increases total welfare.*

The reason for this result is that, in contrast with MFN clauses, this type of RPM solves not only sellers' information problems, but also consumers', and can thus be used to replicate the equilibrium of the public-contract case.

Our results suggest that price-forcing contracts are advantageous to the platform and to consumers as well because they allow the platform to commit to low seller prices, and hence they stimulate platform adoption. This result may explain why price-forcing contracts are common in two-sided markets, as in the cases of iTunes and Netflix. Netflix, in particular, commits to zero price for movie plays, which exactly replicates the price structure of Section 4. To the best of our knowledge, ours is the first paper providing a rationale for the use of price-forcing clauses in a two-sided market setting. Use of such clauses can be welfare-enhancing in cases of dominant platforms such as Netflix or iTunes, both if there are many or few sellers.

These findings can also shed some light on the private and social desirability of what is known as the "agency model of sales" (Johnson, 2014), which is used, for example, by Apple.¹⁴ In the current case, this contractual approach would correspond to the platform provider contracting upon the prices to be charged by sellers and committing to these prices when consumers decide whether to join the platform. The platform would find it dominant to force sellers to sell at a price of 0 and thus obtain the same outcome as in Section 4. Referring to the approach followed by the platform in Section 4 as the wholesale model of sales, we therefore have the following result.

¹⁴The agency model of sales is a business model in which the platform acts as a sale agent of the sellers (Johnson, 2014), so it is not to be confused with a principal-agent problem. Note that in our model sellers and the platform bargain over sellers' prices under the assumption that the platform makes take-it-or-leave-it offers to sellers.

Corollary 1. *Platform profits and consumer welfare are both higher when the platform controls sellers' prices directly than when it simply controls their royalty fees, so the agency model of sales is socially preferred over the wholesale model of sales.*

7.2. Integration between platform and sellers. A natural alternative to contracting with sellers is to acquire one or both. Clearly, integration with both sellers allows the platform to commit to the prices charged by sellers and is therefore a Pareto improvement that results in a situation such as the one in Section 4. However, the incentive to integrate need not be monotonic in the number of sellers acquired by the platform provider. To this end, consider what happens if the platform is integrated with seller 1, but not with seller 2. In the second stage, the platform will choose p_1 to maximize $p_1 q_1(p_1, p_2) + w_2 q_2(p_2, p_1)$, whereas seller 2 will choose p_2 to maximize $(p_2 - w_2) q_2(p_2, p_1)$. It readily follows that prices as a function of w_2 are

$$p_1(w_2) = \frac{(\theta + 2)(1 - \theta) + 3\theta w_2}{(2 - \theta)(2 + \theta)}$$

and

$$p_2(w_2) = \frac{(\theta + 2)(1 - \theta) + (2 + \theta^2)w_2}{(2 - \theta)(2 + \theta)}.$$

If consumers expect that seller 2 pays a royalty fee of w_2^I , their demand equals

$$x_0(w_2^I, p_0) = \frac{2(2 + \theta)(2 + \theta) - 2(1 + \theta)(2 + \theta)^2 w_2^I + (1 + \theta)(4 + 5\theta^2)(w_2^I)^2}{2(1 + \theta)(2 - \theta)^2(2 + \theta)^2} - p_0.$$

Seller 2 anticipates earning $(p_2^*(w_2) - w_2)x_0(w_2^I, p_0)q_2(p_2^*(w_2), p_1^*(w_2)) - f_2$, so the platform chooses w_2 and p_0 to maximize

$$x_0(w_2^I, p_0)[p_0 + p_1^*(w_2)q_1(p_1^*(w_2), p_2^*(w_2)) + p_2^*(w_2)q_2(p_2^*(w_2), p_1^*(w_2))].$$

It is easy to show that

$$w_2^I = \frac{\theta(2 + \theta)^2}{2(4 + 5\theta^2)},$$

which is positive if and only if $\theta > 0$. Also,

$$p_0^I = -\frac{8 - 4\theta + 13\theta^2 + \theta^3}{16(1 + \theta)(4 + 5\theta^2)} < 0$$

and

$$x_0^I = \frac{24 + 4\theta + 23\theta^2 + 3\theta^3}{16(1 + \theta)(4 + 5\theta^2)} > 0,$$

so

$$\pi_0^I = \frac{(24 + 4\theta + 23\theta^2 + 3\theta^3)^2}{256(1 + \theta)^2(4 + 5\theta^2)^2}.$$

Comparing π_0^I with π_0^* , we have that the platform would have an incentive to integrate with seller 1 (say) if and only if $\theta < 0$. When $\theta > 0$, the consumers' fear of being taken advantage of is more mitigated by private contracting than by the platform's control of one of the sellers. In this sense, the loss in the platform's market power vis-à-vis sellers is preferred by consumers over a stricter (but incomplete) control of sellers' pricing behavior. The following proposition summarizes these results.

Proposition 7. *Integration with both sellers makes the platform and consumers better off, so it is always socially desirable. Integration with just one seller makes the platform and consumers better off if $\theta < 0$ and worse off otherwise, so it is socially desirable if and only if $\theta < 0$.*

8. CONCLUDING REMARKS

When contracts between the platform and sellers are private rather than public, we have shown that the pricing structure is basically driven by consumers' fear of being taken advantage of when purchasing the platform. Transparency is beneficial because it allows the platform to commit not to trick consumers into purchasing a platform that will have expensive goods sold by sellers. Another way to make adverse selection disappear is to contract on sellers' prices (as in the case of iTunes). Both the platform and consumers would benefit from the use of such price-forcing contracts. Also, we have shown that integration of the platform with one of the sellers is harmful when sellers sell substitutes, but going all the way and integrating with both sellers would allow the platform to return to the public contracting outcome and do better without harming consumers.

Our results show that giving more market power to a dominant platform (in the form of making private contracts public or allowing for forcing contracts) may make everybody better off because it removes information frictions: everyone benefits from consumers not being so wary about the value delivered by the platform. This insight does not only apply when consumers purchase the platform without observing the costs associated to using it, but rather it is more general. It also holds in cases in which consumers do not observe the quality of the goods sold by sellers before acquiring the platform, or when they do not observe the full variety of goods that will be offered through the platform. This may explain why quality assurance by platforms is common (as is the case for Nintendo). We believe that these topics present an interesting direction for further research.

APPENDIX: PROOFS

Proof of Proposition 3. If $p(w) = \Theta + \Sigma w$ and $B(w) = \Gamma + \Phi w$ for some parameters Θ, Σ, Γ and Φ to be determined, conditions (6) and (9) can be rewritten as

$$(1 - \theta)(1 - \Theta) - 2\Sigma\Gamma + (\Theta + \Sigma\Gamma - \Gamma)2(\Sigma - 1) + [2\Sigma(\theta - \Phi) + \Phi(\Sigma - 1)2(\Sigma - 1)]w_2 = 0$$

and

$$1 - \theta + \theta\Sigma\Gamma - (2 - \theta)\Theta + (1 - 2\Sigma + \theta\Sigma\Phi)w_2 = 0.$$

Since these two conditions should be satisfied for all w_2 , we must have

$$(1 - \theta)(1 - \Theta) - 2\Sigma\Gamma + (\Theta + \Sigma\Gamma - \Gamma)2(\Sigma - 1) = 0, \quad (10)$$

$$2\Sigma(\theta - \Phi) + \Phi(\Sigma - 1)2(\Sigma - 1) = 0, \quad (11)$$

$$1 - \theta + \theta\Sigma\Gamma - (2 - \theta)\Theta = 0 \quad (12)$$

and

$$1 - 2\Sigma + \theta\Sigma\Phi = 0. \quad (13)$$

Rey and Vergé (2004) have already shown that there exists a unique tuple $(\Theta, \Sigma, \Gamma, \Phi)$ that solves these equations and the required second-order conditions for the platform's maximization program, but we will give closed-form solutions that will prove useful later on.

When $\theta = 0$, it is easy to see that there is a unique solution to equations (10)-(13), given by $\Theta = 1/2, \Sigma = 1/2, \Gamma = 0$ and $\Phi = 0$. From (13), one obtains

$$\Phi = \frac{2\Sigma - 1}{\theta\Sigma},$$

since it can be shown that there can be no solution with $\Sigma = 0$. Plugging this value for Φ in (11) allows us to rewrite it as the following cubic equation:

$$\Sigma^3 - \left(\frac{7 - \theta^2}{2}\right)\Sigma^2 + \frac{5}{2}\Sigma - \frac{1}{2} = 0. \quad (14)$$

Letting

$$a \equiv -\frac{7 - \theta^2}{2},$$

$$b \equiv \frac{5}{2},$$

$$c \equiv -\frac{1}{2},$$

$$K \equiv \frac{3b - a^2}{9}$$

and

$$L \equiv \frac{9ab - 27c - 2a^3}{54},$$

the solutions to the cubic equation are the following:

$$\Sigma_k = 2\sqrt{-K} \cos \left(\frac{1}{3} \arccos \left(\frac{L}{\sqrt{-K^3}} \right) + \frac{2\pi k}{3} \right) - \frac{a}{3} \quad (k = 0, 1, 2).$$

The three roots are real, given that the discriminant $K^3 + L^2$ is negative for all $\theta \in (-1, 1)$. Plotting the three roots for all values of θ , it is easy to see that the only one which is equal to $1/2$ when $\theta = 0$ is Σ_2 . Given that the solution must be continuous in θ , we know that $\Sigma = \Sigma_2$, that is,

$$\Sigma = \frac{7 - \theta^2}{6} - \frac{(19 - 14\theta^2 + \theta^4)^{1/2}}{3} \sin \left(\frac{\pi}{6} - \frac{1}{3} \arccos \left(\frac{(1 - \theta^2)(82 - 20\theta^2 + \theta^4)}{(19 - 14\theta^2 + \theta^4)^{3/2}} \right) \right).$$

From equation (12), we obtain

$$\Gamma = \frac{(2 - \theta)\Theta - (1 - \theta)}{\theta\Sigma},$$

so plugging it into (10) and rearranging yields that

$$\Theta = \frac{(1 - \theta)[(6 + \theta)\Sigma - 2(1 + \Sigma^2)]}{4(3 - \Sigma)\Sigma + 2\theta - (3\theta + \theta^2)\Sigma - 4}.$$

It therefore follows from (10) that

$$\Gamma = \frac{(1 - \theta)(2\Sigma - 1)}{4(3 - \Sigma)\Sigma + 2\theta - (3\theta + \theta^2)\Sigma - 4}.$$

Making it explicit that Θ , Σ , Γ and Φ depend on θ by writing Θ_θ , Σ_θ , Γ_θ and Φ_θ , it is easy to plot them and see that $0 \leq \Theta_\theta \leq 1$, $1/2 \leq \Sigma_\theta \leq 1$, $0 \leq \Gamma_\theta \leq 1$ and $-1 \leq \Phi_\theta \leq 1$ for all $\theta \in (-1, 1)$. Note that beliefs must be fulfilled in equilibrium, so $w^* = B(w^*)$ implies that

$$w^* = \frac{\Gamma_\theta}{1 - \Phi_\theta} \geq 0.$$

Also, the platform should find it optimal to choose $p_0 = p_0^*$ and $w_1 = w_2 = w^*$, so $(w^*, w^*, p_0^*) \in \arg\max_{w_1, w_2, p_0} \widehat{\pi}_0(w_1, w_2, p_0)$, where

$$\begin{aligned} \widehat{\pi}_0(w_1, w_2, p_0) = & x(p_0) \left\{ p_0 + w_1 q_1(p_1(w_1), p_2(w_2)) + w_2 q_2(p_2(w_2), p_1(w_1)) \right. \\ & + [p_1(w_1) - w_1] q_1(p_1(w_1), p_2(B(w_1))) \\ & \left. + [p_2(w_2) - w_2] q_2(p_2(w_2), p_1(B(w_2))) \right\}. \end{aligned}$$

Note that the optimal choices of w_1 and w_2 do not depend on the choice of p_0 , so the platform provider can maximize with respect to w_1 and w_2 ignoring the value of

p_0 ; the analysis above leading to expression (9) shows that private offers are chosen optimally, since second-order conditions are satisfied. To see this, note that (8) and the fact that

$$\frac{dq_1(p_1(w_1), p_2(B(w_1)))}{dw_1} = \frac{1}{1 - \theta^2} \left(\frac{dp_1(w_1)}{dw_1} - 1 \right),$$

imply that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} = \frac{2(\Sigma_\theta^2 - 3\Sigma_\theta + 1)}{1 - \theta^2}$$

and

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2} = \frac{2\theta \Sigma_\theta}{1 - \theta^2}.$$

Thus, it follows from the fact that $\Sigma_\theta \geq 1/2$ that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} \leq 0.$$

Also, it holds that

$$\begin{aligned} & \left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} \right)^2 - \left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2} \right)^2 \\ &= \frac{\Sigma_\theta(\Sigma_\theta^2 - 3\Sigma_\theta + 1)(\Sigma_\theta - 1) - (2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) - \theta^2 \Sigma_\theta^2}{\left(\frac{1 - \theta^2}{2} \right)^2}, \end{aligned}$$

which is nonnegative because $1/2 \leq \Sigma_\theta \leq 1$ and $(2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) + \theta^2 \Sigma_\theta^2 = 0$ by (14). Thus, second-order conditions hold.

As for the optimal choice of p_0 given that seller $i \in \{1, 2\}$ receives an offer equal to (w^*, f^*) , we need that

$$x(p_0) + [p_0 + 2p_1(w^*)q_1(p_1(w^*), p_2(w^*))] \frac{dx(p_0)}{dp_0} = 0,$$

so

$$p_0^* = \frac{(1 - w^*)^2}{2(1 + \theta)(2 - \theta)^2} - \frac{2(\Theta_\theta + \Sigma_\theta w^*)(1 - \Theta_\theta - \Sigma_\theta w^*)}{2(1 + \theta)},$$

which is negative for all $\theta \in (-1, 1)$. Finally, note that

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \quad (i \in \{1, 2\}),$$

so $0 \leq p_i^* \leq 1$. It readily follows that

$$q_i^* = \frac{1 - p_i^*}{1 + \theta} > 0,$$

$$\pi_0^* = \left(\frac{1 - (p_i^*)^2}{2(1 + \theta)} \right)^2,$$

and

$$cs^* = \frac{1}{2} \left(\frac{1 - (p_i^*)^2}{2(1 + \theta)} \right)^2.$$

■

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