Customization with Vertically Differentiated Products

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Abstract

We consider a duopoly market with heterogenous consumers. The firms initially produce vertically differentiated standard products located at the end points of the variety interval. Customization provides ideal varieties for consumers but has no effect on quality. The firms first choose whether to customize their products, then engage in price competition. We show that the low quality firm never customizes alone; customization becomes more likely as the difference between the firms’ qualities increases; and less likely as the fixed cost of customization increases.

We extend the base model by relaxing two important assumptions – uniform pricing and exogenous quality. The main conclusions with uniform pricing continue to hold when price customization is allowed. In the second extension the firms’ qualities are endogenously determined. We show that the firms choose to be either substantially differentiated in quality or non-differentiated.

Key words: customization, horizontal differentiation, vertical differentiation

JEL codes: D43, L13, C72

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1 Introduction

Mass customization is a flexible technology designed to produce individually tailored products without significantly compromising cost efficiency. Advances in Internet-based information technologies and improvements in manufacturing flexibility have made customization a reality in many product categories. For example, Dell builds to order notebook and desktop computers; Lands’ End offers custom-crafted pants and shirts; Nike and Adidas allow consumers to create their most preferred pair of athletic shoes; Rug Rats creates rugs and carpets that reflect each customer’s personal needs.

Most of existing theoretical literature on customization adopt one-dimensional horizontal differentiation settings (e.g., Dewan, Jing, and Seidmann 2003, Syam and Kumar 2006, Alexandrov 2008, Mendelson and Parlaktürk 2008, and Xia and Rajagopalan 2009). Customization enables consumers to get their ideal products represented by their locations in the product attribute space. Even though many aspects of customization are captured by these models, important issues have yet to be examined. A number of case studies (see the literature review below) document that the firms that have successfully carried out mass customization are in general high quality firms. The importance of product quality in customization has also been pointed out by several authors in the operations management literature (e.g., Tu, Vonderembse, and Ragu-Nathan 2001, and Broekhuizen and Alsem 2002). In the present paper we fill a gap in the theoretical literature by incorporating quality into product customization competition.

1.1 Overview of Our Model and Results

We consider an industry in which products are characterized by variety and quality. Variety is a horizontal attribute and quality is a vertical attribute. Consumer preferences are heterogeneous in two dimensions. Each consumer has a most preferred variety and a quality valuation. There are two firms that initially produce standard products located at the end points of the variety interval. The firms are asymmetric due to having different (exogenously given) qualities. Customization provides ideal varieties for consumers but has no effect on quality. The firms play a sequential two-stage game. In the first stage, they simultaneously decide whether to customize their products. In the second stage, they simultaneously choose their prices.

In the base model, we assume that customizing firms are restricted to uniform pricing strategies. Obviously, the idea of setting a different price for each variant of a customized product is very appealing. While some firms do engage in price customization (e.g., Dell, Ford, and Rug Rats), many firms practice uniform pricing. For example, Lands’ End charges $74 for a pair of jeans regardless of the options chosen; Timbuk2’s price is not linked to color and fabric selections. The theoretical literature on customization (reviewed below) is divided between models with uniform pricing and models with price customization. In an extension of our base model we allow customizing firms to customize prices.

\footnote{Our model is based on the literature that combines horizontal and vertical differentiation, e.g., Economides (1989) and Neven and Thisse (1990).}
Our equilibrium analysis shows that either both firms customize, only the high quality firm customizes, or no firm customizes. The appearance of this sequence of outcomes is monotone in the fixed cost of customization. The key to this is that the high quality firm always gains more from customization than the low quality firm. As a result, both firms customize when the cost of customization is small, only the high quality firm customizes for intermediate levels of customization cost, and neither firm customizes when the cost is high.

We show that even if customization is costless, each of the three equilibria mentioned above is possible. In particular, both firms choose not to customize when the difference between their qualities is small, only the high quality firm customizes for moderate quality differences, and both firms customize when the quality difference is large. The intuition behind this result is as follows. Customization by one or both firms makes the rivals “closer” to each other, thus intensifying price competition. The smaller is the quality difference, the tougher is price competition. In the extreme case in which the quality difference is zero and both firms customize, price competition results in the Bertrand outcome. Therefore, when the quality difference is small, the firms do not customize their products in order to avoid a price war. When the quality difference is large, the firms customize to take advantage of consumers’ desires for ideal varieties. The intermediate case involves customization by the high quality firm only.

We extend our base model by relaxing two central assumptions – uniform pricing and exogenous qualities. In the first extension, we allow customizing firms to set a different price for each variety. The main conclusions of the base model continue to hold. Namely, customization becomes more likely as the fixed cost of customization decreases and/or the quality difference increases, and the low quality firm never customizes alone. Price customization offers an additional benefit to the customizing firm by enhancing its ability to compete. This benefit is key to three main differences between this extension and the base model. First, while in the base model the firms do not customize when the quality difference is small, in the extension both firms customize provided the fixed cost of customization is low. Second, the minimum quality difference to support customization by the high quality firm only is larger in the extension than in the base model. Finally, while in the base model the equilibrium customization choices are unique, in the extension multiple equilibria exist for a portion of the parameter space.

Our base model points to a tradeoff – firms want to minimize quality difference to avoid customization and hence intensified price competition; meanwhile, a firm wants to excel in quality to benefit disproportionately from customization. It is therefore interesting to investigate what qualities will result if they are determined endogenously. In the second extension, we assume that the firms simultaneously choose their qualities prior to the customization stage. There is a quality-dependent cost that the firms incur in this new stage. We show that in equilibrium the firms are either substantially differentiated in quality or non-differentiated. The former results in customization by one or both firms, the latter leads to no customization.
1.2 Review of Related Literature

In the following we review several strands of the literature that are most relevant to the present paper. We start with case studies. These studies lend support to our key result that higher quality/price firms are more likely to offer customization. We then review a few works in the operations management literature that present success factors for customization. Several of those factors are closely related to product quality. Finally, we review existing theoretical papers and contrast them with our study.

Case Studies and Surveys

There is a number of studies that document customization by individual companies and industries. A well-cited study of the National Bicycle Industrial Company by Kotha (1995) reports that the company successfully implemented customization of its high-end Panasonic brand. In the opinion of Seladurai (2004), the best example of mass customization is Dell. The author states that “Dell can manufacture a build-to-order system of high quality and low cost and does this on a mass production scale to customers all over the world” (p. 296). Cattani, Dahan, Schmidt (2005) explore Timbuk2 that customizes its “good-looking, tough-as-Hell” messenger bags and backpacks. Piller, Moeslein, and Stotko (2004) study customization by fourteen leading companies, including well-known companies like Dell, Lands’ End, Lego, Adidas, and Nike. All these studies indicate that the success stories of customization are in general associated with high quality companies/brands.²

Lihr, Buehlmann, and Beauregard (2008) performed in-depth interviews with furniture industry manufacturers in USA, Canada, and Germany to assess the potential of mass customization. They find that mass customization has been successfully used by domestic producers to compete with low-cost offshore manufacturers (which are in general of lower quality). Among four industry subsectors, kitchen cabinet was found to offer the highest level of customization. This subsector is characterized by lower customization costs for domestic firms.

Several consumer surveys find that consumers are interested in customized goods and are willing to pay a price premium for them. In the EUROSohE market study by Piller and Müller (2004), survey respondents are willing to accept a premium of 10-30% on shoes. Franke and Piller (2004) in their experiments with over 700 participants, find on average a 100% premium for customized watches. An implication of these findings is that higher quality/priced goods are likely to reap higher benefits from customization.

Success Factors

A number of studies in the operations management literature have recognized the importance of product quality for customization. Pine (1993) and Broekhuizen and Alsem (2002) examine the success factors of mass customization. Among them are quality awareness by customers, the levels

²For more examples, see Moser and Piller’s (2006) introduction to the special issue of International Journal of Mass Customization on case studies.
of pre- and post-sale service, and the luxury level of the product. All these are dimensions of product quality.

Tu, Vonderembse, and Ragu-Nathan (2001) emphasize the value to customer in carrying out a successful customization strategy. They define value to customer as measuring “perceptions of the value of product variety, customer satisfaction with product quality and features, and customer loyalty and referrals” (p. 204) and find a positive relationship between value to customer and success in mass customization.

In studying customers’ responses to customized offers, Simonson (2005) finds that “consumers are more likely to accept or act on customized offers and recommendations of high-price, high-quality alternatives. Conversely, customized offers to choose low-price, low-quality products are less likely to cause customers to change their purchase decisions” (p. 38).

**Theoretical Studies**

Among the theoretical studies on customization, the paper that is closest to ours is Syam, Ruan, and Hess (2005). In both papers the consumer space is two-dimensional. Syam, Ruan, and Hess (2005) endow products with two horizontal attributes, for which consumers have heterogeneous preferences. The firms are initially maximally differentiated with respect to both attributes. They first simultaneously choose whether to customize both, one, or none of the attributes, then compete in prices. The key difference between Syam, Ruan, and Hess (2005) and our study is that they work with ex ante symmetric firms and examine how the possibility of customizing multiple attributes affect customization choices, whereas we work with asymmetric firms and focus on the role of quality in customization competition.

Another closely related paper is Bernhardt, Liu, and Serfes (2007), in which ex ante symmetric firms first acquire information about consumers and then customize their products as best as they can to match consumer needs. Similar to our paper, consumer preferences are two-dimensional, corresponding to two attributes of the product, and the second attribute – brand loyalty – cannot be customized. There are two main differences between Bernhardt, Liu, and Serfes (2007) and our study. First, Bernhardt, Liu, and Serfes (2007) emphasize the cost side of customization, whereas our focus is shifted towards the strategic effects of customization. Second, brand loyalty is a horizontal attribute, not vertical as quality in our paper is.

A number of papers have studied customization using a one-dimensional consumer space. Dewan, Jing, and Seidmann (2003) provide a deliberate treatment of customization technology, and assume that customizing firms customize prices, like in our first extension. Syam and Kumar (2006) examine the role of standard products in customization competition. Alexandrov (2008) extends Salop’s (1979) model in which firms can offer interval-long adjustable “fat” products. In a dynamic setting Chen (2006) studies two marketing innovations, one of which is essentially a form of product customization. While these studies assume symmetric firms, in Mendelson and Parlaktürk (2008) one firm has a margin advantage (higher difference between reservation price and unit cost) over the other. As in our paper, Mendelson and Parlaktürk (2008) consider both uniform pricing and price
customization.

Our paper as well as Syam, Ruan, and Hess (2005) model customization as zero-one decisions, so that all consumers get their most preferred varieties when they purchase a customized product. In contrast, all the other papers mentioned above treat customization as continuous choices. Both approaches match aspects of reality and have their advantages. With zero-one decisions, more attention can be devoted to the strategic effects of customization. With continuous customization choices, one can focus on how efficiency considerations determine the range of customization.

The rest of the paper is organized as follows. In the next section we introduce the base model. In Section 3 the pricing stage is analyzed. In Section 4 we study the firms’ customization choices. The two extensions, price customization and endogenous quality, are presented in Section 5. Concluding remarks are provided in Section 6. Proofs of all lemmas and propositions, as well as derivations for some expressions and claims, are relegated to the appendix.

2 The Base Model

Consider a market in which each product \( i \) is characterized by its variety \( x_i \in [0, 1] \) and its quality \( q_i \geq 0 \). The first characteristic corresponds to horizontal differentiation and the second to vertical differentiation. Consumers are heterogeneous in two dimensions. Each consumer has a most preferred variety \( x \in [0, 1] \) and a quality valuation \( y \in [0, 1] \). A consumer of type \( (x, y) \) derives the following utility from buying one unit of product \( i \):

\[
v + q_i y - t|x - x_i| - p_i,
\]

where \( v \) is a positive constant, \( t \) is a preference parameter, and \( p_i \) is the price of product \( i \). Consumers as represented by \( (x, y) \) are uniformly distributed over the unit square \([0, 1] \times [0, 1]\) with a total mass of 1. We assume that \( v \) is large enough for all consumers to find a product that yields positive payoff in equilibrium.

There are two firms, A and B, operating with zero marginal cost of production. Initially, firm A offers a single (standard) product of quality \( q_A \) and variety \( x_A = 0 \), whereas firm B offers a single product of quality \( q_B \geq q_A \geq 0 \) and variety \( x_B = 1 \). That is, firm B is the higher quality firm and the two firms have maximum variety differentiation.

We will normalize \( t \) to 1. This amounts to a monotonic transformation of preferences. The utilities of a consumer of type \( (x, y) \) from buying firm A’s and firm B’s standard products are

\[
v + q_A y - x - p_A \tag{1}
\]

and

\[
v + q_B y - (1 - x) - p_B, \tag{2}
\]

respectively.
Investing $K \geq 0$ into product-customization technology allows a firm to produce a product that exactly matches a given consumer’s preferred variety. The utilities of type $(x, y)$ from buying firm A’s and firm B’s customized products are

$$v + q_{AY} - p_A$$

and

$$v + q_{BY} - p_B.$$ 

The game involves two stages, a customization stage followed by a pricing stage. In the customization stage the firms simultaneously decide whether to customize their products. These decisions become common knowledge after they are made. In the pricing stage the firms simultaneously choose prices. Consumers subsequently decide which products to purchase, and profits are realized. The equilibrium concept employed is subgame perfect Nash equilibrium. The analysis of consumer choices is straightforward. We, therefore, focus on the firms’ choices and proceed using backward induction.

3 Analysis of the Pricing Stage

In this section we investigate the firms’ pricing decisions, taking as given their customization choices in the first stage of the game. There are four subgames to consider, corresponding to the following scenarios: no firm customizes; firm A (the low quality firm) customizes; firm B (the high quality firm) customizes; and both firms customize. We denote these subgames by “SS,” “CS,” “SC,” and “CC,” where “S” and “C” stand for offering standard and customized products, respectively.

3.1 Subgame SS: No Firm Customizes

When no firm customizes, utilities from firm A’s and firm B’s standard products are given by (1) and (2), respectively. Therefore, for a given level of quality valuation $y$, the marginal consumer type in terms of variety $x$ is

$$\hat{x}(y) = \frac{1}{2}(1 - qy + p_B - p_A),$$

where

$$q \equiv q_B - q_A$$

denotes the quality difference between the firms’ products. For any $y \in [0, 1]$, consumers in the interval $x \in [0, \hat{x}(y)]$ will purchase from firm A, whereas those with $x \in (\hat{x}(y), 1]$ will purchase from firm B. There are four possible positions for the indifference line (5), as illustrated in Figure 1. The slope of the indifference line equals $-2/q$. An increase in $q$ makes the line flatter. An increase in $p_B$ (and/or decrease in $p_A$) shifts the line to the right, thereby reducing the market size of firm B.

We acknowledge that in many real-world instances consumers actively participate in the production of customized products. Our model does not make a distinction for this possibility.
Figure 1: Indifference line and market areas

Let $D_A(p_A, p_B)$ and $D_B(p_A, p_B)$ denote the demand functions of firms A and B. The expressions for these functions (detailed in the appendix) depend on the position of the indifference line. The firms choose simultaneously $p_A$ and $p_B$ to maximize their profits,

$$\Pi_A(p_A, p_B) = D_A(p_A, p_B)p_A$$
and

$$\Pi_B(p_A, p_B) = D_B(p_A, p_B)p_B.$$  

**Lemma 1** (Equilibrium prices and profits when no firm customizes). Suppose no firm customizes. The equilibrium prices and profits in the pricing stage are as follows.

(i) If $q \leq 3/2$,

$$\begin{cases} 
  p^\text{SS}_A = 1 - \frac{1}{6}q \\
  p^\text{SS}_B = 1 + \frac{1}{6}q
\end{cases}$$
and

$$\begin{cases} 
  \Pi^\text{SS}_A = \frac{1}{2} (1 - \frac{1}{6}q)^2 \\
  \Pi^\text{SS}_B = \frac{1}{2} (1 + \frac{1}{6}q)^2
\end{cases}$$

(ii) If $q \in (3/2, 3]$,

$$\begin{cases} 
  p^\text{SS}_A = \frac{1}{8} (1 + \sqrt{1+16q}) \\
  p^\text{SS}_B = \frac{1}{8} (-5 + 3\sqrt{1+16q})
\end{cases}$$
and

$$\begin{cases} 
  \Pi^\text{SS}_A = \frac{1}{q} \left(\frac{1+\sqrt{1+16q}}{8}\right)^3 \\
  \Pi^\text{SS}_B = \left(1 - \frac{1}{q} \left(\frac{1+\sqrt{1+16q}}{8}\right)^2\right)^{-5+3\sqrt{1+16q}}
\end{cases}$$

(iii) If $q > 3$,

$$\begin{cases} 
  p^\text{SS}_A = \frac{1}{5}q \\
  p^\text{SS}_B = \frac{2}{5}q
\end{cases}$$
and

$$\begin{cases} 
  \Pi^\text{SS}_A = \frac{1}{5}q \\
  \Pi^\text{SS}_B = \frac{4}{5}q
\end{cases}$$

In Lemma 1, case (i) corresponds to Figure 1(a) in which the quality difference is small. In equilibrium, both firms serve consumers with all quality valuations, and each firm attracts consumers closer to its position on the variety interval. Case (ii) corresponds to Figure 1(b). Firm A attracts only consumers who are close to its variety position and have low quality valuations (i.e., small $x$’s and small $y$’s). Firm B captures all the other consumers. Case (iii) corresponds to Figure 1(c) in which $q$ is large. Firm A serves consumers of all variety preferences, and so does firm B. Firm A
attracts consumers with low quality valuations and firm B attracts consumers with high quality valuations.

Figure 2 plots the equilibrium prices and profits in Lemma 1. The kinks correspond to the two critical values for $q$, $3/2$ and $3$. The intuition behind the shapes of the curves is as follows. An increase in the quality difference has two effects. First, it gives advantage to firm B and hurts firm A. Second, it increases product differentiation, which relaxes competition. Both effects help firm B, resulting in rising price and profit as $q$ increases. As for firm A, its price and profit decrease with $q$ up to the first kink point, and then increase. Intuitively, in a situation of intense competition (small $q$) an increase in $q$ weakens the position of the disadvantaged firm A. When $q$ becomes sufficiently large, the competition is softened enough to enable the weaker firm to raise its price. This explains the initial decrease and eventual increase of firm A’s price and profit.

Figure 2 indicates that for any given value of $q$ the high quality firm B sets a higher price and earns a higher profit than firm A. Additionally, it can be easily verified that firm B always serves a larger market area. It is well established in the literature that, under either price or quantity competition, the firm with quality and/or cost advantages captures a larger market share and earns a higher profit. The results in Lemma 1, in which firm B has a quality advantage over firm A, are in line with the findings in the literature. Note that Figure 1(d) does not arise in equilibrium, because otherwise firm A’s market size would be larger than firm B’s.

3.2 Subgame CS: Firm A Customizes

When only firm A customizes, utilities from firm A’s customized product and firm B’s standard product are given by (3) and (2), respectively. Therefore, for a given $y$, the marginal consumer type in terms of $x$ is

$$\hat{x}(y) = 1 - qy + p_B - p_A,$$

Because of the second effect, the equilibrium indifference line always swivels counter-clockwise inside the unit square, rather than shifts, as $q$ increases. That is, as firm B becomes stronger, firm A does not lose consumers in both dimensions (as a shift of the indifference line would imply).
Lemma 2 (Equilibrium prices and profits when firm A customizes). Suppose firm A customizes. The equilibrium prices and profits in the pricing stage are as follows.

(i) If $q \leq 1$,
\[
\begin{align*}
    & p_{CS}^A = \frac{2}{3} - \frac{1}{6}q \\
    & p_{CS}^B = \frac{1}{3} + \frac{1}{6}q \\
\end{align*}
\]
and
\[
\begin{align*}
    & \Pi_{CS}^A = \left( \frac{2}{3} - \frac{1}{6}q \right)^2 \\
    & \Pi_{CS}^B = \left( \frac{1}{3} + \frac{1}{6}q \right)^2
\end{align*}
\]

(ii) If $q > 1$,
\[
\begin{align*}
    & p_{CS}^A = \frac{1}{3}q + \frac{1}{6} \\
    & p_{CS}^B = \frac{2}{3}q - \frac{1}{6}
\end{align*}
\]
and
\[
\begin{align*}
    & \Pi_{CS}^A = \frac{1}{6} \left( \frac{1}{3}q + \frac{1}{6} \right)^2 \\
    & \Pi_{CS}^B = \frac{1}{6} \left( \frac{2}{3}q - \frac{1}{6} \right)^2
\end{align*}
\]

Case (i) corresponds to Figure 1(a) in which the quality difference is small. Customization enables firm A to overcome its quality disadvantage. Firm A’s equilibrium price, profit, and market size are higher than those of firm B. In equilibrium, both firms serve consumers with all quality valuations; firm A serves consumers with small $x$’s and firm B consumers with large $x$’s. Case (ii) corresponds to Figure 1(c) in which $q$ is large. In this case, customization does not overcome the quality disadvantage of firm A. In equilibrium, firm A’s price, profit, and market size are lower than those of firm B. Firm A serves consumers of all variety preferences and so does firm B. Firm A attracts consumers with low quality valuations and firm B attracts consumers with high quality valuations.

The transition from Figure 1(a) to Figure 1(c) occurs at $q = 1$, at which the market is evenly divided between the two firms by the indifference line that connects the upper-left and lower-right corners of the unit square (the main diagonal). When $q = 1$, customization by firm A exactly offsets firm B’s quality advantage, resulting in the same prices, profits, and market sizes. Note that neither Figure 1(b) nor Figure 1(d) arise in equilibrium, because of a combination of two facts. The
3.3 Subgame SC: Firm B Customizes

When only firm B customizes, utilities from firm A’s standard product and firm B’s customized product are given by (1) and (4), respectively. Therefore, for a given \( y \), the marginal consumer type in terms of \( x \) is

\[
\hat{x}(y) = -qy + p_B - p_A.
\]  

(7)

The next lemma presents the equilibrium prices and profits for subgame SC.

**Lemma 3** (Equilibrium prices and profits when firm B customizes). *Suppose firm B customizes. The equilibrium prices and profits in the pricing stage are as follows.*

(i) If \( q \leq \frac{1}{2} \),

\[
\begin{align*}
p^{SC}_A &= \frac{1}{3} - \frac{1}{6}q \\
p^{SC}_B &= \frac{2}{3} + \frac{1}{6}q
\end{align*}
\]

and

\[
\begin{align*}
\Pi^{SC}_A &= \left(\frac{1}{3} - \frac{1}{6}q\right)^2 \\
\Pi^{SC}_B &= \left(\frac{2}{3} + \frac{1}{6}q\right)^2
\end{align*}
\]

(ii) If \( q \in \left(\frac{1}{2}, 2\right) \),

\[
\begin{align*}
p^{SC}_A &= \frac{1}{3} \sqrt{2q} \\
p^{SC}_B &= \frac{2}{3} \sqrt{2q}
\end{align*}
\]

and

\[
\begin{align*}
\Pi^{SC}_A &= \frac{1}{16} \sqrt{2q} \\
\Pi^{SC}_B &= \frac{9}{16} \sqrt{2q}
\end{align*}
\]

(iii) If \( q > 2 \),

\[
\begin{align*}
p^{SC}_A &= \frac{1}{3} q - \frac{1}{6} \\
p^{SC}_B &= \frac{2}{3} q + \frac{1}{6}
\end{align*}
\]

and

\[
\begin{align*}
\Pi^{SC}_A &= \frac{1}{q} \left(\frac{1}{3} q - \frac{1}{6}\right)^2 \\
\Pi^{SC}_B &= \frac{1}{q} \left(\frac{2}{3} q + \frac{1}{6}\right)^2
\end{align*}
\]
Case (i) corresponds to Figure 1(a), case (ii) to 1(b), and case (iii) to 1(c). Figure 4 plots the equilibrium prices and profits in Lemma 3. The kinks correspond to the two critical values for \( q, 1/2 \) and 2. The properties of these curves are similar to those in Figure 2. The same explanation applies. Compared to subgame SS, firm B’s quality advantage is reinforced by customization, pushing the critical values lower compared to those in Lemma 1.

### 3.4 Subgame CC: Both Firms Customize

When both firms customize, utilities from firm A’s and firm B’s customized products are given by (3) and (4), respectively. Therefore, consumers with \( y \leq (p_B - p_A)/q \) will purchase from firm A, those with \( y > (p_B - p_A)/q \) will purchase from firm B. The firms’ profit functions are

\[
\Pi_A(p_A, p_B) = \frac{1}{q} (p_B - p_A) p_A \quad \text{and} \quad \Pi_B(p_A, p_B) = \frac{1}{q} (q + p_A - p_B) p_B.
\]

The profit maximizing first-order conditions

\[
\begin{align*}
& p_B - 2p_A = 0 \\
& q + p_A - 2p_B = 0
\end{align*}
\]

lead immediately to the following lemma.

**Lemma 4** (Equilibrium prices and profits when both firms customize). *Suppose both firms customize. The equilibrium prices and profits in the pricing stage are given by:*

\[
\begin{align*}
& p_{CC}^A = \frac{1}{3} q \\
& p_{CC}^B = \frac{2}{3} q \\
& \Pi_{CC}^A = \frac{1}{9} q \\
& \Pi_{CC}^B = \frac{4}{9} q
\end{align*}
\]

Graphically, the equilibrium prices and profits increase linearly in \( q \). The equilibrium indifference line is horizontal at 1/3 from the bottom side of the unit square. That is, firm A serves all consumers with quality valuations less than 1/3 and firm B serves the rest. The result here is the same as in the standard model of vertical differentiation.

Having derived the profit functions for the pricing stage, we move one step back to study the customization stage.

### 4 Equilibrium Customization Choices

In the customization stage the firms simultaneously decide whether to customize their products. This is represented by the following matrix.

\[
\begin{array}{c|cc}
\text{Firm A} & S & C \\
\hline
S & \Pi_{SS}^A, \Pi_{SS}^B & \Pi_{SC}^A, \Pi_{SC}^B - K \\
C & \Pi_{CS}^A - K, \Pi_{CS}^B & \Pi_{CC}^A - K, \Pi_{CC}^B - K
\end{array}
\]
It follows that

$$
\begin{align*}
(S,S) \\
(C,S) \\
(S,C) \\
(C,C)
\end{align*}
$$

is a Nash equilibrium if

$$
\begin{align*}
K &\geq \max\{c_1,r_1\} \\
K &\in [r_2,c_1] \\
K &\in [c_2,r_1] \\
K &\leq \min\{c_2,r_2\}
\end{align*}
$$

where

$$
c_1 \equiv \Pi^CS_A - \Pi^SS_A \quad \text{and} \quad c_2 \equiv \Pi^CC_A - \Pi^SC_A
$$

denote firm A’s change in gross profit in the two columns, and

$$
\begin{align*}
r_1 &\equiv \Pi^SC_B - \Pi^SS_B \quad \text{and} \quad r_2 \equiv \Pi^CC_B - \Pi^CS_B
\end{align*}
$$

denote firm B’s change in gross profit in the two rows. These changes represent each firm’s gains from customization, conditional on the choice of the other firm.\(^5\)

**Lemma 5** (Relative gains from customization). *For any value of \(q\), \(\max\{c_1,c_2\} < \min\{r_1,r_2\}\).*

Detailed expressions for \(c_1\), \(c_2\), \(r_1\), and \(r_2\) as functions of \(q\) are provided in the appendix. This lemma stipulates that the high quality firm B always gains more from customization than the low quality firm A. Intuitively, customization enables the customizing firm to charge a higher price and attract more consumers. Hence, the firm with the stronger starting position (firm B) benefits more from customization. This explains the results in Lemma 5.

Lemma 5 has implications for equilibrium customization choices. The condition in the first line of (9) becomes \(K \geq r_1\). Similarly, the condition in the last line of (9) becomes \(K \leq c_2\). Finally, the condition in the second line of (9) never holds. It follows that the customization stage has a unique Nash equilibrium except for \(K = c_2\) and \(K = r_1\). For ease of presentation without affecting the results, we will select \((C,C)\) as the Nash equilibrium at \(K = c_2\) and \((S,C)\) at \(K = r_1\). Accordingly,

$$
\begin{align*}
(S,S) \\
(S,C) \\
(C,C)
\end{align*}
$$

is the Nash equilibrium if

$$
\begin{align*}
K &> r_1 \\
K &\in (c_2,r_1] \\
K &\leq c_2
\end{align*}
$$

Because \(K\) is non-negative, the above discussion implies that the signs of \(c_2\) and \(r_1\) are crucial for equilibrium analysis. Specifically, when both \(c_2\) and \(r_1\) are negative, \((S,S)\) is the Nash equilibrium for any \(K\). When \(c_2 < 0\) and \(r_1 > 0\), either \((S,C)\) or \((S,S)\) is the Nash equilibrium, depending on \(K\). When both \(c_2\) and \(r_1\) are positive, the Nash equilibrium can be \((C,C)\), \((S,C)\), or \((S,S)\).

The next proposition summarizes our main results on the firms’ customization choices.

**Proposition 1** (Equilibrium customization choices). *The following hold for the firms’ equilibrium customization choices.*

(i) *If \(q \leq 0.56\) then the Nash equilibrium is \((S,S)\) for any value of \(K\).*

---

\(^5\)We may also call these changes (conditional) marginal returns to customization.
(ii) If $q \in (0.56, 0.63]$ then the Nash equilibrium is $(S,C)$ for $K \leq r_1$ and $(S,S)$ for $K > r_1$.

(iii) If $q > 0.63$ then the Nash equilibrium is $(C,C)$ for $K \leq c_2$, $(S,C)$ for $K \in (c_2, r_1]$, and $(S,S)$ for $K > r_1$.

Figure 5 plots $c_2$ and $r_1$ as functions of the quality difference. Depending on the values for the parameters $q$ and $K$, either both firms customize, only the high quality firm customizes, or no firm customizes. Proposition 1 implies the following:

- The low quality firm never customizes alone.
- Customization is less likely the higher the fixed cost of customization is.
- For small values of $K$, customization is more likely the higher the quality difference is.
- Even when customization is costless, the firms will not customize in equilibrium if the quality difference is small.

What drives the first two conclusions is the fact that the high quality firm always benefits more from customization than the low quality firm. Whenever the low quality firm has an incentive to customize, so does the high quality firm. Accordingly, the equilibrium changes from $(C,C)$ to $(S,C)$ to $(S,S)$ as the fixed cost of customization increases.

Customization by one or both firms makes the firms’ products less differentiated and intensifies price competition. Competition becomes tougher as the quality difference decreases. In the extreme case in which $q = 0$ and both firms customize, the firms’ products become identical in the eyes of consumers, so price competition results in the Bertrand outcome. This intuition is behind the

---

6While $c_2$ is increasing in $q$, $r_1$ is not. The former means $\Pi_{A}^{C}$ increases faster in $q$ than $\Pi_{A}^{SC}$. The latter means that $\Pi_{B}^{SC}$ does not always go up faster than $\Pi_{B}^{SS}$. The non-monotonicity of $r_1$ is a consequence of kinks in the firms’ equilibrium profit functions, which in turn are artifacts of the $[0,1] \times [0,1]$ consumer space.
last two conclusions above. Specifically, when \( q \) is small, the firms do not customize their products in order to avoid a disastrous price war. When \( q \) is large, the firms customize to take advantage of consumers’ desires for ideal varieties. The intermediate case involves customization by the high quality firm only.\(^7\)

### 5 Extensions

In this section we extend the base model by relaxing two central assumptions – uniform pricing and exogenous product qualities.

When a consumer buys a customized product, his ideal variety \( x \) is revealed to the firm.\(^8\) In the base model we assumed that firms that customize cannot use this information and, hence, are restricted to uniform pricing strategies. In our first extension we allow customizing firms to customize prices, that is, to link prices to \( x \).

As the quality difference is crucial in determining the firms’ equilibrium customization choices, a natural question to ask is: What level of quality difference will emerge if the firms choose their qualities as part of the game? Our second extension endogenizes the firms’ qualities. We model this by assuming that the firms simultaneously select their qualities prior to the customization stage.

#### 5.1 Price Customization

Consider a subgame in which one firm customizes and the other does not. With price customization, the customizing firm competes at each location \( x \). The closer is \( x \) to the standard product of the rival firm, the fiercer is price competition. Hence, we expect that in equilibrium the customizing firm charges higher prices to consumers located further away from the non-customizing firm. Moreover, price flexibility enables the customizing firm to reach consumers of all variety preferences. This is in contrast to the base model, where the customizing firm does not serve all varieties when \( q \) is small (Lemmas 2 and 3).

The above intuition is confirmed below. We need to keep in mind that for price customization to be successful, the price schedule of a customizing firm has to be such that consumers do not have incentives to misrepresent their ideal products.

We start with subgame CS in which firm A customizes and firm B does not. For brevity, the derivations of the equilibrium prices in this subgame are relegated to the appendix.\(^9\) For \( q \leq 5/8 \), the prices are given by

\[
\begin{align*}
P^C_{A}(x) &= \begin{cases} 
1 - \frac{3}{5}q - x, & x \leq 1 - \frac{8}{5}q \\
1/2 + \frac{3}{5}q - 1/2x, & x > 1 - \frac{8}{5}q
\end{cases} 
\end{align*}
\]

and

\[
P^C_{B} = \frac{2}{5}q. \quad (10)
\]

\(^7\)The equilibrium does not always follow the sequence \((S,S)\) to \((S,C)\) to \((C,C)\) as \( q \) increases. As can be seen from Figure 5, for some values of \( K \) the equilibrium changes from \((S,S)\) to \((S,C)\) to \((S,S)\) to \((S,C)\). This is due to the non-monotonicity of \( r_1 \).

\(^8\)We assume that the firm does not know individual consumers’ ideal varieties at the time of its decision to customize.

\(^9\)Since the risk of confusion is minimal, in this subsection we employ the same notation as in the base model.
For $q > 5/8$, the prices are\(^{10}\)

$$p^A_{CS}(x) = \frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x \quad \text{and} \quad p^B_{CS} = \frac{2}{3}q - \frac{1}{6}. \quad (11)$$

The case where $q \leq 5/8$ corresponds to Figure 1(d) and the case where $q > 5/8$ corresponds to Figure 1(c). Hence, firm A serves consumers of all variety preferences. It is worth noting that $p^A_{CS}(x)$ precludes consumers from misrepresenting their ideal products. Indeed, no consumer would misrepresent his ideal product because

$$p^A_{CS}(x) \leq p^A_{CS}(x') + |x' - x|$$

holds for any $x$ and $x' \in [0,1].^{11}$

Next, consider subgame SC in which only firm B customizes. In the appendix we show that for $q \leq 5/4$ the equilibrium prices are given by

$$p^A_{SC} = \frac{1}{5}q \quad \text{and} \quad p^B_{SC}(x) = \begin{cases} \frac{3}{5}q + \frac{1}{2}x, & x \leq \frac{4}{5}q \\ \frac{1}{5}q + x, & x > \frac{4}{5}q \end{cases} \quad (12)$$

For $q > 5/4$, the equilibrium prices are\(^{12}\)

$$p^A_{SC} = \frac{1}{3}q - \frac{1}{6} \quad \text{and} \quad p^B_{SC}(x) = \frac{2}{3}q - \frac{1}{12} + \frac{1}{2}x. \quad (13)$$

The case where $q \leq 5/4$ corresponds to Figure 1(b) and the case where $q > 5/4$ corresponds to Figure 1(c). The customizing firm serves consumers of all variety preferences. Note also that $p^B_{SC}(x)$ precludes consumers from misrepresenting their ideal products.

Finally, consider subgames SS and CC. Obviously, subgame SS is the same as in the base model (Lemma 1). In subgame CC, consumers at each location get their ideal varieties buying from either firm; therefore only vertical differentiation is present. So, at each location the game is equivalent to the standard vertical differentiation game, resulting in each firm charging the same price for all varieties. These prices are as in Lemma 4, i.e.,

$$p^A_{CC}(x) = \frac{1}{3}q \quad \text{and} \quad p^B_{CC}(x) = \frac{2}{3}q.$$ 

Having analyzed the pricing stage, we move to the customization stage. Both the game matrix (8) and the general statement (9) regarding Nash equilibrium still apply. While the diagonal cells (corresponding to subgames SS and CC) remain the same, the off-diagonal cells have changed. Hence, the expressions for $c_1$, $c_2$, $r_1$, and $r_2$ under price customization are different from those in

\(^{10}\)It is easy to verify that for a given value of $q$, $p^A_{CS}(x)$ is continuous in $x$, and that (10) coincides with (11) when $q = 5/8$.

\(^{11}\)Ignoring $x = x'$, $p^A_{CS}(x) = p^A_{CS}(x') + |x' - x|$ for $q \leq 5/8$ and $x < x' \leq 1 - \frac{8}{5}q$, $p^A_{CS}(x) < p^A_{CS}(x') + |x' - x|$ for all other cases.

\(^{12}\)It is easy to verify that for a given value of $q$, $p^B_{SC}(x)$ is continuous in $x$, and that (12) coincides with (13) when $q = 5/4$. 
the base model, implying that the firms’ equilibrium customization choices may not be the same.

We show in the appendix that $c_1 < \min \{r_1, r_2\}$ and $c_2 < r_2$ hold for any value of $q$. Figure 6 depicts the firms’ customization choices in equilibrium. For $q \leq 1.26$, $c_2 \geq r_1$ and

$$(S,S) \text{ and } (C,C)$$
is (are) Nash equilibrium(a) if

$$K > c_2$$
$$K \in (r_1, c_2]$$
$$K \leq r_1$$

For $q > 1.26$, $c_2 < r_1$ and

$$(S,S), (S,C), (C,C)$$
is the Nash equilibrium if

$$K > r_1$$
$$K \in (c_2, r_1]$$
$$K \leq c_2$$

Note that, as in the base model, the equilibrium is determined by $r_1$ and $c_2$ and their relationship with $K$.\footnote{The curve $r_1$ is non-monotonic in $q$ as in the base model. The explanation provided in footnote 6 also applies here.}

The next proposition provides a summary of the above results.

**Proposition 2** (Equilibrium customization choices with price customization). The following hold for the equilibrium customization choices when the firms can customize prices.

(i) If $q \leq 1.26$ then $(C,C)$ is the Nash equilibrium for $K \leq r_1$; both $(C,C)$ and $(S,S)$ are Nash equilibria for $K \in (r_1, c_2]$; and $(S,S)$ is the Nash equilibrium for $K > c_2$.

(ii) If $q > 1.26$ then $(C,C)$ is the Nash equilibrium for $K \leq c_2$; $(S,C)$ is the Nash equilibrium for $K \in (c_2, r_1]$; and $(S,S)$ is the Nash equilibrium for $K > r_1$.\footnote{The curve $r_1$ is non-monotonic in $q$ as in the base model. The explanation provided in footnote 6 also applies here.}

Figure 6: Equilibrium customization choices with price customization
Proposition 2 implies that with customized prices the main conclusions of the base model continue to hold. Namely, the low quality firm never customizes alone; customization becomes less likely as the fixed cost of customization increases; and (for small values of $K$) more likely as the quality difference increases.

Three obvious differences exist between this extension and the base model. With price customization, for any value of $q$ customization by both firms occurs in equilibrium provided that $K$ is sufficiently small, as can be seen from Figure 6. Recall that in the base model no firm customizes when $q < 0.56$ (the last conclusion in the list following Proposition 1). While decreasing product differentiation, customization takes advantage of consumers’ desires for ideal varieties. Price customization offers an additional benefit to the customizing firm by enhancing its ability to compete with the other firm. As a result, in the current extension the firms have incentives to customize even when $q$ is small, provided that the fixed cost of customization is sufficiently small.

Secondly, the minimum quality difference to support customization by the high quality firm only is larger in the extension than in the base model, 1.26 vs. 0.56. Indeed, $(S,C)$ cannot be an equilibrium when $q < 1.26$ because in this range $c_2 > r_1$. That is, whenever firm B finds it profitable to customize, firm A has an incentive to customize as well. The reason for this is the additional benefit to the customizing firm mentioned in the previous paragraph. This benefit hurts the non-customizing firm, increasingly the lower $q$ is (i.e., less product differentiation). As a result, firm A is compelled to customize in order to soften the loss.

Finally, multiple equilibria exist for a portion of the parameter space in the price customization extension, as can be seen from Figure 6.\textsuperscript{14} When $r_1 < c_2$, for $K \in (r_1, c_2)$ (the shaded area in Figure 6) each firm’s best response is to match the other firm’s choice, implying that both $(S,S)$ and $(C,C)$ are Nash equilibria.\textsuperscript{15}

The differences between the base model and the price customization extension are caused by the patterns of $r_1$ and $c_2$ in the two settings. The crucial change in the patterns is that in the base model $r_1$ and $c_2$ start from negative values, whereas the curves in Figure 6 start at the origin. The former is due to intensified competition brought about by customization at small values of $q$.\textsuperscript{16} The latter is driven by price customization which at $q = 0$ enables the customizing firm to break even rather than to lose.\textsuperscript{17}

An important consequence of $r_1$ and $c_2$ starting at the origin is that the region in which $c_2$ lies above $r_1$ is in the first quadrant, whereas in the base model this region is in the fourth quadrant and is therefore irrelevant for equilibrium analysis. This explains multiplicity of equilibria in the price customization extension – the last of the three aforementioned differences between the base model and the extension. Another consequence is that $r_1$ and $c_2$ are positive even for small values of $q$, implying the first of the three differences.

\textsuperscript{14}Obviously, there are mixed-strategy equilibria in this region of the parameter space.

\textsuperscript{15}In the base model it is always the case that at least one firm has a dominant strategy. The same is true in the price customization extension, except for the shaded area. The existence of a dominant strategy in a $2 \times 2$ matrix game always implies a unique Nash equilibrium in pure strategies (see Hamilton and Slutsky 1993).

\textsuperscript{16}In the base model, $r_1 = 4/9 - 1/2 = -1/18$ and $c_2 = 0 - 1/9 = -1/9$ when $q = 0$.

\textsuperscript{17}In the price customization extension, $r_1 = 1/2 - 1/2 = 0$ and $c_2 = 0 - 0 = 0$ when $q = 0$. 

18
5.2 Endogenous Quality

We next extend the base model by allowing the firms to choose simultaneously their qualities prior to the customization stage. As is usually assumed in the literature, the firms incur quality-dependent fixed costs, and the variable costs of production are not affected.

As either firm may become the high quality firm, we label the two firms as firm 1 and firm 2, and their qualities \( q_1 \) and \( q_2 \), which take values in \([0, \infty)\). For any given pair \((q_1, q_2)\), the continuation game is our base model with the quality difference \( q = |q_1 - q_2| \).

To keep our analysis more focused, we will present detailed results for the case \( K = 0 \), and then briefly for a positive \( K \). Based on Lemmas 1 through 4 and Proposition 1, firm \( i \)'s profit as a function of qualities is

\[
\pi_i(q_i, q_j) = \begin{cases} 
\frac{1}{2}(q_j - q_i), & q_i - q_j < -0.63 \\
\frac{1}{16}\sqrt{2(q_j - q_i)}, & q_i - q_j \in [-0.63, -0.56] \\
\frac{1}{2}(1 - \frac{1}{6}(q_j - q_i))^2, & q_i - q_j \in [-0.56, 0] \\
\frac{1}{2}(1 + \frac{1}{6}(q_i - q_j))^2, & q_i - q_j \in [0, 0.56] \\
\frac{9}{16}\sqrt{2(q_i - q_j)}, & q_i - q_j \in (0.56, 0.63] \\
\frac{4}{7}(q_i - q_j), & q_i - q_j > 0.63 
\end{cases}
\]

for \( i = 1, 2 \) and \( j \neq i \). Firm \( i \)'s profit is the same as the low quality firm A’s when \( q_i \leq q_j \) (the first three lines in (14)) and the high quality firm B’s when \( q_i > q_j \) (the last three lines). The first and last lines of (14) correspond to both firms customizing \((q > 0.63)\) and the profits are as in Lemma 4. The second and fifth lines correspond to customization by the high quality firm \((q \in (0.56, 0.63])\) and the profits are as in Lemma 3(ii). The middle two lines correspond to no customization \((q \leq 0.56)\) and the profits are as in Lemma 1(i).

Note that firm \( i \)'s profit \( \pi_i(q_i, q_j) \) depends on the quality difference \(|q_i - q_j|\) and whether \( q_i \) is higher or lower than \( q_j \). The upper curve in Figure 7 depicts firm \( i \)'s profit function when \( q_i > q_j \) and the lower curve when \( q_i < q_j \). The two curves start at the same point that corresponds to equal qualities.

Let \( c(q_i) \) denote the fixed cost firm \( i \) incurs. Firm \( i \)'s best response function is, therefore, given by

\[ q_i(q_j) = \arg \max_{q_i} \pi_i(q_i, q_j) - c(q_i). \]

The firms’ quality choices in equilibrium are determined by the intersection point(s) of the two best response curves.

Due to the complexity of the profit function in (14), we are unable to analytically solve the game, even with a specific cost function. In the following we solve the game through numerical simulations. We use the cost function

\[ c(q_i) = \alpha q_i^2, \]

\[ ^{18} \text{One can easily identify from Figure 5 the two critical values (0.56 and 0.63) used in defining } \pi_i(q_i, q_j). \text{ Note that the critical values change with } K. \]
where $\alpha > 0$ is a parameter.

Proposition 3 summarizes our simulated results. Figure 8 provides a graphical illustration of the firms’ equilibrium quality choices reported in this proposition. The arrows indicate how equilibria change as the cost parameter $\alpha$ increases.

**Proposition 3** (Equilibrium with endogenous qualities). *For $K = 0$, the following hold for the equilibrium quality choices.*

(i) If $\alpha < 0.066$ then there are two asymmetric Nash equilibria, $(0, q^*)$ and $(q^*, 0)$, where $q^* > 3.37$ and decreases in $\alpha$. Both equilibria result in customization by both firms.

(ii) If $\alpha \in [0.066, 0.082]$ then there is one symmetric and two asymmetric Nash equilibria, $(q^\dagger, q^\dagger)$, $(0, q^*)$, and $(q^*, 0)$, where $q^\dagger$ decreases from 1.26 to 1.02 and $q^*$ decreases from 3.37 to 2.71 as $\alpha$ increases from 0.066 to 0.082. In the symmetric equilibrium no firm customizes. In each asymmetric equilibrium both firms customize.

(iii) If $\alpha > 0.082$ then there is one symmetric Nash equilibrium $(q^\dagger, q^\dagger)$, where $q^\dagger < 1.02$ and decreases in $\alpha$. In equilibrium no firm customizes.

When $\alpha$ is small ($< 0.066$), one of the firms sets its quality to zero, while the other chooses positive quality $q^*$. The equilibrium quality difference (equal to $q^*$) is high, leading to customization by both firms. Figure 7 helps in providing intuition for this outcome. The low quality firm does not want to raise its quality to decrease the quality gap. This is because doing so is costly and also decreases its gross profit (see the rising portion of the lower curve). For the high quality firm, the

---

$^{19}$Here $q^*$ is high so that leapfrogging by the low quality firm to become the high quality firm is too costly relative to the gain in profit.
portion of the upper profit curve where \( q > 1.42 \) becomes more attractive than the portion where \( q \leq 0.63 \), justifying \( q^* \) being high \( (>3.37) \).

When \( \alpha > 0.082 \), the firms choose the same qualities \( q^\dag \) in equilibrium, leading to no customization by either firm. Intuitively, with large \( \alpha \), high qualities are not profitable choices. It follows that high quality differences cannot occur in equilibrium. In fact, the equilibrium is symmetric (zero quality difference). Equilibrium value \( q^\dag \) is such that no firm wants to make a small deviation up or down. The profit gain from an upward deviation is less than the increase in cost. The cost saving from a downward deviation is less than the decrease in profit. Moreover, high \( \alpha \) prevents either firm from making a large deviation. In the intermediate case \( (\alpha \in [0.066, 0.082]) \) all forces discussed above are at work. As a result, symmetric and asymmetric equilibria co-exist.

To summarize, Proposition 3 implies that, with endogenous quality determination and zero fixed cost of customization, the firms are substantially differentiated in quality when \( \alpha \) is small and non-differentiated when \( \alpha \) is large. The former results in customization by both firms, while the latter leads to no customization. Note that both \((S,C)\) and \((C,S)\) do not arise in equilibrium.

Our simulations with other values of \( K \) indicate that the result that the firms are either substantially differentiated or non-differentiated is generally true. Sometimes the substantial differentiation leads to customization by the high quality firm alone. In the appendix we report the results for

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\[ \frac{\text{It is obvious from Figure 7 that any quality gap } |q_i - q_j| \in (0.63, 1.42)}{\text{cannot occur in equilibrium. Indeed, if } |q_i - q_j| \in (0.63, 1.42) \text{ then the high quality firm benefits from lowering its quality to reduce the gap to 0.63.}} \]

\[ \frac{\text{Note that } q^\dag \text{ is never zero. This is because at } q_i = q_j = 0 \text{ each firm’s marginal profit is } 1/6 \text{ and marginal cost is } 0.}}{\text{}} \]
The equilibrium pattern of quality choices is similar to Figure 8, with the critical values for $\alpha$, 0.066 and 0.082, replaced by 0.051 and 0.091, respectively. In this example, in all asymmetric equilibria only the high quality firm customizes. As a final note, in all our simulated cases, the high quality firm earns a higher profit (net of all costs) than the low quality firm. This implies that each firm prefers to be the high quality firm in any asymmetric equilibrium.

Comparing our simulated results in Proposition 3 and the discussion above with the results in Proposition 1, we may draw the following conclusions. First, all three outcomes, (S,S), (S,C), and (C,C), can emerge when qualities are chosen endogenously. Second, the outcome (S,S) is always associated with the firms choosing the same quality. Third, in any asymmetric equilibrium the lower quality firm chooses the lowest possible quality.

6 Concluding Remarks

The novelty of our paper is the incorporation of difference in product qualities into customization competition. Customization enables firms to take advantage of consumers’ desires for ideal varieties. However, it makes firms less differentiated and, therefore, intensifies price competition. This intuition is behind much of the results in the theoretical literature on customization, and is shown here to be valid when there is vertical differentiation.

The most important finding in our paper is that quality does play an important role in firms’ strategic decisions concerning customization. We show in our base model that customization can occur in equilibrium only when the quality difference is sufficiently large. Moreover, the high quality firm can always reap a larger benefit from customization than the low quality firm. As a result, the high quality firm may be the only firm customizing in equilibrium, whereas the low quality firm never customizes alone.

Our main result on quality is consistent with many real-world observations. In the National Bicycle Industrial Company example (Kotha 1995) the firm chooses to customize only the high-quality, high-priced Panasonic brand. In the wood furniture market surveyed by Lihra, Buehlmann, and Beauregard (2008) some high quality domestic manufacturers customize, while low quality offshore companies do not. The case study examples in Piller, Moeslein, and Stotko (2004) also support our conclusion that high quality firms are more likely to customize.

Our conclusions from the base model and the two extensions have some obvious managerial implications for firms contemplating customization in competitive situations. First, investments in quality improvement and customization technology can gain strategic advantages in competition. Second, customization is more likely to be a successful strategy for high quality firms and less so for low quality firms. In case of multiple brands the firm should, at least initially, consider customizing only the high-end brands. Third, low quality firms might adopt customization when high quality firms customize and the quality gap is large. Fourth, to engage in price customization, lower prices should be charged to consumers located closer to the rival’s standard product in the attribute space.

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\[ K = 0.115 \] 22 The equilibrium in the base model is (S,S) when \( q < 1 \) and (S,C) when \( q \geq 1 \).
Fifth, if quality is part of a firm’s strategic choices, the firm should try to match the competitor’s quality when the quality-related costs are high, and differentiate itself from the competitor when the quality-related costs are low.

Limitations of our model are also worth mentioning as they may be relevant for managers in making decisions on customization. In our model the customizing firm does not offer its standard product. However, in practice, most customizing firms keep offering their standard products. Our paper assumes that the customization cost is the same for both firms. This obviously need not be true in a real-world situation. In some cases it may even be less costly to customize a high quality product. Such a situation is reported in Lihra, Buehlmann, and Beauregard (2008) for domestic furniture manufacturers. Still, there are other factors not modeled in our paper that should not be overlooked by managers. Some of those factors have been mentioned in Pine (1993) and Broekhuizen and Alsem (2002).

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23Due to those other factors, customizing firms in a market may not be the firms with the highest quality. For example, Tumi, whose messenger bags and backpacks are of superior quality to those of Timbuk2, is not known for customization. We thank an anonymous referee for pointing out this example to us.
References


Appendix

Proof of Lemma 1. Each case is proven in turn.

(i) Consider $q \leq 3/2$ and suppose the indifference line (5) intersects the unit square as shown in Figure 5(a). Straightforward algebra implies

$$D_A(p_A, p_B) = \frac{1}{2} \left( 1 - \frac{1}{2} q + p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{2} \left( 1 + \frac{1}{2} q + p_A - p_B \right)$$

in this case. The profit maximizing first-order conditions yield the equilibrium prices and profits as in part (i) of the lemma. It is left to verify that under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\hat{x}(1) \geq 0$ and $\hat{x}(0) \leq 1$.

$$\hat{x}(1) = \frac{1}{2} \left( 1 - q + p_B^{SS} - p_A^{SS} \right) = \frac{1}{2} \left( 1 - \frac{2}{3} q \right) \geq 0,$$

$$\hat{x}(0) = \frac{1}{2} \left( 1 + p_B^{SS} - p_A^{SS} \right) = \frac{1}{2} \left( 1 + \frac{1}{3} q \right) < 1$$

hold for $q \leq 3/2$. In fact, $\hat{x}(1) = 0$ when $q = 3/2$.

(ii) Consider $3/2 < q \leq 3$ and suppose the indifference line (5) intersects the unit square as shown in Figure 5(b). The firms’ demand functions are

$$D_A(p_A, p_B) = \frac{1}{4q} \left( 1 + p_B - p_A \right)^2 \quad \text{and} \quad D_B(p_A, p_B) = 1 - \frac{1}{4q} \left( 1 + p_B - p_A \right)^2.$$

The first-order conditions yield the equilibrium prices and profits as in part (ii) of the lemma. Under these prices the indifference line intersects the left and bottom sides of the unit square. Algebraically, $\hat{x}(1) \leq 0$ and $\hat{x}(0) \in [0, 1]$. Indeed,

$$\hat{x}(1) = \frac{1}{2} \left( 1 - q + \frac{1}{4} \left( -3 + \sqrt{1 + 16q} \right) \right) < 0,$$

$$\hat{x}(0) = \frac{1}{2} \left( 1 + \frac{1}{4} \left( -3 + \sqrt{1 + 16q} \right) \right) \in (0, 1]$$

hold for $3/2 < q \leq 3$. Note that $\hat{x}(1) = 0$ when $q = 3/2$ and $\hat{x}(0) = 1$ when $q = 3$.

(iii) Consider $q > 3$ and suppose the indifference line (5) intersects the unit square as shown in Figure 5(c). The firms’ demand functions are

$$D_A(p_A, p_B) = \frac{1}{q} \left( p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q} \left( q + p_A - p_B \right).$$

The first-order conditions yield the equilibrium prices and profits as in part (iii) of the lemma. Under these prices the indifference line intersects the right and left sides of the unit square.
Algebraically, \( \hat{x}(1) \leq 0 \) and \( \hat{x}(0) \geq 1 \). Indeed,
\[
\hat{x}(1) = \frac{1}{2} \left( 1 - \frac{2}{3} q \right) < 0 \quad \text{and} \quad \hat{x}(0) = \frac{1}{2} \left( 1 + \frac{1}{3} q \right) > 1
\]
hold for \( q > 3 \). Note that \( \hat{x}(0) = 1 \) when \( q = 3 \).

Proof of Lemma 2. Each case is proven in turn.

(i) Consider \( q \leq 1 \) and suppose the indifference line (6) intersects the unit square as shown in Figure 5(a). The firms’ demand functions are
\[
D_A(p_A, p_B) = 1 - \frac{1}{2} q + p_B - p_A \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{2} q + p_A - p_B.
\]
The first-order conditions yield the equilibrium prices and profits as in part (i) of the lemma. Under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, \( \hat{x}(1) \geq 0 \) and \( \hat{x}(0) \leq 1 \). Indeed,
\[
\hat{x}(1) = 1 - q + p_B^{CS} - p_A^{CS} = \frac{2}{3} - \frac{2}{3} q \geq 0 \quad \text{and} \quad \hat{x}(0) = 1 + p_B^{CS} - p_A^{CS} = \frac{2}{3} + \frac{1}{3} q < 1
\]
hold for \( q \leq 1 \). Note that \( \hat{x}(1) = 0 \) and \( \hat{x}(0) = 1 \) when \( q = 1 \).

(ii) Consider \( q > 1 \) and suppose the indifference line (6) intersects the unit square as shown in Figure 5(c). The firms’ demand functions are
\[
D_A(p_A, p_B) = \frac{1}{q} \left( \frac{1}{2} + p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q} \left( q - \frac{1}{2} + p_A - p_B \right).
\]
The first-order conditions yield the equilibrium prices and profits as in part (ii) of the lemma. Under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, \( \hat{x}(1) \leq 0 \) and \( \hat{x}(0) \geq 1 \). Indeed,
\[
\hat{x}(1) = \frac{2}{3} - \frac{2}{3} q < 0 \quad \text{and} \quad \hat{x}(0) = \frac{2}{3} + \frac{1}{3} q > 1
\]
hold for \( q > 1 \). Note that \( \hat{x}(1) = 0 \) and \( \hat{x}(0) = 1 \) when \( q = 1 \).

Proof of Lemma 3. Each case is proven in turn.

(i) Consider \( q \leq 1/2 \) and suppose the indifference line (7) intersects the unit square as shown in Figure 5(a). The firms’ demand functions are
\[
D_A(p_A, p_B) = -\frac{1}{2} q + p_B - p_A \quad \text{and} \quad D_B(p_A, p_B) = 1 + \frac{1}{2} q + p_A - p_B.
\]
The first-order conditions yield the equilibrium prices and profits as in part (i) of the lemma. Under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, \( \hat{x}(1) \geq 0 \) and \( \hat{x}(0) \leq 1 \). Indeed,

\[
\hat{x}(1) = -q + p_B^{SC} - p_A^{SC} = \frac{1}{3} - \frac{2}{3}q \geq 0 \quad \text{and} \quad \hat{x}(0) = p_B^{SC} - p_A^{SC} = \frac{1}{3} + \frac{1}{3}q < 1
\]

hold for \( q \leq 1/2 \). Note that \( \hat{x}(1) = 0 \) when \( q = 1/2 \).

(ii) Consider \( 1/2 < q \leq 2 \) and suppose the indifference line (7) intersects the unit square as shown in Figure 5(b). The firms’ demand functions are

\[
D_A(p_A, p_B) = \frac{1}{2q} (p_B - p_A)^2 \quad \text{and} \quad D_B(p_A, p_B) = 1 - \frac{1}{2q} (p_B - p_A)^2.
\]

The first-order conditions yield the equilibrium prices and profits as in part (ii) of the lemma. Under these prices the indifference line intersects the left and bottom sides of the unit square. Algebraically, \( \hat{x}(1) \leq 0 \) and \( \hat{x}(0) \in [0, 1] \). Indeed,

\[
\hat{x}(1) = -q + \frac{1}{2} \sqrt{2q} < 0 \quad \text{and} \quad \hat{x}(0) = \frac{1}{2} \sqrt{2q} \in (0, 1]
\]

hold for \( 1/2 < q \leq 2 \). Note that \( \hat{x}(1) = 0 \) when \( q = 1/2 \) and \( \hat{x}(0) = 1 \) when \( q = 2 \).

(iii) Consider \( q > 2 \) and suppose the indifference line (7) intersects the unit square as shown in Figure 5(c). The firms’ demand functions are

\[
D_A(p_A, p_B) = \frac{1}{q} \left( -\frac{1}{2} + p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q} \left( q + \frac{1}{2} + p_A - p_B \right).
\]

The first-order conditions yield the equilibrium prices and profits as in part (iii) of the lemma. Under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, \( \hat{x}(1) \leq 0 \) and \( \hat{x}(0) \geq 1 \). Indeed,

\[
\hat{x}(1) = \frac{1}{3} - \frac{2}{3}q < 0 \quad \text{and} \quad \hat{x}(0) = \frac{1}{3} + \frac{1}{3}q > 1
\]

hold for \( q > 2 \). Note that \( \hat{x}(0) = 1 \) when \( q = 2 \).

\[\square\]

Proof of Lemma 4. The results follow immediately from the first-order conditions. \[\square\]

Proof of Lemma 5. The expressions for \( c_1, c_2, r_1, \) and \( r_2 \) as functions of \( q \) follow immediately
Proofs of (10) and (11).

Proof of Proposition 1. From Lemmas 1 through 4,

\[ c_1 = \Pi_A^{CS} - \Pi_A^{SS} = \begin{cases} \frac{2}{3} - \frac{1}{6}q - \frac{1}{2} (1 - \frac{1}{6}q)^2, & q \leq 1 \\ \frac{1}{9} \frac{1}{q} (\frac{1}{2}q + \frac{1}{6})^2 - \frac{1}{2} (1 - \frac{1}{6}q)^2, & q \in (1, \frac{3}{2}] \\ \frac{1}{9} \frac{1}{q} (\frac{1}{2}q + \frac{1}{6})^2 - \frac{1}{6}q = \frac{1}{9} + \frac{3}{16q}, & q > \frac{3}{2} \end{cases} \]

\[ c_2 = \Pi_A^{CC} - \Pi_A^{SC} = \begin{cases} \frac{1}{9} q - \left(\frac{1}{9} - \frac{1}{6}q\right)^2, & q \leq \frac{1}{2} \\ \frac{1}{9} q - \frac{1}{16}\sqrt{2q}, & q \in \left(\frac{1}{2}, 2]\right] \\ \frac{1}{9} (\frac{2}{3}q + \frac{1}{6})^2 - \frac{1}{2} \left(1 - \frac{1}{6}q\right)^2 = \frac{1}{9} - \frac{1}{36q}, & q > 2 \end{cases} \]

\[ r_1 = \Pi_B^{SC} - \Pi_B^{SS} = \begin{cases} \frac{2}{9} q - \left(\frac{1}{9} + \frac{1}{6}q\right)^2, & q \leq \frac{1}{2} \\ \frac{9}{16} \sqrt{2q} - \frac{1}{2} \left(1 + \frac{1}{6}q\right)^2, & q \in \left(\frac{1}{2}, \frac{3}{2}\right] \\ \frac{9}{16} \sqrt{2q} - \left(1 - \frac{1}{q} \left(1 + \frac{1}{6}q\right)^2\right) = \frac{5}{16} + \frac{3}{36q}, & q \in \left(\frac{3}{2}, 2\right] \\ \frac{1}{4} (\frac{2}{3}q + \frac{1}{6})^2 - \frac{1}{2} \left(1 - \frac{1}{6}q\right)^2 = \frac{2}{9} + \frac{3}{36q}, & q > \frac{3}{2} \end{cases} \]

and

\[ r_2 = \Pi_B^{CC} - \Pi_B^{CS} = \begin{cases} \frac{4}{9} q - \left(\frac{1}{9} + \frac{1}{6}q\right)^2, & q \leq 1 \\ \frac{4}{9} q - \frac{1}{16} \left(\frac{2}{3}q - \frac{1}{6}\right)^2 = \frac{2}{9} - \frac{1}{36q}, & q > 1 \end{cases} \]

All of these four functions are negative on intervals from \( q = 0 \) to some small values of \( q \). Since \( K \geq 0 \), the negative parts of these functions are not relevant for our equilibrium analysis. Replacing the negative parts by zero and performing straightforward numerical calculations confirm that \( \max\{0, c_1, c_2\} \leq \min\{r_1, r_2\} \) for any given value of \( q \).

\( \square \)

**Proof of Proposition 1.** The results follow immediately from Lemma 5 and the discussion preceding Proposition 1.

\( \square \)

**Proofs of (10) and (11).** We will start with the easier case of \( q > 5/8 \), then consider \( q \leq 5/8 \).

(a) Suppose \( q > 5/8 \). Given firm B’s price for its standard product \( p_B^{CS} \), what price should the customizing firm A charge to consumers located at \( x \)? Fraction \( \hat{y}(x) \) of these consumers will purchase from firm A:

\[ v + qAy - p_A = v + qBy - (1 - x) - p_B^{CS} \implies \hat{y}(x) = \frac{1}{q}(1 - x + p_B^{CS} - p_A). \]

The first-order condition for the maximization problem

\[ \max_{p_A} \hat{y}(x)p_A \]

yields

\[ p_A^{CS}(x) = \frac{1}{2}(1 - x + p_B^{CS}). \]
To find firm B’s equilibrium price \( p_{CS}^B \), consider a deviation by firm B,

\[
v + q_A y - \frac{1}{2} (1 - x + p_{CS}^B) = v + q_B y - (1 - x) - p_B \quad \implies \quad \tilde{y}(x) = \frac{1}{2q} (1 - x - p_{CS}^B + 2p_B).
\]

The first-order condition for the maximization problem

\[
\max_{p_B} \int_0^1 (1 - \tilde{y}(x))p_B \, dx = \frac{1}{2q} \left( 2q - \frac{1}{2} + p_{CS}^B - 2p_B \right) p_B
\]

is

\[
2q - \frac{1}{2} + p_{CS}^B - 4p_B = 0.
\]

Substituting \( p_B = p_{CS}^B \) into the above equation yields

\[
p_{CS}^B = \frac{2}{3} q - \frac{1}{6},
\]

Thus, we have proved (11). It is worth noting that \( q > 5/8 \) guarantees

\[
\tilde{y}(x) = \frac{1}{q} \left( 1 - x + p_{CS}^B - p_{A}^{CS}(x) \right) = \frac{1}{q} \left( \frac{5}{12} + \frac{1}{3} q - \frac{1}{2} x \right) \in (0, 1)
\]

for all values of \( x \). In fact, \( \tilde{y}(0) = 1 \) when \( q = 5/8 \).

(b) Consider \( q \leq 5/8 \). In this case

\[
p_{CS}^A(x) = \begin{cases} 1 - x - q + p_{CS}^B, & x \leq 1 - 2q + p_{CS}^B \\ \frac{1}{2} (1 - x + p_{CS}^B), & x > 1 - 2q + p_{CS}^B \end{cases}
\]

implying

\[
\tilde{y}(x) = \begin{cases} 1, & x \leq 1 - 2q + p_{CS}^B \\ \frac{1}{2q} (1 - x + p_{CS}^B), & x > 1 - 2q + p_{CS}^B \end{cases}
\]

To find firm B’s equilibrium price \( p_{CS}^B \), consider a deviation by firm B,

\[
\tilde{y}(x) = \begin{cases} 1, & x \leq 1 - 2q - p_{CS}^B + 2p_B \\ \frac{1}{2q} (1 - x - p_{CS}^B + 2p_B), & x > 1 - 2q - p_{CS}^B + 2p_B \end{cases}
\]

The first-order condition for the maximization problem

\[
\max_{p_B} \int_0^1 (1 - \tilde{y}(x))p_B \, dx = \frac{1}{4q} \left( 2q + p_{CS}^B - 2p_B \right)^2 p_B
\]

is

\[
2q + p_{CS}^B - 6p_B = 0.
\]
Substituting \( p_B = p_{B}^{CS} \) into the above equation yields
\[
p_{B}^{CS} = \frac{2}{5} q.
\]
Thus, we have proved (10).

\[\square\]

**Proofs of (12) and (13).** We will start with the easier case of \( q > 5/4 \), then consider \( q \leq 5/4 \).

(a) Suppose \( q > 5/4 \). Given firm A’s price for its standard product \( p_{A}^{SC} \), what price should the customizing firm B charge to consumers located at \( x \)? Fraction \( 1 - \hat{y}(x) \) of these consumers will purchase from firm B:
\[
v + q_{A}y - x - p_{A}^{SC} = v + q_{B}y - p_{B} \implies \hat{y}(x) = \frac{1}{q} (-x - p_{A}^{SC} + p_{B}).
\]
The first-order condition for the maximization problem
\[
\max_{p_{B}} (1 - \hat{y}(x)) p_{B}
\]
yields
\[
p_{B}^{SC}(x) = \frac{1}{2} (q + x + p_{A}^{SC}).
\]
To find firm A’s equilibrium price \( p_{A}^{CS} \), consider a deviation by firm A,
\[
v + q_{A}y - x - p_{A} = v + q_{B}y - \frac{1}{2} (q + x + p_{A}^{SC}) \implies \hat{y}(x) = \frac{1}{2q} (q - x + p_{A}^{SC} - 2 p_{A}).
\]
The first-order condition for the maximization problem
\[
\max_{p_{A}} \int_{0}^{1} \hat{y}(x) p_{A} \, dx = \frac{1}{2q} \left( q - \frac{1}{2} + p_{A}^{SC} - 4 p_{A} \right) p_{A}
\]
is
\[
q - \frac{1}{2} + p_{A}^{SC} - 4 p_{A} = 0.
\]
Substituting \( p_{A} = p_{A}^{SC} \) into the above equation yields
\[
p_{A}^{SC} = \frac{1}{3} q - \frac{1}{6}.
\]
Thus, we have proved (13). It is worth noting that \( q > 5/4 \) guarantees
\[
\hat{y}(x) = \frac{1}{q} (-x - p_{A}^{SC} + p_{B}^{SC}(x)) = \frac{1}{q} \left( \frac{1}{12} + \frac{1}{3} q - \frac{1}{2} x \right) \in (0, 1)
\]
for all values of \( x \). In fact, \( \hat{y}(1) = 0 \) when \( q = 5/4 \).
Consider $q \leq 5/4$. In this case

$$p^SC_B(x) = \begin{cases} \frac{1}{2} (q + x + p^SC_A), & x \leq q - p^SC_A \\ x + p^SC_A, & x > q - p^SC_A \end{cases}$$

implying

$$\tilde{y}(x) = \begin{cases} \frac{1}{2q} (q - x - p^SC_A), & x \leq q - p^SC_A \\ 0, & x > q - p^SC_A \end{cases}$$

To find firm A’s equilibrium price $p^SC_A$, consider a deviation by firm A,

$$\overline{y}(x) = \begin{cases} \frac{1}{2q} (q - x + p^SC_A - 2p_A), & x \leq q + p^SC_A - 2p_A \\ 0, & x > q + p^SC_A - 2p_A \end{cases}$$

The first-order condition for the maximization problem

$$\max_{p_A} \int_0^1 \overline{y}(x)p_A\,dx = \frac{1}{4q} \left(q + p^SC_A - 2p_A\right)^2 p_A$$

is

$$q + p^SC_A - 6p_A = 0.$$ 

Substituting $p_A = p^SC_A$ into the above equation yields

$$p^SC_A = \frac{1}{5} q.$$ 

Thus, we have proved (12).

\[\square\]

**Proof of Proposition 2.** We will first calculate the firms’ equilibrium profits in subgames CS and SC, then provide expressions for $c_1, c_2, r_1,$ and $r_2$.

(a) Consider subgame CS. For $q \leq 5/8$ the indifference line is

$$\tilde{y}(x) = \begin{cases} 1, & x \leq 1 - \frac{3}{5}q \\ \frac{1}{5} \left(\frac{1}{2} + \frac{1}{5}q - \frac{1}{2}x\right), & x > 1 - \frac{3}{5}q \end{cases}$$

Hence,

$$\Pi^CS_A = \int_0^1 \tilde{y}(x)p^CS_A(x)\,dx = \int_0^{1-\frac{3}{5}q} \left(1 - \frac{3}{5}q - x\right)\,dx + \int_{1-\frac{3}{5}q}^1 \frac{1}{5} \left(\frac{1}{2} + \frac{1}{5}q - \frac{1}{2}x\right)^2\,dx$$

$$= \frac{1}{2} \left(\left(1 - \frac{3}{5}q\right)^2 - q^2\right) + \frac{2}{3q} \left(q^3 - \left(\frac{1}{5}q\right)^3\right)$$

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\[ \Pi^{CS}_B = \int_0^1 (1 - \hat{y}(x))p^CS_B \, dx = \int_0^1 \frac{2}{5} \left( \frac{4}{5}q - \frac{1}{2} + \frac{1}{2}x \right) \, dx = \frac{32}{125}q^2. \]

For \( q > 5/8 \) the indifference line is
\[ \hat{y}(x) = \frac{1}{q} \left( \frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x \right). \]

Hence,
\[ \Pi^{CS}_B = \int_0^1 \frac{1}{q} \left( \frac{5}{12} + \frac{1}{3}q - \frac{1}{2}x \right)^2 \, dx = \frac{2}{3q} \left( \left( \frac{5}{12} + \frac{1}{3}q \right)^3 - \left( \frac{1}{3}q - \frac{1}{12} \right)^3 \right) \]

and
\[ \Pi^{CS}_A = \int_0^1 \frac{1}{q} \left( \frac{2}{3}q - \frac{5}{12} + \frac{1}{2}x \right) \left( \frac{2}{3}q - \frac{1}{6} \right) \, dx = \frac{1}{q} \left( \frac{2}{3}q - \frac{1}{6} \right)^2. \]

(b) Consider subgame SC. For \( q \leq 5/4 \) the indifference line is
\[ \hat{y}(x) = \begin{cases} \frac{1}{q} \left( \frac{2}{3}q - \frac{1}{2}x \right), & x \leq \frac{4}{5}q \\ 0, & x > \frac{4}{5}q \end{cases}. \]

Hence,
\[ \Pi^{SC}_A = \int_0^1 \hat{y}(x)p^SC_A \, dx = \int_0^{\frac{4}{5}q} \frac{1}{q} \left( \frac{2}{3}q - \frac{1}{2}x \right) \, dx = \frac{4}{125}q^2 \]

and
\[ \Pi^{SC}_B = \int_0^1 (1 - \hat{y}(x))p^SC_B \, dx = \int_0^{\frac{4}{5}q} \frac{1}{q} \left( \frac{3}{5}q + \frac{1}{2}x \right)^2 \, dx + \int_{\frac{4}{5}q}^1 \left( \frac{1}{5}q + x \right) \, dx \\
= \frac{2}{3q} \left( q^3 - \left( \frac{3}{5}q \right)^3 \right) + \frac{1}{2} \left( \left( \frac{1}{5}q + 1 \right)^2 - q^2 \right). \]

For \( q > 5/4 \) the indifference line is
\[ \hat{y}(x) = \frac{1}{q} \left( \frac{1}{12} + \frac{1}{3}q - \frac{1}{2}x \right). \]

Hence,
\[ \Pi^{SC}_A = \int_0^1 \frac{1}{q} \left( \frac{1}{12} + \frac{1}{3}q - \frac{1}{2}x \right) \left( \frac{1}{3}q - \frac{1}{6} \right) \, dx = \frac{1}{q} \left( \frac{1}{3}q - \frac{1}{6} \right)^2 \]

and
\[ \Pi^{SC}_B = \int_0^1 \frac{1}{q} \left( \frac{2}{3}q - \frac{1}{12} + \frac{1}{2}x \right)^2 \, dx = \frac{2}{3q} \left( \left( \frac{2}{3}q + \frac{5}{12} \right)^3 - \left( \frac{2}{3}q - \frac{1}{12} \right)^3 \right). \]
(c) We have
\[
c_1 = \begin{cases} 
\frac{1}{2} \left( (1 - \frac{3}{q} q)^2 - q^2 \right) + \frac{2}{3q} \left( q^3 - \left( \frac{4}{q} q \right)^3 \right) - \frac{1}{2} \left( 1 - \frac{1}{q} q \right)^2, & q \leq \frac{5}{8} \\
\frac{2}{3q} \left( \left( \frac{5}{12} + \frac{1}{3} q \right)^3 - \left( \frac{5}{12} q - \frac{1}{12} q^2 \right)^3 \right) - \frac{1}{2} \left( 1 - \frac{1}{q} q \right)^2, & q \in \left( \frac{5}{8}, \frac{3}{2} \right] \\
\frac{2}{3q} \left( \left( \frac{5}{12} + \frac{1}{3} q \right)^3 - \left( \frac{5}{12} q - \frac{1}{12} q^2 \right)^3 \right) - \frac{1}{q} \left( \frac{1 + \sqrt{1+16q}}{8} \right)^3, & q \in \left( \frac{3}{2}, 3 \right] \\
\frac{2}{3q} \left( \left( \frac{5}{12} + \frac{1}{3} q \right)^3 - \left( \frac{5}{12} q - \frac{1}{12} q^2 \right)^3 \right) - \frac{1}{5} q, & q > 3 
\end{cases}
\]
\[
c_2 = \begin{cases} 
\frac{1}{2} \left( \left( \frac{1}{q} q + 1 \right)^2 - q^2 \right) - \frac{1}{2} \left( 1 + \frac{1}{q} q \right)^2, & q \leq \frac{5}{4} \\
\frac{1}{q} \left( \frac{3}{q} q - \frac{1}{q} q^2 \right)^2, & q > \frac{5}{4} 
\end{cases}
\]
\[
r_1 = \begin{cases} 
\frac{2}{3q} \left( q^3 - \left( \frac{3}{q} q \right)^3 \right) + \frac{1}{2} \left( \left( \frac{1}{q} q + 1 \right)^2 - q^2 \right) - \frac{1}{2} \left( 1 + \frac{1}{q} q \right)^2, & q \leq \frac{5}{4} \\
\frac{2}{3q} \left( \left( \frac{2}{3} q + \frac{5}{12} \right)^3 - \left( \frac{2}{3} q - \frac{1}{12} \right)^3 \right) - \frac{1}{2} \left( 1 + \frac{1}{q} q \right)^2, & q \in \left( \frac{5}{4}, \frac{3}{2} \right] \\
\frac{2}{3q} \left( \left( \frac{2}{3} q + \frac{5}{12} \right)^3 - \left( \frac{2}{3} q - \frac{1}{12} \right)^3 \right) - \frac{1}{q} \left( \frac{1 + \sqrt{1+16q}}{8} \right)^3, & q \in \left( \frac{3}{2}, 3 \right] \\
\frac{2}{3q} \left( \left( \frac{2}{3} q + \frac{5}{12} \right)^3 - \left( \frac{2}{3} q - \frac{1}{12} \right)^3 \right) - \frac{4}{5} q, & q > 3 
\end{cases}
\]
and
\[
r_2 = \begin{cases} 
\frac{1}{2} \left( \left( \frac{1}{q} q + 1 \right)^2 - q^2 \right) - \frac{1}{2} \left( 1 + \frac{1}{q} q \right)^2, & q \leq \frac{5}{8} \\
\frac{1}{q} \left( \frac{3}{q} q - \frac{1}{q} q^2 \right)^2, & q > \frac{5}{8} 
\end{cases}
\]

Tedious but straightforward numerical calculations confirm that $c_1 < \min\{r_1, r_2\}$ and $c_2 < r_2$ for any given value of $q$. The results of Proposition 2 follow immediately.

\[
\square
\]

**Proof of Proposition 3.** The three statements in this proposition are based on the equilibrium qualities for various values of $\alpha$ obtained through simulations. In the accompanying graphs, the black and grey curves are firm 1’s and 2’s best response functions, respectively. The intersections of these curves yield Nash equilibrium qualities. The graph for $\alpha = 0.05$ is representative of case (i), $\alpha = 0.08$ of case (ii), and $\alpha = 0.5$ of case (iii).
Equilibrium qualities for $K=0.115$. The counterpart of the profit function in (14) is

$$
\pi_i(q_i, q_j) = \begin{cases} 
\frac{1}{q_i-q_j} \left( \frac{1}{3}(q_j - q_i) - \frac{1}{6} \right)^2, & q_i - q_j < -2 \\
\frac{1}{16} \sqrt{2(q_j - q_i)}, & q_i - q_j \in [-2, -1) \\
\frac{1}{2} \left( 1 - \frac{1}{6}(q_j - q_i) \right)^2, & q_i - q_j \in [-1, 0]\n
\frac{1}{2} \left( 1 + \frac{1}{6}(q_i - q_j) \right)^2, & q_i - q_j \in [0, 1] \\
\frac{9}{16} \sqrt{2(q_i - q_j) - 0.115}, & q_i - q_j \in [0, 1] \\
\frac{1}{q_i-q_j} \left( \frac{2}{3}(q_i - q_j) + \frac{1}{6} \right)^2 - 0.115, & q_i - q_j > 2
\end{cases}
$$

Our numerical simulations show that when $K = 0.115$ the following hold for the firms’ equilibrium quality choices.

(i) If $\alpha < 0.051$ then there are two asymmetric Nash equilibria, $(0, q^*)$ and $(q^*, 0)$, where $q^* > 5.35$ and decreases in $\alpha$.

(ii) If $\alpha \in [0.051, 0.091]$ then there is one symmetric and two asymmetric Nash equilibria, $(q^\dagger, q^\dagger)$, $(0, q^*)$, and $(q^*, 0)$, where $q^\dagger$ decreases from 1.64 to 0.92 and $q^*$ decreases from 4.35 to 2.42 as $\alpha$ increases from 0.051 to 0.091.

(iii) If $\alpha > 0.091$ then there is one symmetric Nash equilibrium $(q^\dagger, q^\dagger)$, where $q^\dagger < 0.92$ and decreases in $\alpha$. 

\[ \Box \]