NET Institute*

www.NETinst.org

Working Paper #19-04

September 2019

**Group Hug:
Platform Competition with User-groups**

Sarit Markovich
Kellogg School of Management, Northwestern University

Yaron Yehezkel
Coller School of Management, Tel Aviv University

* The Networks, Electronic Commerce, and Telecommunications (“NET”) Institute, <http://www.NETinst.org>, is a non-profit institution devoted to research on network industries, electronic commerce, telecommunications, the Internet, “virtual networks” comprised of computers that share the same technical standard or operating system, and on network issues in general.

Group Hug: Platform Competition with User-groups*

Sarit Markovich[†] and Yaron Yehezkel[‡]

September 2019

Abstract

We consider platform competition in the presence of small users and a user-group. One platform enjoys a quality advantage and the other benefits from favorable beliefs. We study whether the group mitigates the users' coordination problem –i.e., joining a low-quality platform because they believe that other users would do the same. We find that when the group is sufficiently large to facilitate coordination on the high-quality platform, the group may choose to join the low-quality one. When the group joins the more efficient platform it does not necessarily increase consumer surplus. Specifically, a non-group user benefits from a group with an intermediate size, and prefers a small group over a large group. The utility of a group user is also non-monotonic in the size of the group.

JEL Classification: L1

Keywords: network extremities, coordination

1 Introduction

In markets with network effects, platforms aim at attracting a large number of users as a large consumer base allows for more interactions across users and thus gives users

*For helpful comments and discussions we thank Toker Doganoglu, Gary Biglaiser, Bruno Julien, Massimo Motta, Yossi Spiegel, Mark Tremblay and seminar participants at the Bergamo 2018 Workshop on Advances in Industrial Organization, the St. Gallen 2018 IO Workshop, the 2019 Paris Conference on Digital Economics, the 2019 IIOC in Boston, the CRESSE 2019 in Rhodes, and the 2019 EARIE conference in Barcelona. For financial support, we thank the Collier Foundation, the NET Institute–www.NETinst.org, the Henry Crown Institute and the Eli Hurvitz Institute for Strategic Management.

[†]Kellogg School of Management, Northwestern University (s-markovich@kellogg.northwestern.edu)

[‡]Collier School of Management, Tel Aviv University (yehezkel@tauex.tau.ac.il)

higher benefits. Users would like to join the same platform that other users adopt in order to benefit from network effects. This dynamics may create a coordination problem and inefficiencies where users join the “wrong” platform, e.g., a low-quality platform, simply because they expect that other users will do the same. In such a case, each user is too small to affect the decisions of other users, and as market expectations are for all other users to join the low-quality platform, all users do the same.

Yet, in many markets, some users join a platform as a group rather than each user joining individually. Moreover, platforms compete on small users as well as large users, where the latter can affect the former’s decisions as to which platform to join. This raises the question of whether the presence of a user-group or a large user can mitigate users’ coordination failure. This paper studies platform competition between a high and a low-quality platform. Because of customers’ inertia or incumbency advantage, users expect that other users join the low-quality platform. A subset of users belong to a group that makes a collective decision about which platform to join. We ask when the presence of a user-group enhances efficiency, and how the size of the group affects profits and users’ surplus of users inside and outside the group.

User-groups are common in many markets for platforms. For example, when launching iTunes, Steve Jobs first approached Warner Music, and other big labels, like Universal and Sony. Each of these big labels brought with it contracts with a large number of artists that joined the iTunes platform as a group.¹ Other examples include marketplace lenders who aim at attracting both private and large, institutional investors. These platforms connect investors and borrowers. Institutional investors are organizations that pool capital from many smaller investors and choose how to, collectively, invest on behalf of these investors. Investor composition on lending platforms like LendingClub and Prosper has significantly evolved since the platforms’ inception in late 2000s. While, initially, investors were composed of only small private investors that were looking for new investment opportunities, nowadays institutional investors represent a large share of investors on these platforms.² Likewise, a large restaurant chain like McDonalds would likely make a collective decision to all its franchisees whether to join the Apple Pay platform. In contrast, small merchants, such as family-owned restaurants, make

¹<https://www.rollingstone.com/culture/culture-news/itunes-10th-anniversary-how-steve-jobs-turned-the-industry-upside-down-68985/>

²In the early days of marketplace lending, private investors were browsing borrowers and individually choosing the loans they want to fund and the amount they want to invest in each and every loan they fund. Institutional investors, in contrast, choose borrowers for the investors that are part of their fund and typically fund the entire loan.

an individual decision on whether to join Apple Pay. In the market for mobile operating systems, Apple and Google are competing on both small and large application developers. For example, EA Sports holds a high market share in the market for online sports gaming.³ Likewise, GetTaxi is attempting to democratize the taxi rides market and may attract individual drivers or contract with large taxi providing companies. Note that user-groups are not necessarily very large. For example, a student club may choose to join a peer-to-peer (P2P) payment network (e.g., Venmo) in order to simplify money transfer across the club members. Furthermore, the same type of effects may arise with a large user, as opposed to a user group. For example, accredited investors (investors with net worth above \$1M) may invest large amounts on a lending platform, facilitating the same type of network effects that institutional investors generate.

In general, user-groups create high value to a platform as the group creates network effects not only to its members but also to users outside of the group. The group's choice of which platform to join then may affect the platform's ability to attract other users that do not belong to the group. For example, a popular artist who is part of the Warner Music group attracts users to iTunes and thereby creates value for other artists that are part of the Warner Music group as well as for other independent artists that join the iTunes platform. As such, just as Jobs hoped, iTunes' contract with Warner Music affected individual artists' and listeners' decision to join iTunes. Similarly, institutional investors bring with them large amount of capital which is highly desirable for the lending platform. An institutional investor's decision to join a specific marketplace lending platform makes it more likely that small individual investors as well as borrowers would join that same lending platform. It is, therefore, likely that platforms would compete over attracting user-groups to join their network. The student club decision to join Venmo creates value to all club members who can now easily transfer money to one another but also creates value to other users who can now easily transfer and/or receive payments from this group of college students.

A network structure where some users join as part of a group while others join individually brings up some interesting questions. In particular, does the student club's decision to join Venmo as opposed to Zelle, for example, affect students that are not part of the club choice of which P2P payment network to join? While users want to join the platforms other users join, if the student club is relatively small and chooses the non-focal platform, other students may opt to join the other platform they believe

³For data on EA's market share, see: https://csimarket.com/stocks/compet_glance.php?code=EA.

most other users would adopt. Furthermore, does the presence of user-groups affect the likelihood that the higher quality, yet maybe not focal platform, becomes the dominant platform? That is, does an institutional investor choice of a lending platform makes it easier for the market to coordinate on the “better” platform? Does the presence of large application developers make it easier for the market to coordinate on the better mobile operating system? Intuitively, if the group size is small, it might not have any effect on other users’ decisions. Yet, a large group may affect the decisions of individual users. In the latter case, the question is whether, when the group anticipates that its choice would affect the choice of individual users, it chooses the platform with the better quality. This question is important for policy with regards to the size of such large users. In particular, should antitrust authorities allow mergers between users – such as application developers or institutional investors – into a large user-group with market power to affect the identity of the dominant platform? In markets for networks, mergers between users may have the welfare-enhancing effect of facilitating coordination on the right platform. At the same time, one must consider the effects of a large user-group on small users outside the group, as well as on individual users inside the group.

Another interesting question is the incentives to be part of the user-group. While the user-group may achieve better prices from the platforms that try to attract it, as the size of the group increases the value the group gets is split over a larger number of users—decreasing the value of being part of the group as well as decreasing the group incentives to accept additional users. Finally, it is not clear how the answers to these questions change once users can multi-home and join more than one platform.

To study these questions, we develop a model of platform competition in a market with network effects with two types of users: a set of users that join the platform as a group—a user-group, and individual users. We assume a game where platforms first compete over attracting the group and then set prices to attract the non-group users. Competition in prices implies that a platform may be willing to pay the user-group to become part of its network. In this setting, we examine the effect of the proportion of the user-group, out of the total size of the market, on market structure, profits, and market efficiency.

In order to capture the effect of network effects and users’ beliefs, we assume a market with two competing platforms where one has an advantage in terms of the value it offers at the outset while the other platform enjoys focality—meaning, users believe that it would be able to attract other users and become the dominant platform

in the market. As such, the two platforms differ in terms of the overall value they create and thus in terms of the value they are able and willing to share with the user-group. Note that since the platforms compete first over the user-group and then over the rest of the users, beliefs about focality affect only the decision of the non-group users.

We find that there is a threshold group size where groups that are smaller than the threshold cannot affect the equilibrium winning platform. The intuition is simple. As long as the group is a small proportion in comparison to the size of the market, it does not create large enough network effects to attract the non-group users and thus to affect their decision. Thus, the focal platform always wins the non-group users. If, however, the user group is larger than the threshold, the group creates large network effects that affect the decision of the non-group users. Specifically, if the non-focal platform is able to attract the group, it also wins the non-group users. In this latter case, we say that the group is *pivotal*. The cutoff above which the group is pivotal decreases with the quality advantage that platform enjoys—the larger the quality gap across the platform, the smaller the group size needed to make the group pivotal.

These results imply that a proportionally large user-group has the ability to facilitate coordination on the more efficient platform and thereby solve the coordination failure. This, however, does not necessarily imply that a pivotal group joins the high-quality platform. We find that under some market conditions, a pivotal group chooses to join the low-quality focal platform, dragging non-group users to join the low-quality platform as well. Only when the pivotal group is sufficiently large, it facilitates coordination on the high-quality platform. The intuition for this result is that the focal platform can extract the network effects that non-group users gain from both group users and non-group users. However, the non-focal platform can only extract the former value. Hence, when the group is not too large, the focal platform can transfer this benefit to the group, making it more beneficial for the group to join the low-quality platform, on the expense of the non-group users. When size of the group is relatively large, the value the focal platform can transfer to the group is not large enough to attract the group. In this case, the group joins the more efficient platform.

A large group may not necessarily increase consumer surplus. Specifically, we find that the utility of a non-group user is increasing and then decreasing with the proportion of the group. In total, a non-group user prefers a small group over a large group, and prefers the most an intermediate group size. The utility of a group user is also non-monotonic in the proportion of the group. If the group is large, a non-group user that

joins the group decreases the utility of each group user. Moreover, group users may gain lower utility than non-group users. Combining the utility of group and non-group users, an increase in the size of the group has conflicting effects on total consumers' surplus and the profits of the two platforms. Yet, total welfare increases discontinuously with the proportion of the group, at the point where the group switches from the low-quality to the high-quality platform.

Finally, we study factors that affect the group's ability to help the high-quality, non-focal platform win. First, when the group can multi-home, in equilibrium the group joins both platforms but doing so makes it more difficult for the high-quality platform to win the market. Second, a group that can directly affect the beliefs of non-group users and consequently the platforms' focal position is more likely to choose the high-quality platform. Third, when the group joins a platform before non-group users but the group cannot coordinate the decisions of its members, the group may join the low-quality platform regardless of its size.

For policy, these results suggest that user mergers, or large users, can indeed mitigate users' coordination problem. However, antitrust authorities should not adopt a too lenient approach towards user-merger for two reasons. First, a large user group may not facilitate coordination, even when it has the ability to do so. Second, a large user group can indirectly extract utility from non-group users, through the subsidy offered by the platform, which in total can harm consumers. Qualitatively, intermediate sized user-groups seem to have positive effects on both non-group and group users. However, large user groups can be harmful, sometimes to both types of users.

Our paper contributes to the literature on platform competition with a coordination problem, where consumers need to form expectations concerning the decisions of other consumers. Katz and Shapiro (1986) study platform competition in a sequential game. They assume that consumers coordinate on the Pareto outcome and thus their model does not exhibit coordination failures. Caillaud and Jullien (2001; 2003) assume that an incumbent platform benefits from a focality advantage: if there is an equilibrium in which consumers join the incumbent's platform, then consumers play this equilibrium, even if there is a second equilibrium in which consumers join the entrant.⁴ They find that the focal platform earns positive profits, even though at the outset the platforms are homogeneous. Hagiu (2006) extends the focality approach to a setting where plat-

⁴In the terminology of Caillaud and Jullien, the incumbent benefits from "favorable beliefs": consumers expect other consumers to join it, whenever it is rational for them to do so.

forms first compete on one of the sides and then compete on the other side. Jullien (2011) assumes a multi-sided market where one of the platforms offers a superior base quality. He finds that a focal platform can dominate the market even when competing against a higher-quality platform. This result occurs when the focality advantage outweighs the platform’s quality advantage. Hałaburda and Yehezkel (2013) consider focality advantage when users are ex-ante uninformed about their benefits from joining a platform, and become privately informed once they join. Hałaburda and Yehezkel (2016) and Hałaburda and Yehezkel (2019) extends the concept of focality to a partial degree of focality. Accordingly, users adjust their beliefs concerning the winning platform depending on the price gap between the two platforms.

In the context of a dynamic game, Halaburda et al. (2018) consider a repeated platform competition between a high and a low quality platform. Users take the winning platform in the previous period to be focal in the current period. They find that dynamic considerations may increase the ability of a low-quality platform to maintain its focal position. Biglaiser and Crémer (2018) consider dynamic platform competition on two groups of consumers that differ in their network effects. At the beginning of each period, users are attached to the platform that they joined in the previous period, and engage in a sequential, non-cooperative migration path to the winning platform in the current period.

A common feature of all of the above papers is that users are too small to affect the winning platform. Each user takes the focal position, and hence the decisions of other users, as given. We contribute to this literature by considering platform competition in a market with both a user-group (or a large user) and a set of small users.

Several papers looked at platform competition with an large user. Biglaiser et al. (2019) survey reasons for incumbency advantage in platform competition, and speculate that a pivotal buyer may be a mitigating factor on a platform’s incumbency advantage. Our paper contributes to this idea by identifying how a pivotal buyer may, in fact, preserve the incumbency advantage of the low-quality platform. Akerlof et al. (2018) consider a market with network effects, allowing both for a focality advantage and product differentiation, such that a non-focal platform gains a small, yet positive share of the market. In the context of a monopolistic platform, they show that an “influencer” buyer, that can affect the decisions of other buyers, can be in a pivotal position. We contribute to this paper by considering competition for the pivotal buyer, who internalizes the ability to affect the winning platform. Carroni et al. (2019) consider platform

competition on small users and a “superstar”, which provides higher network effects than the small users. Their paper focuses on horizontally differentiated platforms and thus do not examine coordination failures like excess inertia where the inefficient platform wins the market. Consequently, in their model, small users always benefit from the presence of a superstar. Our paper contributes to this idea by showing that when platform competition involves a coordination problem, a very large user may harm small users.

A second strand of relevant literature to our paper concerns entry deterrence and naked exclusion. Rasmusen et al. (1991) and Segal and Whinston (2000) consider competition between an incumbent and a more efficient entrant. The latter needs a sufficient scale (i.e., number of buyers) in order to enter the market. This creates a buyers’ coordination problem: when buyers expect other buyers to join the incumbent, the entrant cannot profitably enter the market. In a closely related paper to ours, Karlinger and Massimo (2012) study naked exclusion when buyers differ in size. The paper focuses mainly on small buyers, such that no individual buyer is large enough to provide the entrant with the sufficient scale needed for entry. In this case, there are exclusionary equilibria in which the inefficient incumbent dominates the market. Yet, the paper finds that when there is a sufficiently large buyer that can meet the entrant’s scale requirement, or when a group of buyers can take a joint decision, such inefficient exclusionary equilibria do not arise. Consequently, in the context of naked exclusion, buyer-groups have pro-competitive effects. Our paper contributes to these findings by showing that in the alternative context of platform competition, a pivotal user-group may not choose the efficient platform. Moreover, an increase in the proportion of the user-group may decrease the utilities of users in and outside the group. The intuition behind the difference in results is that in the context of naked exclusion there are no direct network effects. Buyers’ utility then depends on the decisions of other buyers only through the effect of other buyers’ decision on the identity of the winning platform. In contrast, under platform competition with network effects, users gain direct positive network effects as more users join the platform. This enables the focal platform to win the group and non-group users even when the group is pivotal. More specifically, in our settings, the group internalizes the effect its decision has on non-group users as it can extract some of this value through the price it pays. In the case of naked inclusion, there are no incentives for the group to internalize the effect of its decision on other buyers.

2 The model

Consider two platforms, platform A and platform B , and a mass 1 of identical users. The utility of a consumer from joining platform i ($i = A, B$) is $V_i(n_i) - p_i$, where p_i is the price of platform i and n_i is the number of users in platform i . $V_A(n)$ and $V_B(n)$ are two (different) continuous and twice differentiable functions with the following features:

Assumption 1: $V'_i(n) > 0$

Assumption 2: $V_B(n) > V_A(n)$, $\forall n \in [0, 1]$

Assumption 3: $V_{A1} \equiv V_A(1) > V_{B0} \equiv V_B(0)$

Assumption 1 indicates that there are positive network effects: a consumer benefits the more other users are joining the same platform as the consumer does. For now, we allow for $V_i(n)$ to be concave or convex. Later on, we show that our main results hold as long as $V_i(n)$ is not too concave. Assumption 2 means that given the same number of users on each platform, platform B offers higher value than platform A . This higher value can be due to superior quality, such that $V'_B(n) = V'_A(n)$, and/or from platform's B superior ability to connect between users (i.e., $V'_B(n) > V'_A(n)$). Yet, by assumption 3, a consumer prefers to join platform A when all other users are joining it, over joining an empty platform B . That is, network effects (meeting other members) are more important to users than the quality gap between the two platforms. Finally, we normalize $V_A(0) = 0$.

Out of the mass of 1 users, a fraction x ($0 \leq x \leq 1$) belong to a user-group. The remaining users, $1 - x$, are “non-grouped”. We can interpret x as a group of small users that makes a collective decision, such as institutional investors in marketplace lending. Alternatively, x can measure the relative size of a large user, such as a large application developer in the market for mobile operating systems, or a potential merger between several small application developers.⁵ In order to focus on the effect of users making a collective rather than an individual decision, we assume that the per-user value of joining a platform is the same for users in the user-group and non-group users; i.e., $V_i(n)$.⁶ Hence, the utility of the entire group from joining platform i is

⁵Notice that we measure the size of the group in proportional terms. Alternatively, one could assume a market with n_G group members and n_{NG} non-group users where $n_G + n_{NG} \leq 1$. In this case, however, changes in the size of the group are confounded with changes in network effects. Since we are interested in separating these two effects, we assume that $n_G + n_{NG} = 1$. Moreover, looking at the the proportion of the group enables us to study the effect of a user-merger.

⁶In our motivating examples, members of the group may have higher utility than regular users. For example, in marketplace lending, institutional investors may enjoy higher returns as compared to

$xV(n_i) - p_i^G$, where p_i^G is the price that platform i charges the group. We assume that x is exogenous, that users (both the group and non-group) can only join one platform (i.e., “single-home”),⁷ and that the group cannot divide its members across the two platforms.

The timing of the model is as follows. In the first stage, the two platforms compete by simultaneously setting prices to the group, p_A^G and p_B^G , and the group chooses a platform. We denote the group’s decision by $J = \{A, B\}$. In the second stage, the two platforms compete by setting prices to the non-group users, p_A and p_B . In the third stage, non-group users observe J , p_A and p_B , and decide simultaneously and non-cooperatively which platform to join. Notice that we assume that the platforms have market power on both the group and non-group users. Our qualitative results in sections 3 and 4 hold when the group has the market power to make take-it-or-leave-it offers to the two platforms (or any Nash-bargaining combination).

As is typically the case when markets exhibit network effects, expectations play an important role. Consequently, given some values of J , p_A and p_B , the third stage may have two equilibria: one in which each non-group user expects that all other users join A , in which case everyone joins A . In the second equilibrium, for the same values of J , p_A and p_B , all non-group users join platform B , expecting that other users will do the same. Users play one of these equilibria, based on their beliefs concerning the platforms’ ability to attract other users.

An incumbent platform, or a platform that dominated the market in the past, may benefit from favorable beliefs, as users may expect it to maintain its dominance and thus for other users to join it. These beliefs may make it difficult for a new, yet of higher quality, platform to gain a foothold in the market.⁸ In order to model such beliefs advantage, in what follows, we assume that platform A is *focal*: whenever both outcomes are possible, non-group users join platform A , expecting that other users will do the same. Users join platform B only if it is a dominant strategy for them to do so. We assume that platform A ’s focal position is independent of whether the group joined

independent investors as they are more diversified or may have better information on the loans. Still, we assume that the platform offers identical value to the group and non-group users in order to focus on the net strategic effect of the size of the group.

⁷We relax these two assumptions in section 6

⁸We focus on outcomes in which all users join the same platform. All equilibria in which some users join platform A while others join platform B are not stable. To see why, notice that in such an equilibrium all users have to be indifferent between joining A or B . Hence, if a consumer of mass ε switches from platform i to j , then now all users gain a higher utility in platform j than in i and all users will switch.

A or B . That is, the group is too small to affect users' beliefs.

To illustrate the role of focality, consider a benchmark case in which all users are non-group: $x = 0$. In stage 3, when users decide which platform to join given p_A and p_B , there is an outcome in which all users join platform A if:

$$V_{A1} - p_A \geq V_{B0} - p_B \iff V_{A1} - V_{B0} \geq p_A - p_B. \quad (1)$$

Likewise, there is an outcome in which all users join platform B if

$$V_{B1} - p_B \geq V_{A0} - p_A \iff p_A - p_B \geq V_{A0} - V_{B1}. \quad (2)$$

Since $V_{A1} > V_{A0} = 0$ and $V_{B0} < V_{B1}$, $V_{A1} - V_{B0} > V_{A0} - V_{B1}$, implying that for $V_{A1} - V_{B0} > p_A - p_B > V_{A0} - V_{B1}$, both outcomes are possible. By the assumption that platform A is focal, users join A if $V_{A1} - p_A \geq V_{B0} - p_B$, and join platform B if $V_{A1} - p_A < V_{B0} - p_B$, expecting that others will do the same. Focality can emerge because of incumbency advantage and users' inertia. If platform A was the first in the market, users may expect that other users will continue to join the old platform, even though there are better, new alternatives. These beliefs can be rational, given that high network effects keep users on platform A .⁹

We demonstrate the coordination problem by looking at the case where platform A is focal and $x = 0$. In this case, platform A always wins the market. To see why, platform A charges p_A such that equation (1) holds in equality, while the losing platform B charges the lowest price possible, $p_B = 0$. Substituting $p_B = 0$ in (1), platform A charges $p_A = V_{A1} - V_{B0} > 0$ and earns positive profit, where the inequality holds by Assumption 3. In a putative equilibrium in which the non-focal platform B wins, if such an equilibrium were to exist, platform B needs to charge p_B such that (1) holds in equality given $p_A = 0$, but then $p_B = -(V_{A1} - V_{B0}) < 0$, again from Assumption 3, implying that platform B cannot profitably win the market. We summarize these results in the following lemma:

Lemma 1. *(with no group, the inefficient platform always wins) When all users are non-group ($x = 0$), the low-quality but focal platform A wins the market and earns $V_{A1} - V_{B0}$.*

Intuitively, focality means that platform A can collect the users' network effects

⁹For a formal definition of focality, and its potential sources, see Hałaburda and Yehezkel (2019).

because users expect that other users join A . Platform B can only collect its quality advantage. Yet, network effects are more important to users than the quality advantage, by Assumption 3, resulting in an equilibrium in which platform A wins. That is, the inability of users to coordinate their choices creates a miss-coordination in which they all join the inefficient platform. This raises the question of when and how the group can correct this market failure.

3 Competition on the non-group

We start by solving the second and third stages: platforms' competition on the non-group users, given that the group already joined a platform. The main result of this section is that a large group can determine which platform wins the non-group users. Yet, when the group is relatively small, the focal platform A always wins the non-group users. All proofs can be found in the appendix.

We study the two cases where the group joins platform A and platform B in turn. Suppose first that $J = A$. An equilibrium in which platform A wins the non-group users satisfies the following conditions. First, prices for the non-group users are :

$$V_{A1} - p_A \geq V_{B0} - p_B, \quad p_B = 0 \implies p_A = V_{A1} - V_{B0}. \quad (3)$$

That is, platform A charges the highest price that ensures that non-group users prefer joining the focal platform A over joining platform B , given that all other non-group users (and all the group users) are on platform A . Platform B charges the lowest price that ensures non-negative profits. The second condition is that platform A earns positive profit from attracting the non-group users. Let $\pi_i(x; J) = p_i(1 - x)$ denote the profit of platform i from the non-group users given x and the decision of the group, J . Using (3), we have:

$$\pi_A(x; A) = (1 - x)p_A = (1 - x)(V_{A1} - V_{B0}) > 0, \quad (4)$$

where the inequality follows from Assumption 3. Hence, given $J = A$, there is an equilibrium in which platform A wins the non-group. To see that given $J = A$ there is no equilibrium in which platform B wins the non-group, note that if such equilibrium were to exist, $p_A = 0$ and $V_{A1} - p_A = V_{B0} - p_B$, implying that $p_B = V_{B0} - V_{A1}$ and platform B earns: $\pi_B(x; A) = (1 - x)(V_{B0} - V_{A1}) < 0$. Hence, when $J = A$, platform

A always wins the non-group.

Suppose now that the group joins platform B . An equilibrium in which platform A wins the non-group users satisfies the following conditions. First, platform A charges the highest price possible that induces the non-group users to join it, given that they know that the group joined platform B , yet still expect that all other non-group users join platform A . Unable to attract the non-group users, platform B again charges 0. Hence:

$$V_A(1-x) - p_A \geq V_B(x) - p_B, \quad p_B = 0 \implies p_A = V_A(1-x) - V_B(x). \quad (5)$$

As before, the equilibrium requires that platform A earns positive profit from the non-group users: $\pi_A(x, B) = (1-x)p_A > 0$, where using (5):

$$\pi_A(x, B) = (1-x)(V_A(1-x) - V_B(x)). \quad (6)$$

Likewise, in an equilibrium in which platform B wins, it charges and earns, respectively,

$$p_B = V_B(x) - V_A(1-x), \quad \pi_B(x, B) = (1-x)(V_B(x) - V_A(1-x)). \quad (7)$$

Hence, platform A wins iff $V_A(1-x) \geq V_B(x)$. Let $\hat{x}$ denote the solution to:

$$V_A(1-\hat{x}) = V_B(\hat{x}). \quad (8)$$

The following lemma summarizes the results in this section:

Lemma 2. (*The effect of x on the winner of the non-group*) When $J = A$, there is a unique equilibrium where platform A always wins the non-group users and earns from the non-group $\pi_A(x; A) = (1 - x)(V_{A1} - V_{B0})$.

When $J = B$, there is a threshold, $\hat{x}$, where $\hat{x}$ is the solution to $V_A(1 - \hat{x}) = V_B(\hat{x})$, and $0 < \hat{x} < \frac{1}{2}$ such that:

1. when $x \in [0, \hat{x}]$, platform A wins the non-group and earns from the non-group:

$$\pi_A(x; B) = (1 - x)(V_A(1 - x) - V_B(x));$$
2. when $x \in [\hat{x}, 1]$, platform B wins the non-group and earns from the non-group:

$$\pi_B(x; B) = (1 - x)(V_B(x) - V_A(1 - x)).$$

Lemma 2 shows that a small group has no effect on the winning platform: platform A wins the non-group users due to its focal position, regardless of whether the group joins it or not. Yet, once the group is sufficiently large, it becomes *pivotal* in the sense that the group determines the winning platform. By choosing to join the non-focal but more efficient platform B , the group provides platform B with a network effect advantage to win the non-group users. That is, the group creates large enough network effects such that the value for a single non-group user from joining platform B is larger than the value from platform A when only the non-group users join it— $V_B(x) > V_A(1 - x)$.

The result that, depending on its size, the group may solve the inefficiency created by platform A 's focal position raises the question: under what market conditions the group makes the efficient choice of joining the higher quality platform B . We study this below.

4 Competition on the group

Consider now the first stage, where platforms compete on attracting the group. The group joins the platform that provides it with the highest benefit as a group, $xV_i(n_i) - p_i^G$. Hence, when making a decision, the group takes into account the platforms' qualities, prices, and how the group's decision affect the decision of the non-group. The latter case depends on whether the group's size is smaller or larger than $\hat{x}$. We study each possibility in turn.

The main results of this section is that when the group is not pivotal—i.e., it cannot affect the winning platform—it always chooses the inefficient outcome and joins platform

A . Nevertheless, being large enough to become pivotal does not guarantee an efficient outcome. Specifically, an intermediate sized group may still choose to join the inefficient platform A . It is only when the group is large enough that the group makes the efficient decision

4.1 The group is not pivotal: $x < \hat{x}$

Suppose first that the group is too small to affect the winning platform: platform A always wins the non-group. An equilibrium in which platform A wins the group has to satisfy the following conditions. First, platform B charges the lowest price that ensures non-negative profit and platform A charges the highest price possible that still compels the group to join it, given that it also wins the non-group:

$$xV_{A1} - p_A^G \geq xV_B(x) - p_B^G, p_B^G = 0 \implies p_A^G = x(V_{A1} - V_B(x)). \quad (9)$$

Second, it is advantageous for platform A to win the group: $\pi_A(x; A) + p_A^G \geq \pi_A(x; B)$. Denoting platform i 's total profit when $J = i$ by $\Pi_i(x; i) \equiv \pi_i(x; i) + p_i^G$ and substituting (4) and (9) into $\Pi_A(x; A) = \pi_A(x; A) + p_A^G$, we have:

$$\Pi_A(x; A) = V_{A1} - (1 - x)V_{B0} - xV_B(x), \quad (10)$$

In this case, there is an equilibrium in which platform A wins the group iff: $\Pi_A(x; A) > \pi_A(x; B)$. The following proposition shows that this is always the case:

Proposition 1. (*When not pivotal, the group always chooses the inefficient platform*) Suppose that $x < \hat{x}$. Then, there is a unique equilibrium in which platform A wins the group and non-group users. Platform A charges the group and non-group users: $p_A = V_{A1} - V_{B0}$ and $p_A^G = x(V_{A1} - V_B(x))$, respectively, and earns $\Pi_A(x; A) = V_{A1} - (1 - x)V_{B0} - xV_B(x)$ while platform B earns 0.

Intuitively, since focality implies that non-group users expect other non-group users to join platform A , platform A can offer the group a higher value than platform B . The group does not create large enough network effects and thus cannot help platform B win the non-group. These network effects are also not large enough for its group members to prefer the non-focal platform, and is therefore better off joining A .

4.2 The group is pivotal: $x \geq \hat{x}$

Suppose now that the group is large enough to determine the winning platform: $x \geq \hat{x}$. In this case, the group gains a utility of $xV_{i1} - p_i^G$ from joining platform i .

Consider an equilibrium in which platform A wins the group (and consequently the non-group). The lowest price that platform B is willing to charge the group is its profit from winning the non-group. Platform A charges the highest price possible that induces the group to join it, given that the non-group users will follow. Hence:

$$xV_{A1} - p_A^G \geq xV_{B1} - p_B^G, \quad p_B^G = -\pi_B(x, B). \quad (11)$$

Notice that the group gains a higher gross utility from joining platform B : $xV_{B1} > xV_{A1}$. However, the group would join platform A if platform A sets a sufficiently low price. Substituting (7) into (11),

$$p_A^G = -(1-x)(V_B(x) - V_A(1-x)) - x(V_{B1} - V_{A1}). \quad (12)$$

Note that both platforms set negative prices for the group. The logic is similar to the “divide-and-conquer” strategy (Caillaud and Jullien (2003)), where platforms compete in subsidizing one set of users in order to attract another set. Here, platforms compete on attracting the group because the group determines the platform that wins the non-group.

The second condition for an equilibrium in which platform A wins is that platform A earns positive total profit. Substituting (4) and (12) into $\Pi_A(x; A) = \pi_A(x; A) + p_A^G$,

$$\Pi_A(x; A) = (V_{A1} - V_{B0}) - x(V_{B1} - V_{B0}) - (1-x)(V_B(x) - V_A(1-x)). \quad (13)$$

Using similar logic, the equilibrium in which platform B wins the group and the non-group satisfies $p_A^G = -\pi_A(x; A)$ and $xV_{A1} - p_A^G = xV_{B1} - p_B^G$, hence:

$$p_B^G = x(V_{B1} - V_{B0}) - (V_{A1} - V_{B0}). \quad (14)$$

Notice that while $p_A^G < 0$, p_B^G can be negative (if x is close to $\hat{x}$) or positive (if x is sufficiently close to 1). Intuitively, once the group size is close to 1, the superior utility that platform B offers the group, xV_{B1} , is sufficiently high to enable platform B to attract the group with a *positive* price, even though platform A charges the

group a negative price. An equilibrium in which platform B wins exists if $\Pi_B(x; B) = \pi_B(x; B) + p_B^G > 0$. Substituting (7) and (14) into $\Pi_B(x; B) = \pi_B(x; B) + p_B^G$, and rearranging the terms we get that $\Pi_B(x; B) = -\Pi_A(x; A)$.

Hence, if (13) is positive, there is a unique equilibrium in which platform A wins the group and the non-group users. Otherwise, there is a unique equilibrium in which platform B wins. To evaluate the sign of (13), we start with the two extreme cases of $x = \hat{x}$ and $x = 1$:

Proposition 2. (*The group is pivotal*) Suppose that $x > \hat{x}$, then

1. if x is close to $\hat{x}$, $V_A(n)$ is convex or linear in n , and $V_{B1} - V_{B0} \leq V_{A1}$, there is a unique equilibrium in which platform A wins the group and the non group .
2. if x is close to 1, there is a unique equilibrium in which platform B wins the entire market.

Proposition 2 shows that when the group is large enough to determine the identity of the winning platform but not too large, the outcome may be inefficient as the group may choose to adopt the low-quality platform, A . This result holds when $V_A(n)$ is convex or linear in n and $V_{B1} - V_{B0} \leq V_{A1}$, and may also hold when $V_B(n)$ is concave in n as long as $V_B(n)$ is not too concave, and $V_{B1} - V_{B0} > V_{A1}$ as long as the gap between $V_{B1} - V_{B0}$ and V_{A1} is not too large. The market ends up with the efficient outcome only when the group is substantially large, in which case it chooses the high-quality platform B .

The intuition behind this result is nicely represented by the following equation, which results from rearranging terms in equation (13).

$$\underbrace{(1-x)(V_{A1} - V_{B0})}_{A's \text{ gain from non-group}} + \underbrace{xV_{A1}}_{\text{Group's gain at } A} < \underbrace{(1-x)(V_B(x) - V_A(1-x))}_{B's \text{ gain from non-group}} + \underbrace{xV_{B1}}_{\text{Group's gain at } B} . \quad (15)$$

The left hand side in (15) represents the value to A from winning the group—the network effect A extracts from the non-group users—and the value the group gains from adopting platform A . Together these represent the joint value for the platform and the group, if A wins the market. Similarly, the right hand side represents the joint value to the platform and the group, if B wins. Equation (15) then implies that for a platform

to win the market it must be the case that the joint value on that platform is higher than the joint value on the competing one. The group's base utility is always higher when the group joins platform B : $xV_{B1} > xV_{A1}$. Still, platform A may gain more from the non-group as its focal position enables it to collect the utility of each non-group user from meeting other non-group users as well as from meeting group users. In contrast, the non-focality of platform B implies that it can only collect the non-group's utility from meeting the group. By joining platform A , the pivotal group drive non-group users to join platform A as well, which can collect their network effects and share them with the group by offering the group a larger subsidy: $-p_A^G > -p_B^G$.

The intuition for the conditions in Proposition 2 are the following. First, when the number of non-group users is large enough (i.e., x is close to $\hat{x}$), the focality advantage of platform A dominates platform B 's superior quality and A can attract the group. If, however, the proportion of the non-group is small, platform A cannot extract enough network effects to compensate the group for the lower quality A offers. Second, a convex $V_A(n)$ implies that marginal network effects are increasing with n . Consequently, given that at least x users – the group – join platform A , the marginal network effects from an additional $1-x$ users – the non-group – should be sufficiently large to provide platform A with a sufficient network effect advantage. In contrast, when $V_B(n)$ is highly concave, the marginal network effect from the non-group may be too small to sustain platform A 's advantage. The importance of the condition $V_{B1} - V_{B0} \leq V_{A1}$ has similar intuition. This condition implies that total network effect on platform A , $V - V_A(0)$ (recall that we set $V_A(0) = 0$), should not be much lower than the the total network effects on platform B , $V_{B1} - B_{B0}$. Otherwise, platform A 's advantage, which emerges from its focal position through the network effects, may be too small for it to win.

Proposition 2 details the equilibrium in the extremes: when x is close to $\hat{x}$ and when x is close to 1. The switch from an equilibrium where platform A wins on one end of the range to an equilibrium where platform B wins on the other end, raises the question of whether the range of $[\hat{x}, 1]$ contains a unique threshold value of x such that platform A wins when x is below the threshold and platform B wins otherwise. Alternatively, it is possible that there are multiple cutoffs such that there are several ranges within this range where each platform wins.

Using the definition of $\Pi_A(x; A)$, let $\tilde{x}$ denote the solution to:

$$(V_{A1} - V_{B0}) - \tilde{x}(V_{B1} - V_{B0}) - (1 - \tilde{x})(V_B(\tilde{x}) - V_A(1 - \tilde{x})) = 0. \quad (16)$$

The following proposition provides a condition for a unique cutoff value of $\tilde{x}$:

Proposition 3. (*Minimum group size for efficient platform to win*) Suppose that $x > \hat{x}$ and the conditions in Proposition 2 hold. Then, there is a unique cutoff, $\hat{x} < \tilde{x} < 1$, such that, if $\pi_B(x; B)$ is concave in the number of non-group users :

$$-2(V'_B(x) + V'_A(1-x)) + (1-x)(V''_B(x) - V''_A(1-x)) < 0, \quad (17)$$

platform A wins the group and non-group when $x < \tilde{x}$ and platform B wins when $\tilde{x} < x$.

Proposition 3 shows that when platform B 's revenue function from serving the non-group (given $J = B$) has the standard concavity feature, the model has a unique cutoff in the size of group (or non-group) users such that platform B wins if the size of the group (non-group) is larger (smaller) than this cutoff. From (7), an increase in the number of non-group users (a decrease in x) decreases the price that platform B charges them and therefore has a conflicting effect on total revenues. as a result, firm B 's revenues are concave in the number of non-group users where the increase in the number of non-group users - quantity effect - is stronger (weaker) than the decreased in price- price effect when the number of non-group members is large (small).¹⁰ The first term in (17) is negative because $V'_i(n) > 0$. Therefore, (17) always holds when $V_i(n)$ are linear and also holds when $V_A(n)$ ($V_B(n)$) is convex (concave), or not too concave (convex).¹¹

Note that equations (15) and (17) imply that the threshold $\tilde{x}$ does not depend on whether the group or the platform have the bargaining power in the negotiation. That is, just like the Nash bargaining outcome, $\tilde{x}$ is determined by the joint value of the group and the platform and bargaining would only affect the share each party can capture.

Corollary 1. (*$\tilde{x}$ does not depend on bargaining power*) The minimum group size needed for the group to choose the efficient outcome, $\tilde{x}$, does not depend on which party has the negotiating power.

In what follows, we assume that the conditions in Propositions 2 and 3 hold. To conclude this section, we find that for $x \in [0, \hat{x}]$, the group cannot affect the winning

¹⁰To see how this condition translates to the standard concavity assumption, let $Q \equiv 1 - x$ denote the number of non-group users and $P(Q) \equiv V_B(1 - Q) - V_A(Q)$ denote the price charged by platform B from the non-group, by replacing Q and $x = 1 - Q$ into (2). Then, platform B 's revenues as a function of Q are $QP(Q)$, and they are concave when $2P'(Q) + QP''(Q) < 0$, which is consistent with (17).

¹¹Using numerical simulations, we find that condition (17) holds, for example, when $V_A(n) = \lambda n^\alpha$ and $V_B(n) = V + \lambda n^\alpha$, at least when $0 < V < \lambda < 1$ and $0 < \alpha \leq 2$.

platform and platform A wins. For $x \in [\hat{x}, \tilde{x})$, the group is pivotal, yet, joins platform A . When $x \in [\tilde{x}, 1]$, the group is still pivotal and chooses platform B .

4.3 Example

To illustrate the main results of this section, suppose in this subsection that $V_A(n_A) = \lambda n_A$ and $V(n_B) = V + \lambda n_B$. The parameter λ represents the network effect and V the relative quality advantage platform B offers. We assume that $0 < V < \lambda$ such that Assumptions 1 – 3 hold.

Given this functional form, $\hat{x} = \frac{1}{2} - \frac{V}{2\lambda}$ and $\tilde{x} = 1 - \frac{1}{4\lambda} \left(V + \sqrt{(8\lambda + V)V} \right)$. We start by looking at how platform B 's quality advantage affects the thresholds $\hat{x}$ and $\tilde{x}$ (see Figure 1). If $V = 0$, $\hat{x} = \frac{1}{2}$. In this case, both platforms offer the same quality. Platform A 's focality then implies that as long as the group is smaller than half the mass of users, non-group users will always prefer platform A over platform B . Once the group is larger than half the mass of users, it is large enough to “override” platform A 's focality and become pivotal. As platform B 's quality advantage increases ($V \uparrow$), $\hat{x}$ decreases and reaches zero when $V = \lambda$. It is easy to show that $\hat{x}$ increases with λ —as the focality advantage increases, a larger group is needed to make the group pivotal.

The threshold $\tilde{x}$ starts at 1 when $V = 0$ and reaches 0 when $V = \lambda$. Moreover, $\tilde{x}$ decreases in V —as platform B 's advantage increases, the threshold required for B to win the market decreases. In contrast, as the network effect becomes more important ($\lambda \uparrow$), it becomes harder for B to win the market so $\tilde{x}$ increases in λ .

As Figure 1 shows, as the quality gap between the platforms increases, the range within which the group is pivotal yet chooses the inefficient platform, $[\hat{x}, \tilde{x})$, becomes smaller and the range of group sizes that result in an efficient choice of platform B increases.

5 How the size of the group affects profits and users' surplus

This section studies how the ratio of the group, x , affects the platforms' profits and users' utility. Recall that x measures the ratio of group users out of the total number of users, and not their absolute size. An increase in x implies that a marginal user switches from the non-group to the group. This modeling enable us to study how

Figure 1: $\hat{x}$ and $\tilde{x}$ as a function of V

a user-merger (e.g., a merger between two application developers) affects individual utilities and welfare.

The main conclusion of this section is that a large group may not always be beneficial to users. In particular, the utilities of both a non-group and a group user are non-monotonic in the proportion of the group.

Note that while the qualitative results in Section 4 do not depend on the platforms' bargaining power over the group, the welfare analysis is sensitive to this assumption. We focus on the case where the platforms make offers to the group in order to give the non-group users the same level of market power as the group users. Hence, when the marginal user joins the group, his bargaining power over the platforms remains the same as a non-group user.

The next subsection studies the effect of an increase in x on an individual non-group user. We then look at the effect of x on an individual group user. Finally, we show how x affects total consumer's surplus and firms' profits.

5.1 The effect of the size of the group on an individual non-group user

The utility of each non-group user is $u(x) = V_{A1} - p_A$ if A wins, and $u(x) = V_{B1} - p_B$ if B wins, where p_A is given by (3) and p_B is given by (7). Putting this together, we get the following utility function for the non-group users:

$$u(x) = \begin{cases} V_{B0}, & \text{if } x \in [0, \tilde{x}), \\ V_{B1} - (V_B(x) - V_A(1 - x)), & \text{if } x \in [\tilde{x}, 1]. \end{cases} \quad (18)$$

The following proposition describes how x affect the utility of a non-group user:

Proposition 4. (*Effect of the proportion of the group on an individual non-group user*)

1. When $x \in [0, \tilde{x})$, changes in the proportion of the group do not affect the utility of a non-group user
2. At $x = \tilde{x}$, there is a discontinuous climb in the utility of a non-group user
3. When $x \in [\tilde{x}, 1]$, the utility of a non-group user decreases with the proportion of the group and equals 0 at $x = 1$.

Figure 2 illustrates the general results of 4 using the example in subsection 4.3. Proposition 4 shows that when the group joins platform A , the proportion of the group does not affect the utility of a non-group user. Since non-group users believe that platform A is focal, A can extract the network effects non-group users generate to each other. Consequently, the non-group users are agnostic about whether users join platform A individually or as part of the group. This is not the case when platform B wins the market. When the size of the group reaches $\tilde{x}$, there is a discontinuous climb in the utility of a non-group user because of two reasons. First, the user gets to join the superior quality platform. Second, platform B is non-focal and therefore cannot extract the network effects that non-group users generate to each other. As the proportion of the group increases further above $\tilde{x}$, the proportion of non-group users becomes negligible and the second effect becomes weaker—decreasing the utility of non-group users. Finally, the last marginal user that is still outside the group (when $x \rightarrow 1$) earns 0. Notice that this implies that a non-group user may prefer a small group that joins the “wrong” platform A , rather than a large group that helps the high-quality platform win the market. Moreover, the optimal size of the group from the viewpoint of a non-group user is of intermediate level: $\tilde{x}$.

5.2 The effect of the size of the group on an individual group user

The utility of an individual group user is $u^G(x) = V_{A1} - \frac{p_A^G}{x}$ if A wins, and $u^G(x) = V_{B1} - \frac{p_B^G}{x}$ if B wins, where p_A^G is given by (9) when $x \in [0, \hat{x})$, (12) when $x \in [\hat{x}, \tilde{x}]$, and p_B^G is given by (14). Hence,

Figure 2: Non-group users' utility as a function of x
Non-group users' utility when $\frac{V}{X} = 0.5$

$$u^G(x) = \begin{cases} V_B(x), & \text{if } x \in [0, \hat{x}), \\ V_{B1} + \frac{(1-x)(V_B(x) - V_A(1-x))}{x}, & \text{if } x \in [\hat{x}, \tilde{x}), \\ V_{B0} + \frac{V_{A1} - V_{B0}}{x}, & \text{if } x \in [\tilde{x}, 1]. \end{cases} \quad (19)$$

It is easy to see from equation 19 that when $x \leq \hat{x}$ ($x > \tilde{x}$), the utility of a group user increases (decreases) with the proportion of the group. The effect of the proportion of the group when $x \in [\hat{x}, \tilde{x}]$ is more subtle. We first characterize the effect of x on the utility of an individual group user and then further discuss below.

Proposition 5. (*Effect of the proportion of the group on an individual group user*)

1. When $x \in [0, \hat{x}]$, the utility on an individual group user is increasing in x , with a discontinuous jump at $x = \hat{x}$.
2. Evaluated at x slightly above $\hat{x}$, the utility on an individual group user is increasing in x . Further increases in x at the interval $x \in [\hat{x}, \tilde{x}]$ has an ambiguous effect on the utility of a group user.
3. When $x \in [\tilde{x}, 1]$, the utility on an individual group user is decreasing in x .

Figure 3 illustrates the general results of proposition 5 using the example in subsection 4.3. Proposition 5 shows that the utility of a group user increases with the proportion

Panel (a): small V ($V = 0.05$)

Panel (b): large V ($V = 0.3$)

The figure shows the utility of an individual group user as a function of the size of the group.

Figure 3: Group users' utility

of the group when $x \leq \hat{x}$, with a discontinuous climb at $x = \hat{x}$. Intuitively, as the group becomes larger, its alternative option of joining platform B increases, forcing platform A to offer the group better terms. At $x = \hat{x}$, the group becomes pivotal which triggers more intense competition between the two platforms, increasing the group's surplus. A further increase in x has two conflicting effects on $u^G(x)$. First, the group's alternative option increases with its size. At the same time, the decrease in the proportion of non-group users reduces the platforms' incentive to compete on the pivotal group (notice that this effect is insignificant when the group is not pivotal). For the general utility function we can only infer that $u^G(x)$ increases with x at $x = \tilde{x}$. Using the example in subsection 4.3, we find that when $x \in [\hat{x}, \tilde{x}]$, $u^G(x)$ is an inverse U-shape of x if $V < \lambda(3\sqrt{3} - 5) \cong 0.196\lambda$, and strictly increasing otherwise. We illustrate these two options in panels (a) and (b) of Figure 3.

Proposition 5 also reveals that when $x \in [\tilde{x}, 1]$, a group member is hurt when a non-group member joins the group. In this parameter space the group wants platform B for its superior quality while platform B needs the group for its pivotal position that enables it to attract the non-group users. As x increases, the first effect becomes stronger and the second effect becomes weaker—decreasing the value the group can extract from platform B .

These results indicate that a group user benefits when the group is at least pivotal, i.e., $x > \hat{x}$. Yet, beyond $\hat{x}$, additional users that join the group do not necessarily benefit existing group members.

5.3 The effect of the size of the group on total profits and total users' surplus

In this subsection we evaluate the effect of the proportion of the group on total profits, consumers' surplus and welfare. Recall that when $x \in [0, \hat{x})$ and $x \in [\hat{x}, \tilde{x})$, platform A wins and earns (10) and (13) respectively. When $x \in [\tilde{x}, 1]$, platform B wins and earns (15). Therefore, total profit as a function of x is:

$$\Pi(x) = \begin{cases} V_{A1} - (1-x)V_{B0} - xV_B(x), & \text{if } x \in [0, \hat{x}), \\ (V_{A1} - V_{B0}) - x(V_{B1} - V_{B0}) - (1-x)(V_B(x) - V_A(1-x)), & \text{if } x \in [\hat{x}, \tilde{x}), \\ x(V_{B1} - V_{B0}) + (1-x)(V_B(x) - V_A(1-x)) - (V_{A1} - V_{B0}), & \text{if } x \in [\tilde{x}, 1]. \end{cases}$$

Let $W(x) = V_{i1}$ denote total welfare as a function of x given that platform $i = \{A, B\}$ wins (notice that i is a function of x). The users' surplus – total users' utility given the winning platform i – is $CS(x) = V_{i1} - \Pi(x)$. The following proposition describes how x affects $\Pi(x)$ and $CS(x)$:

Proposition 6. (*A large group may harm users*)

1. When $x \in [0, \tilde{x})$, $\Pi(x)$ is decreasing with x , with a discontinuous drop at $x = \hat{x}$. When $x \in [\tilde{x}, 1]$, $\Pi(x)$ is an inverse U-shape function of x .
2. When $x \in [0, \tilde{x})$, $CS(x)$ is increasing with x , with discontinuous climbs at $x = \hat{x}$ and $x = \tilde{x}$. When $x \in [\tilde{x}, 1]$, $CS(x)$ is a U-shape function of x . Moreover, when x is close to 1, $CS(x)$ is lower than $CS(x)$ when x is slightly higher than $\tilde{x}$.

Figure 5.3 illustrates the results in Proposition 6. The bold line represents $CS(x)$, the double line represents $W(x)$, and $\Pi(x)$ is the gap between the two.¹² The figure shows that an increase in x is not always beneficial to users. For $x \in [0, \tilde{x})$, users' surplus is increasing in x and jumps at $x = \tilde{x}$ where it reaches its maximal level. Recall that for these group sizes, the utility of both non-group and group users is either increasing in x , or, increasing (non-group users) and slightly decreasing (group users). Then, for $x > \tilde{x}$, users' surplus first decreases with x because the decrease in both group and non-group users' utilities. Then, when the “last” non-group users join the group, they gain an increase in their own utilities, which is larger than the decrease in the group member.

¹²The first part of $CS(x)$ (when $x \in [0, \hat{x})$) is concave (convex) in x when $V_B''(x) < 0$ ($V_B''(x) > 0$).

Figure 4: Consumers' surplus ($CS(x)$) and welfare ($W(x)$) as a function of x

This last result implies that users may be better off under the inefficient outcome where the group chooses to join the low-quality platform as compared to the efficient case where the group chooses the high-quality platform. The size of the group affects the platforms' profits in opposite directions. Platform A 's profit is always decreasing with x , while platform B 's profit is first increasing and then decreasing with x .

6 Factors affecting the group's incentives to choose the efficient outcome

In this section we relax some of the model's assumptions to study the effect these assumptions have on the group's incentives to choose the efficient platform, B . We first look at the case where the group can multi-home. We then relax the focality assumption and study the case where the group can affect the non-group users' beliefs about the winning platform. Finally, we look at the case where the group's choice is not cooperative, yet the group still makes its platform choice before the non-group users do. This allows us to disentangle the effect of a cooperative decision from the effect the sequential decision making has on our main results.

6.1 Multi-homing

In many of the examples we introduced above users multi-home-i.e., choose to be affiliated with more than one platform. For example, many consumers use both Zelle

and Venmo, McDonalds accepts Apple Pay and Google Pay, and EA Sports' games are compatible both with the iOS and Android mobile operating systems. In this section, we account for the possibility of multi-homing and examine the effect of multi-homing on the coordination failure. Since we are interested in the effect on coordination failures, we look at the case where the group can multi-home while the non-group users single home.

As before, we start by solving the second and third stages—platforms' competition on the non-group users—and then consider the first stage where the platforms compete on the group users.

Starting with the second stage, since section 3 studies the case where the group single homes, we need only to analyze the non-group users' choice when the group chose to multi-home. Suppose the group joins platforms A and B . In this case, non-group users compare their value from choosing A , i.e., $V_{A1} - p_A$, with their value from choosing B , i.e., $V_B(x) - p_B$. In an equilibrium where platform A wins the non-group users, A charges the highest price that ensures the non-group users prefer joining its focal platform over joining platform B — $V_{A1} - p_A \geq V_B(x) - p_B$. Platform B then charges the lowest price that ensures non-negative profits. That is, prices will be set to $p_A = V_{A1} - V_B(x)$; $p_B = 0$. Using the same logic, in an equilibrium where the group multi-homes and B wins the non-group users, prices would be set to $p_A = 0$; $p_B = V_B(x) - V_{A1}$. Let $\pi_A(x; AB)$ denote the profit of platform i from the non-group users given that the group is in both platforms. We have $\pi_A(x; AB) = -\pi_B(x; AB) = (1 - x)(V_{A1} - V_B(x))$ where platform A wins the non-group users iff $\pi_A(x; AB) > 0$. The following lemma summarizes this result:

Lemma 3. *(The effect of x on the winner of the non-group when the group multi-homes) When the group joins both platforms, there is a unique threshold, $\hat{x}_M$, where $\hat{x}_M$ is the solution to $V_{A1} = V_B(\hat{x}_M)$, such that when $x \in [0, \hat{x}_M]$, platform A wins the non-group; otherwise, platform B wins the non-group. Moreover, $\hat{x}_M > \hat{x}$.*

Note that the threshold found in Lemma 3 is different than the one identified in Lemma 2 as well as the threshold identified in Proposition 3. The intuition behind the result that $\hat{x}_M > \hat{x}$ is that when the group multi-homes, the non-group users can “meet” the group either on platform A or B . As a result, the group has a weaker effect on which platform wins the market.

We now consider the first stage where platforms compete on attracting the group. We assume that when the group joins both platforms, each group user receives the

network effects from other users only in one of the platforms. This means that under multi-homing, we need to redefine the utility of a group user to account for the fact that users may interact with some users on one platform and other users on another platform. To do so, we make the simplifying assumption that the quality gap between platforms is independent of the size of network effects. That is, $V_B(n) - V_A(n)$ is constant and equals to V_{B0} for all n . This assumption implies that when the group joins both platforms, regardless of what platform the non-group users join, a group-user gains V_{B1} .¹³ Given that the group can multi-home, the price a platform charges in the first stage should make the group indifferent between joining only the competing platform and joining both platforms. The following lemma shows that in equilibrium, the group always joins both platforms. Intuitively, platform B can always attract the group by charging it at most platform B 's base quality. Likewise, platform A can attract the group by charging at most 0.

Lemma 4. (*The group always multihome*) *In equilibrium, the group always joins both platforms. The equilibrium prices are:*

1. *when $x \in [0, \hat{x}]$, $p_A^G = x(V_{B1} - V_B(x))$ and $p_B^G = xV_{B0}$;*
2. *when $x \in [\hat{x}, \hat{x}_M]$, $p_A^G = 0$ and $p_B^G = xV_{B0}$;*
3. *when $x \in [\hat{x}_M, 1]$, $p_A^G = 0$ and $p_B^G = xV_{B0}$.*

Given that the group always multi-homes, it is the non-group users' decision that determines whether the market ends up with the efficient outcome. According to Lemma 3, we know that the non-group users join platform B iff $\hat{x}_M < x$. That is, whether multi-homing improves or worsens the coordination failure depends on whether $\hat{x}_M$ is larger or smaller than $\tilde{x}$ —the threshold identified under the single-homing case. As shown in the following proposition, we find that $\tilde{x} < \hat{x}_M$ —i.e., multi-homing worsens the coordination failure problem.

Proposition 7. (*Under multihoming, larger group required for efficient outcome*) *As compared to single-homing, when the group can multi-home, the group always multi-homes and a larger group size is required for the more efficient platform, B , to win the market; i.e., $\tilde{x} < \hat{x}_M$.*

¹³Our results hold if instead we assume that the group users get their network effects from the platform the non-group users join.

The intuition for this result is that when the group multi-homes it has a lower ability to affect the winning platform (as shown by lemma 3). In this case, the two platforms have less of an incentive to compete on attracting the group. This goes against platform B that generally earns less from attracting the group than platform A . Taken together, the Proposition implies that multi-homing reduces the group’s ability to mitigate the coordination problem and help the market achieve the more efficient outcome.

The result in Proposition 7 differs from Doganoglu and Wright (2010) who consider platform competition between an incumbent and a more efficient entrant. The incumbent platform can make an introductory offer to a subset of users, which can multi-home i.e., join the incumbent’s and the entrant’s platforms. They find that if the incumbent cannot impose exclusive contracts on the initial set of users, the market overcomes the coordination failure problem and both the initial set and the remaining users choose to join the more efficient entrant. We contribute to this literature by showing that when the remaining users’ decision depends not only on the initial users’ decisions but also on their beliefs about what other users within their group choose, multi-homing does not mitigate the coordination failure problem and the more efficient platform cannot overcome its competitive disadvantage. In fact, multi-homing makes the coordination problem more severe.

6.2 The group affects the non-group users’ beliefs

Our base model assumes that the group cannot affect the beliefs of non-group users. This assumption is reasonable when the group is of small proportion. Indeed, recall that the base model’s results hold even when $x < \frac{1}{2}$. Yet, a large enough group may affect the beliefs of the non-group users where the larger the group, the stronger the effect.

This subsection considers the case where the group can directly affect the beliefs of the non-group users. We find that the stronger the effect of the group on the beliefs of the non-group, the more likely the group to join the high-quality platform. To illustrate this argument, we consider the extreme case in which the group determines the beliefs of the non-group: the platform that wins the group becomes focal.

When the platform that wins the group becomes focal, by definition, the group is always pivotal (i.e., for all values of $0 \leq x \leq 1$). In this case, according to Lemma 2, if platform A wins the group, it wins the non-group and earns from the non-group $\pi_A(x; A) = (1-x)(V_{A1} - V_{B0})$. Similarly, when platform B wins the group and benefits

from the focal position, it wins the non-group and earns $\pi_B(x; B) = (1 - x)V_{B1}$. The following proposition shows that in this case, platform B always wins the group and non-group users:

Proposition 8. *Suppose that the platform that wins the group becomes focal. Then, for all $0 \leq x \leq 1$, platform B wins the group and non-group users.*

The intuition for this result is that when the platform that wins the group becomes focal, both platforms can use the group in order to collect the network effects the non-group users create to each other. In this case, the joint value of the group and the platform under platform B is always larger than under platform A — $V_{B1} > V_{A1}$. That is, platform B can use its superior quality in order to attract the group and consequently win the non-group. This result implies that the stronger the group’s ability to affect the beliefs of the non-group users, the more likely the group to make the efficient choice and join platform B .

Halaburda and Yehezkel (2019) consider platform competition when one of the platforms benefits from a partial degree of focality. Using their notion of focality, the group in our model may have a partial effect on the degree of focality, which is an increasing function of the size of the group. In such a case, platform B may not always win the market. Still, its ability to win the market increases as the size of the group has a stronger effect on the degree of focality.

6.3 The group cannot make a collective decision

In this section we ask how the group’s ability to make a collective decision affects the results. Recall that the group makes its platform decision before the non-group users. This raises the question whether the group’s ability to mitigate the coordination problem is mainly driven by the coordinated decision or the sequential aspect of our game. To study this we look at how the results change when an uncoordinated group chooses a platform before the (uncoordinated) non-group users. The main conclusion of this section is that when the group cannot make a collective decision, a pivotal uncoordinated group is more likely to join platform A . Moreover, if the quality gap between platforms is not too large, then an uncoordinated group joins platform A for all group sizes, $x \in [0, 1]$. These results imply that the group has a stronger ability to help the high-quality platform win when it can coordinate the decisions of its group members.

Suppose that in the first stage, platforms A and B compete on a group of users with proportion x by charging each user the prices ρ_A and ρ_B , respectively. Group-users make individual decisions: each group user takes the decisions of other group users as given and believes that platform A is focal. In the second stage, platforms compete on non-group users.

The second stage is identical to our base model: the group is pivotal iff $x > \hat{x}$ and the profits of the two platforms from non-group users are given by lemma 2. Turning to stage 1, consider the case where the platform that wins the group wins the non-group: $x > \hat{x}$. Even though the group is pivotal in the sense that it can affect the winning platform, an uncoordinated group-user does not internalize this effect. Instead, the group-user expects that all other group users join the focal platform A and consequently platform A wins the non-group users. In an equilibrium in which platform A wins the group, prices are:

$$V_{A1} - \rho_A \geq V_{B0} - \rho_B \quad , \rho_B = -\frac{\pi_B(x; B)}{x} \implies \rho_A = V_{A1} - V_{B0} - \frac{\pi_B(x; B)}{x}. \quad (20)$$

Comparing (20) with the equilibrium prices when the group makes a collective decision (equation (3)), notice that an uncoordinated group-user expects to gain the quality advantage, V_{B0} , by joining the non-focal platform B (the right-hand side of (20)), while when the group makes a collective decision, the group expects to gain the network effects as well—a per-user utility of V_{B1} (the right-hand side of (3)); as the group moves together and non-group users follow. Consequently, platform A can charge each uncoordinated group-user a higher price than a coordinated group-user: $\rho_A = V_{A1} - V_{B0} - \frac{\pi_B(x; B)}{x} > V_{A1} - V_{B1} - \frac{\pi_B(x; B)}{x} = \frac{p_A^G}{x}$. The same argument holds when the group is not pivotal: $x < \hat{x}$. An uncoordinated group user expects to gain V_{B0} by joining the non-focal platform B , while if the group were to move together, each group-user would have earned $V_B(x)$. This again enables platform A to charge uncoordinated group-users a higher price than a coordinated group.

For tractability, we solve this case based on the linear utility considered in subsection 4.3. We find that when the quality gap in favor of platform B is not too high, $V < 0.657\lambda$, platform A wins for all values of $x \in [0, 1]$. When platform B is of significantly higher quality, $V > 0.657\lambda$, there are two cutoffs, $\tilde{x}_1$ and $\tilde{x}_2$ where $\tilde{x} < \tilde{x}_1 < \tilde{x}_2 < 1$ such that platform B wins for $\tilde{x}_1 < x < \tilde{x}_2$. Figure 5 illustrates these cutoffs.

Intuitively, the two platforms compete on two distinct subgroups of users and can

Figure 5: The cutoff values of $\tilde{x}$, $\tilde{x}_1$ and $\tilde{x}_2$ as a function of V

price discriminate between them. As suggested by Caillaud and Jullien (2001; 2003), for a non-focal platform to win the market, the platform needs to adopt a divide-and-conquer strategy: the platform can subsidize the first group of users and then collect the revenues from the other group. Figure 5 shows that when the first group cannot make a collective decision, the ability of the non-focal platform B to win the market with a divide-and-conquer strategy either substantially decreases (when $V > 0.657\lambda$) or completely vanishes (when $V < 0.657\lambda$). Without coordination, group users expect other group users to join the focal platform and do not internalize their ability to affect the decisions of non-group users. This makes it more profitable for the focal platform to maintain its dominant position.

This result implies that the less the group can coordinate the decisions of its members, the less the high-quality but non-focal platform can utilize the group to win the market.

7 Conclusion

This paper considers platform competition when some users belong to a user group. The group makes a collective decision which platform to join, while small, non-group users make individual decision. One of the platforms is of low-quality, but benefits from a focality advantage: non-group users expect that other non-group users will join

it. Such focality can emerge from incumbency advantage or users' inertia towards a platform that users joined in the past.

The model reveals two main results. First, a proportionally large group can solve users' coordination problem by joining the high-quality platform. Yet, such a pivotal group will choose to join the low-quality platform unless its proportion is large enough. This happens because the low-quality focal platform is better positioned to subsidize the group, as the focal platform can extract the network effects the non-group users create to each other. When the proportion of the group is large, the pivotal group joins the high-quality platform. In this case, the quality advantage is more important to the group than earning the focal platform's revenues from serving the non-users.

The second main result is that an increase in the proportion of the group has a non-monotonic effect on the individual utility of group and non-group users. When the size of the group is small, the group is not pivotal and an increase in the proportion of the group increases the utility of each of its members, while keeping the utility of non-group members fixed. When the group becomes pivotal, each of its members earns additional payoff, even when it continues to choose the low-quality platform. When the group chooses the high-quality platform, a further increase in its proportion decreases the utilities of each member inside and outside the group. For group users, the alternative of joining the low-quality platform becomes less attractive, making it possible for the high-quality platform to attract it with a higher price. The utility of a non-group user decreases with the proportion of group users because the non-focal platform can extract the network effects that the group provides to the non-group users.

These results suggest that mergers between users may not always be beneficial for users. Both group and non-group users would rather have a group of intermediate size. Interestingly, non-group users prefer a small group over a large group, while the opposite holds for group users.

Our model makes two simplifying assumptions. First, users are homogeneous. This assumption combined with the presence of network effects imply that in equilibrium all users join the same platform and the competing platform remains with no users. In contrast, some of our motivating examples exhibit markets with more than one active platform. Intuitively, multiple platforms can emerge because users may differ in their subjective preferences over the competing platforms. Armstrong (2006) shows that when users have heterogeneous preferences that are stronger than network effects, the equilibrium involves two active platforms. In such markets, coordination problems and

beliefs do not play a significant role. Since our paper focuses on beliefs, we assume that network effects are more important than heterogeneous preferences, and we take it to the extreme by assuming that users are homogeneous. In the background of our model, the losing platform may still be active and serve only users that have strong preferences towards it, or operate in a different market.¹⁴

Our second simplifying assumption is that we focus on markets with direct network effects. However, some of our motivating examples concern two-sided markets. As Caillaud and Jullien (2001) and (2003) show, in a two-sided market, platform competition is typically asymmetric: platforms compete more aggressively on one of the sides. This raises the question of how the presence of a user-group affects such asymmetric competition. Since the focus of this paper is on coordination and focality, we leave this question for future research.

¹⁴Halaburda and Yehezkel (2016) consider platform competition when some consumers are “loyal” to each platform, while other consumers are non-loyal. They apply the concept of focality to the non-loyal consumers and show that qualitative features of focality are not affected by the presence of loyal consumers.

Appendix

We prove Lemma 1 in the text. Below are the proofs of all other Lemmas and propositions in the text.

Proof of Lemma 2:

To complete the analysis preceding Lemma 2, we only need to prove that $V_A(1 - x) > V_B(x)$ iff $x < \hat{x}$ where $0 < \hat{x} < \frac{1}{2}$. Evaluating $V_A(1 - x) - V_B(x)$ at $x = 0$, $V_A(1 - 0) - V_B(0) = V_{A1} - V_{B0} > 0$, where the last inequality follows from Assumption 3. Evaluating $V_A(1 - x) - V_B(x)$ at $x = \frac{1}{2}$, $V_A(1 - \frac{1}{2}) - V_B(\frac{1}{2}) = V_A(\frac{1}{2}) - V_B(\frac{1}{2}) < 0$, where the last inequality follows from Assumption 2. Since by Assumption 1, $V_A(1 - x)$ is decreasing with x and $V_B(x)$ is increasing with x , there is a unique $x < \frac{1}{2}$ such that $V_A(1 - x) > V_B(x)$ iff $x < \hat{x}$. ■

Proof of Proposition 1:

We first show that, when $x < \hat{x}$, there is always an equilibrium in which platform A wins the group. Then, we show that it is unique. To do this, following the analysis preceding the proposition, we need to show that $\pi_A(x; A) + p_A^G - \pi_A(x; B) \geq 0$. Plugging in equations (6) and (10), we have:

$$\begin{aligned}
 & \pi_A(x; A) + p_A^G - \pi_A(x; B) \\
 &= \Pi_A(x; A) - \pi_A(x; B) \\
 &= V_{A1} - (1 - x)(V_{B0} + V_A(1 - x)) + (1 - 2x)V_B(x) \\
 &> V_{A1} - (1 - x)(V_{B0} + V_A(1 - x)) + (1 - 2x)V_{B0} \\
 &> V_{A1} - (1 - x)(V_{B0} + V_{A1}) + (1 - 2x)V_{B0} \\
 &= x(V_{A1} - V_{B0}) \\
 &> 0,
 \end{aligned} \tag{21}$$

where the first inequality follows because $V_B(x) > V_{B0}$ and from Lemma 1, $x < \hat{x} < \frac{1}{2}$. The second inequality follows because $V_A(1 - x) < V_{A1}$, and the last inequality follows from Assumption 3. Hence, there is an equilibrium in which platform A wins the group.

Next, consider a putative equilibrium in which platform B wins the group (and platform A wins the non-group). In such an equilibrium, if it were to exist, platform A charges the highest price that makes it indifferent between winning and not winning the group: $p_A^G = -(\pi_A(x; A) - \pi_A(x; B))$. The highest price that platform B can charge

the group solves

$$xV_B(x) - p_B^G \geq xV_{A1} - p_A^G,$$

hence, $p_B^G = -(\pi_A(x; A) - \pi_A(x; B)) - x(V_{A1} - V_B(x))$. Platform B earns $\Pi_B(x; B) = p_B^G$, because B cannot win the non-group even when $J = B$. Yet, notice that $\Pi_B(x; B) = \pi(x; A) + x(V_{A1} - V_B(x)) - \pi_A(x; B) < 0$, were the inequality follows from (21), implying that there is no equilibrium in which platform B wins. ■

Proof of Proposition 2:

We prove the proposition by showing that at the top ($x = 1$), platform A 's profits are negative while at the bottom ($x = \hat{x}$), platform A enjoys positive profits. Evaluating (13) at $x = 1$:

$$\begin{aligned} \Pi_A(1; A) &= (V_{A1} - V_{B0}) - (V_{B1} - V_{B0}) \\ &= V_{A1} - V_{B1} \\ &< 0, \end{aligned}$$

where the inequality follows from Assumption 3. Since $\Pi_B(x; B) = -\Pi_A(x; A)$, it follows that $\Pi_B(1; B) > 0$.

Next, evaluating (13) at $x = \hat{x}$:

$$\begin{aligned} \Pi_A(\hat{x}; A) &= (V_{A1} - V_{B0}) - (1 - \hat{x})(V_B(\hat{x}) - V_A(1 - \hat{x})) - \hat{x}(V_{B1} - V_{B0}) \\ &= (V_{A1} - V_{B0}) - \hat{x}(V_{B1} - V_{B0}) \\ &\geq (1 - \hat{x})V_{A1} - V_{B0} \\ &\geq V_A(1 - \hat{x}) - V_{B0} \\ &= V_B(\hat{x}) - V_{B0} \\ &> 0, \end{aligned}$$

where the equality in the second line follows because by definition, $V_B(\hat{x}) = V_A(1 - \hat{x})$, the inequality in the third line follows since $V_{B1} - V_{B0} \leq V_{A1}$ (and rearranging), the inequality in the fourth line follows because the convexity of $V_A(n)$ together with $V_{A0} = 0$ imply that $(1 - \hat{x})V_{A1} = \hat{x}V_A(0) + (1 - \hat{x})V_A(1) \geq V_A(\hat{x}0 + (1 - \hat{x})1) = V_A(1 - \hat{x})$, the equality in the fifth line follows again because $V_B(\hat{x}) = V_A(1 - \hat{x})$ and the inequality in the last line follows from Assumption 1. Since the last inequality is strong, $\Pi_A(\hat{x}; A) > 0$

also holds when $V_A(n)$ is linear or concave in n , as long as it is not too concave, and when $V_{B1} - V_{B0} > V_{A1}$ as long as the gap is not too large. ■

Proof of Proposition 3:

The plan of the proof is as follows. First, we show that $\frac{d\Pi_A(x;A)}{dx}|_{x=\hat{x}} < 0$ and $\frac{d\Pi_A(x;A)}{dx}|_{x=1} > 0$. Second, recalling that from Proposition 2, $\Pi_A(\hat{x}; A) > 0$ and $\Pi_A(1; A) < 0$, we have that if $\frac{d^2\Pi_A(x;A)}{d^2x} > 0$, there is a cutoff, $\tilde{x}$, where $\Pi_A(\tilde{x}; A) = 0$, such that $\Pi_A(x; A) > 0$ iff $x < \hat{x}$. We therefore show that $\frac{d^2\Pi_A(x;A)}{d^2x} > 0$ whenever condition (17) holds. Finally, since $\Pi_B(x; B) = -\Pi_A(x; A)$, we have that the same cutoff satisfies that $\Pi_B(x; B) > 0$ iff $x > \hat{x}$.

We start with $\frac{d\Pi_A(x;A)}{dx}$:

$$\frac{d\Pi_A(x; A)}{dx} = -(V_{B1} - V_{B0}) + (V_B(x) - V_A(1 - x)) - (1 - x)(V'_B(x) + V'_A(1 - x)). \quad (22)$$

Evaluated at $x = \hat{x}$, the term in the second large brackets in (22) disappears because by definition, $V_B(\hat{x}) - V_A(1 - \hat{x}) = 0$, we get

$$\left. \frac{d\Pi_A(x; A)}{dx} \right|_{x=\hat{x}} = -(V_{B1} - V_{B0}) - (1 - \hat{x})(V'_B(\hat{x}) + V'_A(1 - \hat{x})) < 0,$$

where the inequality follows because $V'_i(n) > 0$. Evaluated at $x = 1$:

$$\left. \frac{d\Pi_A(x; A)}{dx} \right|_{x=1} = -(V_{B1} - V_{B0}) + (V_{B1} - 0) = V_{B0} > 0.$$

Next, differentiating (22) with respect to x yields that $\frac{d^2\Pi_A(x;A)}{d^2x} > 0$ when condition (17) holds (notice that when $\pi_B(x; B)$ is concave in x , $\Pi_B(x; B)$ is also concave in x and therefore $\Pi_A(x; A)$ is convex in x). ■

Proof of Proposition 4:

When $x < \tilde{x}$, it follows immediately from (18) that $u(x)$ is independent of x . Next consider the case where $x \geq \tilde{x}$. We start by evaluating $u(\tilde{x})$. Extracting $V_B(\tilde{x})$ from the definition of $\tilde{x}$ (equation (16)) and substituting into the second line in (18), we can rewrite $u(x \rightarrow \tilde{x}^+) = V_{B0} + \frac{V_{B1} - V_{A1}}{1 - \tilde{x}}$. Recalling that $u(x \rightarrow \tilde{x}^-) = V_{B0}$, we have that $u(x \rightarrow \tilde{x}^+) - u(x \rightarrow \tilde{x}^-) = \frac{V_{B1} - V_{A1}}{1 - \tilde{x}} > 0$, where the inequality follows from Assumption 2. Next, we have that for $x > \tilde{x}$, $u'(x) = -V'_B(x) - V'_A(1 - x) < 0$ where the inequality follows from Assumption 1. Finally, $u(1) = V_{B1} - (V_B(1) - V_A(1 - 1)) = 0$. ■

Proof of Proposition 5:

Starting with part 1, it follows from Assumption 1 that when $x \in [0, \hat{x})$, $u^G(x) = V_B(x)$ is increasing with x .

Moving to part 2, we first show that there is a discontinuous jump in $u^G(\hat{x})$. To this end, recall that $V_B(\hat{x}) = V_A(1 - \hat{x})$. Substituting $V_B(\hat{x}) = V_A(1 - \hat{x})$ into the second line in (19) yields that $u^G(x \rightarrow \hat{x}^+) = V_{B1} > V_B(\hat{x}) = u^G(x \rightarrow \hat{x}^-)$, where the inequality follows because $\hat{x} < 1$ and $V_B(x)$ is increasing with x . Next we turn to showing that evaluated at $x = \hat{x}$, $u^G(x)$ is increasing in x . To this end, differentiating the second line of (19) with respect to x :

$$u^{G'}(x) = \frac{[V_A(1 - x) - V_B(x)] + x(1 - x)[V'_A(1 - x) + V'_B(x)]}{x^2}.$$

Evaluated at $V_B(\hat{x}) = V_A(1 - \hat{x})$, the term in the first squared brackets vanishes. The term in the second squared brackets is positive by Assumption 1. Hence, $u^{G'}(\hat{x}) > 0$.

Turning to part (3), we have:

$$u^{G'}(x) = -\frac{V_{A1} - V_{B0}}{x^2} < 0,$$

where the inequality follows from Assumption 3. ■

Proof of Proposition 6:

Part (i): We start with the first line of $\Pi(x)$: $x \in [0, \hat{x})$. We have $\Pi(0) = V_{A1} - V_{B0}$. Moreover,

$$\Pi'(x) = V_{B0} - V_B(x) - xV'_B(x) < 0,$$

where the inequality follows because for all $x > 0$, $V_B(x) > V_{B0}$ and $V_B(x)$ increases with x , (notice that $\Pi''(x) = -V'_B(x) - xV''_B(x)$). Evaluating $\Pi(x)$ at $x \rightarrow \hat{x}^-$ (first line in $\Pi(x)$) and $x \rightarrow \hat{x}^+$ (second line in $\Pi(x)$), we have $\Pi(x \rightarrow \hat{x}^-) = V_{A1} - V_{B0} - \hat{x}(V_B(\hat{x}) - V_{B0})$ and $\Pi(x \rightarrow \hat{x}^+) = (V_{A1} - V_{B0}) - \hat{x}(V_{B1} - V_{B0})$ (where recall that $V_A(1 - \hat{x}) = V_B(\hat{x})$). The gap:

$$\Pi(x \rightarrow \hat{x}^-) - \Pi(x \rightarrow \hat{x}^+) = \hat{x}(V_{B1} - V_B(\hat{x})) > 0.$$

Hence, there is a discontinuous drop in $\Pi(\hat{x})$.

Next, consider the second line of $\Pi(x)$: $x \in [\hat{x}, \tilde{x})$. From the proof of Proposition 3, $\Pi(x) = \Pi_A(x; A)$ is decreasing and convex in x and $\Pi(\tilde{x}) = 0$.

Next, consider the third line of $\Pi(x)$: $x \in [\tilde{x}, 1]$. Again from the proof of Proposition

3, $\Pi(\tilde{x}) = \Pi_B(\tilde{x}; B) = 0$, hence, $\Pi(x)$ is continuous at $\hat{x}$. Moreover, $\Pi(x) = \Pi_B(x; B)$ is concave in x . Recall also that the proof of Proposition 3 shows that $\frac{d\Pi_A(x; A)}{dx}|_{x=\tilde{x}} < 0$ and $\frac{d\Pi_A(x; A)}{dx}|_{x=1} > 0$. As $\Pi_B(x; B) = -\Pi_A(x; A)$, we have that $\frac{d\Pi_B(x; B)}{dx}|_{x=\tilde{x}} > 0$ and $\frac{d\Pi_B(x; B)}{dx}|_{x=1} < 0$, implying that $\Pi(x) = \Pi_B(x; B)$ is an inverse U-shape function of x .

Part (ii): Starting with $x \in [0, \tilde{x})$, we have $CS(0) = V_{A1} - \Pi(0) = V_{B0}$. Because $\Pi(x)$ is decreasing in x , and has discontinuous decline at $x = \hat{x}$, $CS(x)$ is increasing in x and has a discontinuous climb at $x = \hat{x}$. At $x = \tilde{x}$, $\Pi(\tilde{x}) = 0$, but total utility increases from V_{A1} to V_{B1} , hence, there is a discontinuous climb in $CS(\tilde{x})$. Finally, at $x \in [\tilde{x}, 1]$, $\Pi(x) = \Pi_B(x; B)$ is an inverse U-shape function of x , hence $CS(x)$ is a U-shape function of x . Finally, $CS(1) = V_{B1} - \Pi_B(1, B) = V_{A1} < V_{B1} = CS(\hat{x})$. ■

Proof of Lemma 3:

The proof that $V_{A1} > V_B(x)$ iff $x < \hat{x}_M$ is identical to the proof of Lemma 2. To show that $\hat{x}_M > \hat{x}$, recall that $V_A(1 - \hat{x}) = V_B(\hat{x})$ and $V_{A1} - V_B(\hat{x}_M) = 0$. Hence $V_{A1} - V_B(\hat{x}) = V_{A1} - V_A(1 - \hat{x}) > 0$, implying that at $x = \hat{x}$ $\pi_A(x; AB) > 0$ and $\hat{x}_M > \hat{x}$. ■

Proof of Lemma 4

We prove that the group always joins both platforms by showing that for all three parts of the Lemma, both platforms can profitably attract the group.

Part (i): $x < \hat{x}$. The two platforms should set prices such that the group is indifferent between multi-homing and single-homing on the competing platform. Since when $x < \hat{x}$, the non-group users choose platform A , regardless of the group's choice, platform A should set its price such that the group is indifferent between joining only platform B and gaining $xV_B(x) - p_B^G$ and joining both platforms and gaining $xV_{B1} - p_A^G - p_B^G$.¹⁵ Similarly, platform B sets its price to make the group indifferent between joining only platform A and gaining $xV_{A1} - p_A^G$ and joining both platforms. To summarize, the two platforms set their price as follows: $p_A^G = x(V_{B1} - V_B(x))$ and $p_B^G = x(V_{B1} - V_{A1}) = xV_{B0}$ and the group joins both platforms. Note that both $p_A^G > 0$ and $p_B^G > 0$.¹⁶

Part (ii): $\hat{x} < x < \hat{x}_M$. Consider first platform A . Given that under single-homing the group is pivotal ($x > \hat{x}$), if the group joins only B , it gains $xV_{B1} - p_B^G$. As before, if

¹⁵Recall that we assume that the quality gap between platforms is independent of the size of the network effects and thus, regardless of the non-group users' choice, under multi-homing a group-user gains V_{B1} .

¹⁶To see why $p_A^G > 0$, we have $p_A^G = x(V_{A1} + V_{B0} - V_B(x)) > x(V_{A1} + V_{B0} - V_A(1 - x)) > 0$, where the first inequality follows because $V_A(1 - x) > V_B(x)$ and the second inequality follows because $V_{A1} > V_A(1 - x)$.

the group multi-homes, the group gains $xV_{B1} - p_A^G - p_B^G$. That is, A sets its price to zero. While this may suggest that A might not find it optimal to attract the group, recall that if A does not attract the group, the group joins B and A loses the non-group users. If, however, A attracts the group, according to Lemma 3, it wins the non-group users, and since $p_A > 0$ earns positive profit. Hence, A strongly prefers attracting the group.

Platform B 's decision is identical to the case where $x < \hat{x}$, as in both cases if the group joins both platforms, A wins the non-group. So, as before, B finds it optimal to attract the group and sets its price to $p_B^G = xV_{B0}$. To summarize, prices are $p_B^G = xV_{B0}$; $p_A^G = 0$ and the group joins both platforms.

Part (iii): $\hat{x}_M < x < 1$. Since in this region x is still larger than $\hat{x}$, just like before, if the group only joins platform B it gains $xV_{B1} - p_B^G$. Also as before, if the group multi-homes it gains $xV_{B1} - p_A^G - p_B^G$. That is, here again the most platform A can charge is $p_A^G = 0$. In contrast to the previous case, regardless of whether platform A attracts the group, according to Lemma 3, the non-group users choose platform B . Consequently, A is indifferent between attracting and not attracting the group as in both cases it gains 0.

Platform B still needs to set a price that makes the group indifferent between joining only A and gaining $xV_{A1} - p_A^G$ (as the non-group would join A as well) and multi-homing and gaining $xV_{B1} - p_A^G - p_B^G$. So, again, B finds it optimal to attract the group and sets its price to $p_B^G = xV_{B0} > 0$. To summarize, prices are $p_A^G = 0$ and $p_B^G = xV_{B0}$ and the group joins both platforms. ■

Proof of Proposition 7:

We proved that the group always multi-home in Lemma 4. Thus, we only prove here that the threshold $\hat{x}_M$ that solves $(1-x)(V_{A1} - V_B(x)) = 0$ is higher than the threshold $\tilde{x}$ that solves $(1-x)(V_A(1-x) - V_B(x)) + (V_{A1} - V_{B0}) - x(V_{B1} - V_{B0}) = 0$. To this end, we first need to show that evaluated at $\hat{x}_M$, the equation $\tilde{x}$ solves is negative; i.e.:

$$(1 - \hat{x}_M)(V_A(1 - \hat{x}_M) - V_B(\hat{x}_M)) + (V_{A1} - V_{B0}) - \hat{x}_M(V_{B1} - V_{B0}) < 0$$

We know that

$$(1 - \hat{x}_M)(V_A(1 - \hat{x}_M) - V_B(\hat{x}_M)) + (V_{A1} - V_{B0}) - \hat{x}_M(V_{B1} - V_{B0}) <$$

$$(1 - \hat{x}_M)(V_{A1} - V_B(\hat{x}_M)) + (V_{A1} - V_{B0}) - \hat{x}_M(V_{B1} - V_{B0}) =$$

$$= (V_{A1} - V_{B0}) - \hat{x}_M(V_{B1} - V_{B0})$$

where the first inequality follows because $V_{A1} > V_A(x)$ for all $x < 1$ (Assumption 1); and the second inequality follows because $V_{A1} = V_B(\hat{x}_M)$. Moreover:

$$(V_{A1} - V_{B0}) - \hat{x}_M(V_{B1} - V_{B0}) = (V_B(\hat{x}_M) - V_{B0}) - \hat{x}_M(V_{B1} - V_{B0})$$

again, because $V_{A1} = V_B(\hat{x}_M)$. We rearrange the equation to receive:

$$\begin{aligned} V_B(\hat{x}_M) - (\hat{x}_M V_{B1} + (1 - \hat{x}_M)V_{B0}) &= V_B(\hat{x}_M) - (\hat{x}_M V_B(1) + (1 - \hat{x}_M)V_B(0)) \leq \\ V_B(\hat{x}_M) - V_B(1 * \hat{x}_M + (1 - \hat{x}_M) * 0) &= V_B(\hat{x}_M) - V_B(1) < 0 \end{aligned}$$

where the first equality re-arranges terms and the second inequality results from the convexity of $V_B(x)$. The last inequality follows from Assumption 1. Note that the result also holds if $V_B(x)$ is linear or not too concave. Since $\tilde{x}$ valued at the same equation is zero, and since the function is decreasing in x , it follows that there is a unique $\hat{x}_M > \tilde{x}$. ■

Proof of Proposition 8:

When the platform that wins the group wins focality, the group joins platform B when: $xV_{B1} - p_B^G \geq xV_{A1} - p_A^G$. Then, in an equilibrium in which platform B wins, $p_A^G = -\pi_A(x; A) = -(1 - x)(V_{A1} - V_{B0})$. Hence, platform B sets $p_B^G = x(V_{B1} - V_{A1}) - (1 - x)(V_{A1} - V_{B0})$ and earns $\Pi_B(x; B) = \pi_B(x; B) + p_B^G = (1 - x)V_{B1} + x(V_{B1} - V_{A1}) - (1 - x)(V_{A1} - V_{B0}) = V_{B1} - V_{A1} + (1 - x)V_{B0} > 0$, where the inequality follows because $V_{B1} > V_{A1}$ and $X \leq 1$.

Finally, to show that there is no equilibrium in which platform A wins, in this equilibrium $p_B^G = -\pi_B(x; B) = -(1 - x)V_{B1}$. Hence, platform A sets $p_A^G = -x(V_{B1} - V_{A1}) - (1 - x)V_{B1}$ and earns $\Pi_A(x; A) = \pi_A(x; A) + p_A^G = (1 - x)(V_{A1} - V_{B0}) - x(V_{B1} - V_{A1}) - (1 - x)V_{B1} = -(V_{B1} - V_{A1} + (1 - x)V_{B0}) < 0$. Consequently, platform A cannot profitably win the market. ■

References

- [1] Akerlof, R., Holden, R., Rayo, L., 2018. “Network externalities and market dominance.” working paper.
- [2] Armstrong, M., 2006a. “Competition in two-sided markets.” *The RAND Journal of Economics* 37 (3), 668–691.
- [3] Armstrong, M., 2006b. “Competition in two-sided markets.” *The RAND Journal of Economics* 37 (3), 668–691.
- [4] Biglaiser, G., Calvano, E., Crémer, J., 2019. “Incumbency advantage and its value.” *Journal of Economics and Management Strategy* 28 (1), 41-48.
- [5] Biglaiser, G., Crémer, J., 2018. “The value of incumbency in heterogenous networks.” working paper.
- [6] Caillaud, B., Jullien, B., 2001. “Competing cybermediaries.” *European Economic Review* 45 (4-6), 797–808.
- [7] Caillaud, B., Jullien, B., 2003. “Chicken & egg: Competition among intermediation service providers.” *The RAND Journal of Economics*, 309–328.
- [8] Carroni, E., Madio, L., Shekhar, S., 2019. “Superstars in two-sided markets: exclusives or not?” working paper.
- [9] Doganoglu, T., Wright, J., 2010. “Exclusive dealing with network effects.” *International Journal of Industrial Organization* 28, 145-154.
- [10] Hagiu, A., 2006. “Pricing and commitment by two-sided platforms.” *The RAND Journal of Economics* 37 (3), 720–737.
- [11] Halaburda, H., Yehezkel, Y., 2013. “Platform competition under asymmetric information.” *American Economic Journal: Microeconomics* 5 (3), 22–68.
- [12] Halaburda, H., Yehezkel, Y., 2016. “The role of coordination bias in platform competition.” *Journal of Economics and Management Strategy* 25 (2), 274–312.
- [13] Halaburda, H., Yehezkel, Y., 2019. “How beliefs affect platform competition.” *Journal of Economics and Management Strategy* 28 (1), 49-49.

- [14] Halaburda, H., Jullien, B., Yehezkel, Y., 2018. “Dynamic platform competition: why history matters?” working paper.
- [15] Jullien, B., 2011. “Competition in multi-sided markets: Divide and conquer.” *American Economic Journal: Microeconomics* 3 (4), 186–220.
- [16] Karlinger, L., Massimo, M., 2012. “Exclusionary pricing when scale matters.” *The Journal of Industrial Economics* 60 (1), 75–103.
- [17] Katz, M., Shapiro, C., 1986. “Technology adoption in the presence of network externalities.” *Journal of Political Economy* 94, (4), 822-841.
- [18] Rasmusen, E. B., Ramseyer, J. M., Wiley, J. S., 1991. “Naked exclusion.” *American Economic Review* 81, 1137–1145.
- [19] Segal, I. R., Whinston, M. D., 2000. “Naked exclusion: Comment.” *American Economic Review* 90, 296 – 309.