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Abstract

One emerging business model for Internet service providers (ISPs) is to allow content providers (CPs) to subsidize Internet access for end consumers. In the present study, we develop a game-theoretical model to analyze the effects of this sponsorship of consumer data usage. The findings indicate that for an ISP, its optimal network management choice of data sponsorship largely hinges on specific market conditions such as the revenue rates of CPs and the fit cost for consumers. If the fit cost is low, the ISP will either allow both CPs to subsidize consumers' Internet access, or allow only the more competitive CP to subsidize, depending on the CPs' per-consumer revenue generation rates. If the fit cost is high, it is in the ISP's interest not to allow any subsidization. The study also identifies the conditions under which an ISP's network management choices of data sponsorship deviate from the social optimum. By identifying additional revenue models, these findings have direct implications for the telecom industry, for online content providers competing for customer loyalty, and for policymakers vested in this issue.

Keywords: Internet service provider, online content provider, usage subsidization, consumer surplus, social welfare

JEL Classification: C72, D43, L44

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1. Introduction

Consumers are increasingly migrating online to procure a variety of digitized content. According to Cisco's visual networking index, mobile Internet traffic has increased from 820 petabytes to 1.5 exabytes per month from 2012 to 2013 as consumers gained access to faster networks and created or downloaded richer media such as high-definition videos and video conferencing (Cisco 2015). In response to this growing traffic, some Internet service providers (ISPs) have proposed charging a fee to content providers (CPs) for the right to prioritize content. Under this scheme, paying CPs would have their content prioritized over that of competitors, thus ensuring a smoother online experience for their consumers. Given the relative lack of competition in local geographies, ISPs would have both the incentive and the ability to discriminate among online content from different CPs. This has become a highly controversial issue in recent years, bringing to the fore the ongoing debate on net neutrality (Choi and Kim 2010).

Recently, ISPs have come up with a different payment scheme for online CPs and end users. Online consumers in the US typically subscribe to data plans with a cap on the amount of data they can use during a payment plan period. While this is encountered more often on mobile broadband networks, ISPs on fixed broadband networks are increasingly instituting such caps as well. Sensing an opportunity, AT&T developed a plan (which was quickly imitated by its competitors) whereby a particular CP could effectively subsidize the consumption of its own content: when ISP subscribers consume content from that particular CP, the downloaded content (capped at an amount negotiated between the ISP and the CP) is not counted towards the consumer's ISP subscription cap (Sharma et al. 2014). Under this arrangement, eligible data usage charges are billed directly to the sponsoring company. Data sessions are identified by a "sponsored data" icon on the end user's device, with the usage itemized separately on the end

user's monthly invoice. This new scheme allows CPs to pay for the bandwidth that end users consume. A Seattle-area startup, Syntonic Wireless, is an example of one such subsidizing CP. Under the data subsidization plan, any content from Syntonic Wireless that a consumer watches on her smartphone through a cellular connection would not count toward her monthly data cap. Syntonic is one of several sponsored data CPs listed under AT&T's program, which also includes Aquoto, 5 Screen Media and DataMi, among others (Sharma, Ante, and Troianovski, 2014). This arrangement is beneficial to the CP because a consumer who consumes its content effectively precludes herself from consuming content from a rival provider. Moreover, CPs can generate revenue from such consumers both directly (through subscription or on-demand fees) and indirectly (through advertising). CPs that are effective in generating revenue are in a better financial position to subsidize consumption of their content than CPs who generate less revenue (e.g., startups without an established consumer base).

Proponents of net neutrality have argued that such arrangements contradict the spirit of their cause, which holds that all content transmitted over the Internet should be treated equally (i.e., Internet service providers should be neutral in their treatment of every data packet, regardless of its origin, destination, or content). They argue that subsidization plans effectively grant preferential treatment to the subsidized packets, since consumers have more incentive to consume 'free' packets rather than unsubsidized ones (Knutason and Gryta, 2014; Kravets, 2014). The telecom companies, however, have argued that these plans do not violate the principle of net neutrality since the sponsored data will be delivered at the same speed and performance levels as non-sponsored data. They also maintain that the plan affects only the CP paying for the sponsored traffic; it does not prioritize that traffic, nor does it throttle usage of non-sponsored content (Knutason and Gryta, 2014). However, some CPs have contested this view. YouTube, which is

not a member of T-Mobile's Binge On program, recently complained that its videos were not being streamed at HD quality, which is a requirement of T-Mobile for its Binge On partners (Kastrenakes 2015).

U.S. regulators are still exploring the welfare implications of these and similar new data subsidization plans. In letters to AT&T and T-Mobile, the Federal Communications Commission previously wrote: "We want to ensure that we have all the facts to understand how these services (data subsidization) relate to the commission's goal of maintaining a free and open Internet while incentivizing innovation and investment from all sources" (Kang 2015). Additionally, while FCC Chairman Tom Wheeler has said the commission would be "keeping an eye on" the data subsidization programs, he has also praised them as "highly innovative and highly competitive" (Melendez 2015). In light of this ongoing evaluation, it is critical that policymakers be equipped with an analytical framework for gauging the impact of the new data subsidization plans on social welfare and understanding the consequential policy implications.

Without broaching the issue of whether such plans violate the principle of net neutrality, our research aims to analyze the subsidization plans within an economic framework. First, we identify the incentives of the different players (the ISP, CPs, and consumers) in order to analyze the dominating equilibrium outcomes. We then compare the consumer surplus and social welfare under such equilibria with the baseline scenario in which the ISP does not implement a subsidization plan.

The remainder of this paper is arranged as follows. We first review the literature that motivated our research. We then explain our model, analyze it under four subsidization scenarios, and discuss some of our expected results. Finally, we analyze the impact of such results, discuss their possible policy implications, and provide some insights on future research.

2. Literature Review

Our research deploys the general analysis framework for network management with three types of players. There are two streams of existing literature on network management. The first focuses on the economic analysis of net neutrality within a two-sided market framework and how preferential access fees from CPs influence the preferential delivery of digital content to consumers (Choi and Kim 2010; Cheng et al. 2011).

The second stream looks at subsidization plans in communications technology, which includes both hardware and software products. One of the reasons subsidization plans have recently drawn research and media attention is the growth of household budgets allocated for cell phone service. Specifically, after the introduction of Apple's iPhone, average annual household expenditures on cell phone service jumped by 10% (Troianovski 2012). Inoue et al. (2011) showed how customers care more about the monthly cost of their mobile carrier service contract than they do about discounts offered on the phone itself, suggesting how consumers prioritize content subsidization. Marcus (2014) suggested that ISPs can improve earnings without sacrificing revenue by having CPs subsidize a portion of the monthly online traffic cost for consumers. Economides and Hermalin (2014) used online traffic congestion to examine the effect of CPs' subsidies on their users. Ma (2014), meanwhile, analyzed competition among CPs to subsidize data usage costs. The latter analysis assumed that an ISP did not charge CPs, and that CPs voluntarily subsidized their users' cost for Internet connectivity, thus affecting the users' choice of data plan. None of these papers address the issues raised by the recent plans of some ISPs to deliver sponsored (subsidized) data at the same speed as non-sponsored data, which would allow CPs to subsidize their users' monthly costs through payments to ISPs without affecting access speed. Our study considers the following phenomenon: a contract between ISPs

and CPs that does not discriminate in terms of online traffic speed but nonetheless affects consumers' CP choice.

Therefore, we examine a subsidization plan that delivers both sponsored and non-sponsored content at the same speed. In addition, we model the CPs to participate in sponsored-data allocation by deciding on the amount of sponsored data, which also mirrors reality.

In the next section, we present our model and investigate various ways for an ISP to allow CPs to sponsor consumer data usage for end users. We then analyze how these proposals affect ISP and CP profits, consumer surplus, and social welfare.

3. Baseline Model

Our model for analyzing the subsidization plan assumes three types of players engaged in the delivery of digital content: a monopolist ISP, two competing CPs, and a unit mass of consumers. We assume the monopolist ISP has complete market coverage and delivers content from the CPs by providing Internet access and charging a usage-based price p to end consumers. The two CPs, L and H , compete for the end consumers. The end consumers pay the ISP for Internet access in order to consume digital content from CPs, under the assumption that they are heterogeneous in their CP preference. For a uniform distribution of consumer preference on a line segment $[0, 1]$, we set the preference for L at 0 and the preference for H at 1.

1) Internet Service Provider (ISP)

As mentioned above, the monopolist ISP charges usage-based price p per packet to consumers seeking Internet connectivity. For ease of exposition, we assume that CPs wishing to subsidize their users' data usage are also charged the same usage-based price p . Consumption is measured in packets of content (λ) and the ISP is assumed to have zero marginal cost. Accordingly, the

ISP's profit is formulated as $\pi_{ISP} = p \cdot \lambda$ and is directly related to the amount of content consumed.

Table 1 provides a list of the notations used in our model.

2) Content Provider (CP)

We use an indicator function to track whether the ISP allows CPs to subsidize consumers' usage. The indicator function is set at 1 if the ISP allows subsidization, and 0 otherwise. The indicators, I_L and I_H , are endogenous variables decided by the ISP according to its profit maximization scheme. We assume that CPs provide consumers with content for free and earn revenue through advertising (Choi and Kim 2010; Cheng et al. 2011). The amount of revenue that a CP earns from a consumer through advertising is directly related to the amount of data the latter consumes. Without loss of generality, we assume that H has a better ability to attract the right consumers for its advertisers; consequently it can charge higher fees to advertisers and has a higher revenue-generating rate, $r_H \geq r_L$, than L . If CPs are allowed to subsidize consumer usage, the ISP charges usage-based price p per packet per consumer to CPs, with the following profit equations:

$$\pi_L = r_L \lambda x - I_L \cdot p \lambda_{s,L} x \quad (1)$$

$$\pi_H = r_H \lambda x - I_H \cdot p \lambda_{s,H} x \quad (2)$$

3) Consumers

In this model, we consider a unit mass of consumers who access broadband content. The consumers have a gross valuation for the content, $V(\lambda)$ and decide whether to purchase content based on their net utility. Net utility consists of three components – the aforementioned gross value function, the cost caused by the difference between consumers' preferred and consumed content (i.e., the fit cost), and the cost of broadband access. To illustrate deviation from

consumers' ideal content, we introduce a fit cost t from the Hotelling framework (Hotelling 1929). When a consumer accesses the Internet to acquire content, they pay usage-based price p per packet to the ISP. If L or H subsidizes $\lambda_{s,L}$ or $\lambda_{s,H}$ packets, respectively, for their consumers, consumer cost decreases by $p\lambda_{s,L}$ or $p\lambda_{s,H}$, respectively. Therefore the utility function for an arbitrary consumer is interpreted as $x \in [0, 1]$ is $U_L(x) = V(\lambda) - tx - p\lambda + I_L \cdot p\lambda_{s,L}$ if the consumer utilizes content from L , or $U_H(x) = V(\lambda) - (1 - t)x - p\lambda + I_H \cdot p\lambda_{s,H}$ if the consumer utilizes content from H .

Sequence of Events in the Game

In this model, the three players – the ISP, CPs (CP L and CP H), and consumers – make decisions to maximize their utility or profits through a set sequence of events. The sequence of decisions is illustrated in Figure 1.

In stage 1, the ISP announces p and offers a contract allowing either CP to subsidize its consumers' data usage (determining the value of I_L and I_H). In stage 2, L and H , which have contracted with the ISP, decide how much content to subsidize for their users. In stage 3, consumers choose a CP. We use backward induction to deduce the subgame perfect equilibrium of the players' actions in each stage. In stage 3, consumers choose their preferred CP, after factoring in the ISP's usage-based price p per packet, the CPs' plan to subsidize their users with the amount of packets $\lambda_{s,L}$ and $\lambda_{s,H}$ and their intrinsic preference for L and H . Consumers' decisions can be demonstrated by the marginal consumer x^* , who is indifferent between L and H . That is, consumers located in $[0, x^*]$ choose L and consumers located in $[x^*, 1]$ choose H .

Moving back to stage 2, let us examine the CPs' decisions. Given the consumer information from stage 3, the CPs will decide on the number of user packets to subsidize to maximize their profits. We use the first order condition to solve the CPs' profit maximization problem. Finally, we analyze the ISP's decision in stage 1. The ISP sets a usage-based price p and decides whether to contract with L or H to maximize profits. Because of the full market coverage assumption and participation constraints, the ISP will set its prices and network management policies so that both the consumers' utility and the CPs' profits are non-negative. Considering these constraints, the ISP's profit maximization problem can be expressed as follows:

$$\begin{aligned} & \max_p \pi_{ISP} \\ \text{s.t.} \quad & U_H(x^*) \geq 0, U_L(x^*) \geq 0 \end{aligned} \tag{3}$$

$$\pi_L(\lambda_{s,L}^*) \geq 0, \pi_H(\lambda_{s,H}^*) \geq 0 \tag{4}$$

$$0 \leq x^* \leq 1 \tag{5}$$

$$0 \leq \lambda_{s,L}^* \leq \lambda, \quad 0 \leq \lambda_{s,H}^* \leq \lambda. \tag{6}$$

Constraints (3) and (4) specify the participation constraints of the marginal consumer and the two CPs. Constraint (5) is self-explanatory, while (6) specifies that the CPs will at most subsidize the entirety of content consumed by a consumer.

To analyze all the potential network management options of the ISP, we consider the four possible outcomes: the ISP does not allow either CP to subsidize packets (Case 1); the ISP allows L to subsidize packets (Case 2); the ISP allows H to subsidize packets (Case 3); and the ISP allows both L and H to subsidize packets. Table 2 presents these four network management options.

4. Results

As outlined in the previous section, we use backward induction to find the equilibrium amount of subsidized data, $\lambda_{s,L}^*$ and $\lambda_{s,H}^*$, and the usage-based price p^* in each case; the results are summarized in Tables 3, 4, and 5. Given the values of the exogenous parameters λ and $V(\lambda)$, Tables 3, 4, and 5 show that the equilibrium values of $\lambda_{s,L}^*$, $\lambda_{s,H}^*$ and p^* are dependent on the revenue generation rates of the CPs, r_L and r_H , and consumers' fit cost, t .

Proposition 1: (The equilibrium results). Depending on the values of the different parameters, there are several possible equilibria. When there is no subsidization (Case 1), the resultant price is given as $p^* = \frac{2V(\lambda) - t}{2\lambda}$. There are three possible equilibria in Case 2, depending on the value of r_L , which are summarized in Table 3. In Case 3, there are three possible equilibria, depending on the value of r_H , with results shown in Table 4. Finally, Case 4 encompasses five possible equilibria, depending on the values of both r_L and r_H , summarized in Table 5. All proofs are relegated to the Appendix.

As can be observed from these results, whether the CPs sponsor (subsidize) the consumption of content by their consumers hinges on the amount of consumer-generated revenue. When the revenue generation rate falls below a certain threshold, CPs do not have an incentive to sponsor traffic. However, when the rate exceeds that threshold, CPs begin partially sponsoring connectivity to content. Finally, when the rate reaches a second, higher threshold, CPs have the incentive to fully sponsor their consumers' traffic.

For a given set of parameter values, the ISP determines (a) whether to enter into a contract with one (L or H), both or neither of the CPs, (i.e., Cases 1, 2, 3 or 4, respectively) and (b) the equilibrium price that would maximize its profits in each scenario. Unfortunately, the surfeit of parameters makes it impossible to analytically derive the dominating equilibrium in many cases,

necessitating numerical analyses hereunder. Based on the equilibrium results, this section presents the preferred cases in terms of ISP profits, consumer surplus, and social welfare, deploying different parameter values in each case.

Consumer choice depends on the fit cost t : when t is high, consumers are more loyal to their preferred CP, requiring higher amounts of subsidization to induce them to migrate to another CP. A low t , on the other hand, makes it easier for CPs to induce consumers to switch from their (the CPs') rivals. Our numerical analyses thus clarify the effect of differing levels of fit cost t .

Another important parameter is the ratio of the revenue generation rate of the two CPs (r^L/r_H). Amid comparable revenue generation rates, the ISP will find it preferable to have similar policies regarding both CPs (since either one of them will react similarly to any proposed contract to subsidize content for their consumers). However, if the revenue generation rates are considerably different, the two CPs might find it advantageous to react differently to a contract proposed by the ISP. In our numerical analyses, the second factor that we examine is the effect of changing the ratio of the revenue generation rates of the two CPs (i.e., r^L/r_H).

In conducting the numerical analysis, we need to specify the values of three exogenous parameters, λ , $V(\lambda)$, and r_H . Meanwhile, in order to have complete market coverage in equilibrium in Cases 2 and 3, the upper bound of the second condition must be the same as the lower bound of the third condition in Tables 3 and 4. Specifying $\lambda = 1$ ensures that said upper and lower bounds are always the same, regardless of fit cost t and step-size of the numerical analysis. Further, in order to satisfy all necessary conditions for the existence of equilibrium in Cases 2, 3 and 4 (detailed within the proofs in the Appendix), we choose $V(\lambda) = 10$, and $r_H = 2$. Our results below are derived using these parameter values. To test the validity of our

numerical analyses, we experimented with other combinations of λ , $V(\lambda)$, and r_H under the feasible regions of equilibrium results in all cases; the results and propositions that follow hold for all such variances in parameter values.

Proposition 2 illustrates the interplay between the two main factors driving the equilibrium results: the fit cost t and the ratio of the revenue generation rates of the two CPs, r_L/r_H .

Proposition 2 (ISP's Network Management Choices). If the fit cost is low, the ISP will choose one of the equilibria in Case 3 (H is allowed to subsidize its packets) or one of those in Case 4 (H and L are allowed to subsidize their packets), depending on the ratio of the revenue generation rate of the two CPs. However, if the fit cost is high, the ISP will not enter into any contract that allows subsidization of content by the CPs.

In practical terms, an ISP will enter into a contract with a CP to subsidize online consumption for the latter's consumers only if those consumers are not very loyal to their preferred CP.

Moreover, if the consumers are less loyal to the CPs, depending on whether the ratio of the revenue generation rate (r_L/r_H) is low or high, the ISP will enter into a contract either solely with CP H or with both CPs L and H .

Figure 2 illustrates the ISP's network management choices under various parameter values. The ISP's profit is always maximized in either Case 3 or Case 4 if the fit cost is low. However, if the fit cost is high, the ISP allows no subsidization (Case 1). The following observations offer some inferences for Proposition 2. First, we observe that in cases where the value of r_L is relatively low, CP L cannot participate in any subsidization plan because it cannot afford to pay the connectivity cost for its users. Second, if the fit cost is low, consumers may switch from one CP to another to take advantage of the subsidization. As r_L increases and subsidization becomes

more affordable, CP L will also begin to subsidize its content to attract more consumers. If the ISP allows both L and H to subsidize consumer usage, p increases amid competition between L and H : both CPs would rather pay the ISP than lose consumers to rivals, thereby bidding up p . In effect, the CPs are caught in a classic “Prisoner’s Dilemma”, which the monopolist ISP can exploit to its advantage. Accordingly, the ISP reaps the largest profit under Case 4. Third, if the fit cost between the CPs is high, consumers will remain loyal to their preferred CP and will not easily abandon it, even if the rival CP subsidizes content.

Next, we consider the conditions that effectively maximize consumer surplus and social welfare, as well as the conditions that maximize the CPs’ profits. Proposition 2 establishes how, depending on parameter values, the ISP might prefer outcomes under Cases 1, 3 or 4. However, the policymaker’s perspective does not contemplate ISP profit alone. Indeed, policymakers consider other factors as well, specifically CPs’ profit, consumer surplus and social welfare. Propositions 3, 4, and 5 are formulated with those factors in mind.

Proposition 3 (CPs’ profit). CP L maximizes its profit under Case 1 or Case 3. The result is illustrated in Figure 3, which shows CP L ’s preference while considering the different fit cost and the competitiveness of two CPs (L and H). The result is interpreted as follows: regardless of the fit cost or the revenue generation rate, CP L prefers not to subsidize its consumers’ access costs due to the adverse effect that subsidization would exert on its profit. If CP L has a low revenue generation rate, then it does not have the capability for subsidization. Moreover, even if the rate increases, CP L prefers to forego competing with CP H since the subsidization would increase the profit of the ISP and not that of CP L .

Figure 3 illustrates how CP L maximizes its profit under either Case 3 or Case 4. The result shows that if CP L is less competitive, CP H has incentive to attract consumers who would

choose a CP based on subsidization. Hence, CP H maximizes its profit under Case 4 if both the fit cost and the revenue generation rate are low (i.e., if CP L is less competitive). However, if the competitiveness between the two CPs rises (i.e., CP L becomes more efficient at revenue generation), subsidization would not increase the profit of CP H ; rather, the ISP would maximize its profit by exploiting the competition between the CPs. Therefore, if either the fit cost or the revenue generation rate is high, CP H prefers not to subsidize its consumers' usage (Case 1). We now consider consumer surplus in order to evaluate the influence that digital contents exert on end users when subsidization is allowed. Consumer surplus is the sum of consumers' utility and is calculated as follows:

$$\text{Consumer Surplus} = \int_0^{x^*} U_L(x)dx + \int_{x^*}^1 U_H(x)dx \quad (7)$$

Proposition 4 (Consumer Surplus). Consumer surplus is maximized under Cases 2 or 4 when the fit cost is low. However, if that cost is high, consumers prefer to forego subsidization.

Figure 5 illustrates the proposition graphically.

It is instructive to analyze why such a result holds. As Figure 6 illustrates, the ISP charges the lowest usage-based price p under Case 1 (i.e., no subsidization). One might therefore expect that Case 1 would maximize consumer utility. However, under the specifications of Case 2, wherein the ISP charges the second-lowest price to consumers and CPs, consumers enjoy the lowest net cost to get online if they choose their content from CP L (thanks to L 's subsidization). The next consideration is the fit cost. A small fit cost implies that consumers will easily switch from their preferred CP to a rival CP. In the process, they incur a low disutility cost but can reduce their connectivity costs (from CP subsidization), which increases overall utility. But if consumers have a strong preference for their CP (high fit cost), they will likely not switch providers, electing to pay a little more to continue using their preferred CP. Under such a scenario, they

would prefer Case 4, which offers lower connectivity costs (due to CP subsidization) as well as the opportunity to continue enjoying content from their preferred CP. However, when the fit cost is prohibitively high, the CPs balk at paying the high price required to entice marginal consumers away from their rival. Thus, for very high fit costs, consumer surplus is maximized under Case 1. Along with ISP profit maximization and consumer satisfaction, we also examine which subsidization plan options are socially optimal. We define social welfare to be the sum of the ISP's profit, the CPs' profits, and the consumer surplus.

Proposition 5 (Social Welfare). Social welfare is maximized under Case 1 or Case 4.

Figure 8 illustrates the proposition graphically. When the fit cost is high, the profits of the ISP and the CPs, as well as consumer surplus, are maximized under Case 1. Accordingly, Case 1 maximizes social welfare under conditions of high fit cost. When the fit cost is relatively low, CP *L*'s profit is maximized when it does not subsidize its users (see Figure 3). In addition, CP *H* seeks to subsidize content when CP *L* does the same (Case 4) or when neither CP subsidizes content (Case 1) (see Figure 6). Cumulatively, overall welfare is highest in Case 4 when the fit cost is low. We further examine the components of social welfare in the following discussion section.

5. Discussion

The results in the previous section are useful in determining equilibrium scenarios, but a policymaker would presumably seek clarification of the distribution of the social surplus. For example, a scenario where social surplus is not maximized (even while approaching maximization) but the surplus is more equitably distributed among the various actors would perhaps be preferable to a welfare-maximizing option where the surplus is mostly directed

towards just one actor (or an option with minimal consumer surplus). We consider such scenarios in this section, highlighting four potential cases that illustrate how the ISP's decisions influence its profit, the consumer surplus, social welfare, and the CPs' profits. In this analysis, we exclude the instance with a very high fit cost, as Case 1 (no subsidization) would then be the most preferred scenario for all three types of players. The aforementioned four scenarios are as follows: (1) t is small and r_L/r_H is small (Figure 8); (2) t is small and r_L/r_H is large (Figure 9); (3) t is large and r_L/r_H is small (Figure 10); and (4) t is large and r_L/r_H is large (Figure 11). The stacked bars in Figures 8 through 11 represent social welfare as the sum of the profit of CP H , the profit of CP L , the consumer surplus, and the ISP's profit.

Not surprisingly, if CP H generates revenue much more efficiently than CP L (i.e., r_L/r_H is small, Figures 8 and 10), the two large contributors to social welfare are the profits of H and of the ISP. However, when the CPs are relatively competitive with each other (in terms of their revenue generation rates), we see that the ISP profit makes the biggest contribution to social welfare (Figures 9 and 11). In other words, if the CPs are more or less equally competitive, their profits are mostly extracted by the ISP. At the same time, consumer surplus also suffers, as compared to the case under scenarios 1 and 3. Additionally, under most conditions (except for that of a very high fit cost), social welfare is maximized in Case 4, which is consistent with the results discussed in Section 4. As a general rule, consumer surplus is higher when consumers are more loyal (high fit cost). We also find that minimizing the net price charged to consumers maximizes consumer surplus and social welfare, regardless of the fit cost and the ratio of revenue generation rates between the two CPs. If the equitable distribution of the cumulative surplus is a concern,

these guidelines should provide a basis for deciding whether there is a need for market intervention if the ISP decides to propose a data subsidization plan to the CPs.

6. Conclusion

Subsidizing consumers for broadband usage is fast becoming a way for content providers to get a leg up on the competition. In this paper, we examined the strategies that a monopolist ISP might exploit for profit maximization in the course of allowing CPs to subsidize consumer data usage. As smartphones become bigger and more powerful, consumers are increasingly using them to watch videos, listen to streaming music and operate other bandwidth-hungry applications. ISPs must therefore rapidly upgrade their broadband infrastructures in order to meet this exploding demand for bandwidth (Fung 2014). For many households, the cost of Internet access already represents a significant portion of their budget (Troianovski 2012). ISPs are thus trying to solve the challenge of effectively monetizing broadband traffic while increasing infrastructure capital expenditures without passing the cost onto their consumers (Cisco 2015). Convincing CPs to subsidize bandwidth access may prove key to solving this challenge without contravening the principle of net neutrality.

It is this phenomenon that motivated our research and led us to examine which method of subsidization would maximize an ISP's profit in the presence of two competing CPs. To review, our model considers two pricing policies. First, we assume that an ISP charges the same price p to consumers as well as to CPs. ISPs would gain access to a new revenue stream, and consumers would potentially reduce their cost of Internet access. However, if consumers were very loyal to their preferred CP, they might end up spending more to access their preferred content due to the increase in the price p . However, scenarios where ISPs maximize profits do not always result in maximization of consumer surplus and/or social welfare. We find that consumer surplus and

social welfare are maximized when the net price charged to consumers is minimized, regardless of the fit cost and the ratio of revenue generation rates between the two CPs. Our analysis should thus serve as a cogent reference for policymakers as they study this phenomenon.

Several managerial implications emerge from our research. For an ISP, the best subsidization plan will involve inducing the dominant CP to pay for content subsidization if the revenue generation rate of the latter is much higher compared to its competition and if consumers are easily persuaded to switch their CP loyalties. However, in a content market dominated by two relatively comparable competitors, the ISP would prefer a subsidization plan that induces both CPs to retain their customers and subsidize their content for consumption.

There are several possible extensions to our research. Guo et al. (2013) considered heavy consumers and light consumers to capture patterns of heterogeneous data usage. Going forward, we intend to extend our research to study consumers who have different data usage patterns. Curien and Moreau (2007) researched complementary strategies between ISPs and CPs, in which access providers participate in the production and distribution of digital content. They found that these strategies benefit both access providers and CPs: the former acquire additional profits from co-producing content to generate more data consumption, while the latter are able to exploit the relatively larger market of the access providers and crack down on the illegal consumption of their digital content. Our model could be extended to incorporate this circumstance in which the ISP is merged with a CP to provide both Internet connectivity as well as digital content. Prominent examples of such mergers can be found in the United States: Comcast, for example, has a majority stake in NBC Universal, while Google is reconnoitering the ISP market with its rollout of Google Fiber (Brodkin 2015; Snider et al. 2015). The implications of subsidization in

such contexts should provide additional insights on the rapid evolution of the production and delivery of digital content.

Tables

Table 1. List of Notation.

Notation	Description
p	Usage-based price charged to consumers and CPs L and H
t	Fit cost parameter for an end consumer away from the content which is perfectly fitted with her preference
λ	Poisson arrival rate of content requested from each consumer in packets per unit of time
$\lambda_{s,L}, \lambda_{s,H}$	Poisson arrival rate of content subsidized for each consumer in packets per unit of time
$\lambda_{s,L}^*, \lambda_{s,H}^*$	Poisson arrival rate of CP-subsidized content for each consumer in packets per unit of time, in equilibrium
r_L, r_H	CP's revenue rate per packet request for content
x	An arbitrary consumer on $[0, 1]$
x^*	Marginal consumer who is indifferent between L and H in equilibrium
$V(\lambda)$	Gross value function of retrieving content for each consumer
I_L, I_H	Function to indicate whether ISP allows CP L or H to subsidize the usage of their consumers
U_L, U_H	Consumers' utility function of subscribing to L and H
π_{ISP}	ISP profit
π_L, π_H	CP profit with a certain subsidy plan i
CS	Consumer surplus
SW	Social welfare

Table 2. ISP's network management options.

Case 1 ISP does not allow either CP to subsidize.	Case 2 ISP allows only L to subsidize packets.
Case 3 ISP allows only H to subsidize packets.	Case 4 ISP allows both L and H to subsidize packets.

Table 3. Equilibrium in Case 2.

Conditions	$\lambda_{s,L}^*$	p^*
$r_L < \frac{t}{\lambda}$	0	$\frac{2V(\lambda) - t}{2\lambda}$
$\frac{t}{\lambda} \leq r_L < \frac{4\lambda V(\lambda) + (2\lambda - 3)t}{\lambda(-1 + 2\lambda)}$	$\frac{2\lambda(r_L\lambda - t)}{4V(\lambda) - 3t + r_L\lambda}$	$V(\lambda) + \frac{r_L\lambda - 3t}{4\lambda}$
$r_L \geq \frac{4V(\lambda) + t(\lambda - 2)}{\lambda^2}$	λ	$\frac{2V(\lambda) - t}{\lambda}$

Table 4. Equilibrium in Case 3.

Conditions	$\lambda_{s,H}^*$	p^*
$r_H < \frac{t}{\lambda}$	0	$\frac{2V(\lambda) - t}{2\lambda}$
$\frac{t}{\lambda} \leq r_H < \frac{4\lambda V(\lambda) + (2\lambda - 3)t}{\lambda(-1 + 2\lambda)}$	$\frac{2\lambda(r_H\lambda - t)}{4V(\lambda) - 3t + r_H\lambda}$	$V(\lambda) + \frac{r_H\lambda - 3t}{4\lambda}$
$r_H \geq \frac{4V(\lambda) + t(\lambda - 2)}{\lambda^2}$	λ	$\frac{2V(\lambda) - t}{\lambda}$

Table 5. Equilibrium in Case 4.

Conditions		$\lambda_{s,L}^*$	$\lambda_{s,H}^*$	p^*
$r_L \leq \frac{t}{\lambda}$	$r_H \leq \frac{t}{\lambda}$	0	0	$\frac{2V(\lambda) - t}{2\lambda}$
$(r_H + 2r_L) \leq \frac{3t}{\lambda}$	$\frac{t}{\lambda} < r_H \leq \frac{4V(\lambda) - t}{\lambda}$	0	$\frac{2(-t\lambda + r_H\lambda^2)}{4V(\lambda) - 3t + r_H\lambda}$	$\frac{4V(\lambda) - 3t + r_H\lambda}{4\lambda}$
$\frac{-2V(\lambda) + 2t}{\lambda} \geq r_L$	$\frac{4V(\lambda) - t}{\lambda} < r_H$	0	λ	$\frac{2V(\lambda) - t}{\lambda}$
$\frac{3t}{\lambda} < (2r_L + r_H)$	$(r_H - r_L) \leq \frac{6V(\lambda) - 3t}{\lambda}$	$\frac{2\lambda(-3t + (r_H + 2r_L)\lambda)}{3(2V(\lambda) - 3t + (r_H + r_L)\lambda)}$	$\frac{2\lambda(-3t + (2r_H + r_L)\lambda)}{3(2V(\lambda) - 3t + (r_H + r_L)\lambda)}$	$\frac{2V(\lambda) - 3t + (r_H + r_L)\lambda}{2\lambda}$
$\frac{-2V(\lambda) + 2t}{\lambda} < r_L$	$(r_H - r_L) > \frac{6V(\lambda) - 3t}{\lambda}$	$\frac{\lambda(2V(\lambda) - 2t + r_L\lambda)}{-3t + 4V(\lambda) + r_L\lambda}$	λ	$\frac{4V(\lambda) - 3t + r_L\lambda}{\lambda}$

Figures

Figure 1. The sequence of events in the game.

Figure 2. Internet Service Provider's network management choices ($V(\lambda) = 10, \lambda = 1, r_H = 2$).

Figure 3. CP L 's profit maximizing network management options ($V(\lambda) = 10, \lambda = 1, r_H = 2$).

Figure 4. CP H 's profit maximizing network management options ($V(\lambda) = 10, \lambda = 1, r_H = 2$).

Figure 5. Network management options that maximize consumer surplus ($V(\lambda) = 10, \lambda = 1, r_H = 2$).

Figure 6. Usage-based price p in terms of r_L/r_H and t ($V(\lambda) = 10, \lambda = 1, r_H = 2$).

Figure 7. Social welfare maximizing network management options ($V(\lambda) = 10, \lambda = 1, r_H = 2$).

Figure 8. Scenario 1 (t is small and r_L/r_H is small)

Figure 9. Scenario 2 (t is small and r_L/r_H is large)

Figure 10. Scenario 3 (t is large and r_L/r_H is small)

Figure 11. Scenario 4 (t is large and r_L/r_H is large)

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Mathematical Appendix

Appendix A: Baseline Problem

Case 1 (Benchmark Case). The ISP allows no CPs to subsidize their consumers' data usage.

In Case 1, the timeline of the game is given as follows: in stage 1, the ISP announces p , which is the usage-based price for consumers; in stage 2, the end consumers choose content from either L or H . We solve this game-theoretical model using the backward induction.

In stage 2, the price charged by the ISP, p , is given. To find out the market share of both CPs, we identify the location of marginal consumers who are indifferent between choosing content from L and from H : $U_L(x^*) = U_H(x^*)$, where $U_L(x) = V(\lambda) - p\lambda x - t$ and $U_H(x) = V(\lambda) - p\lambda - t(1 - x)$. This leads to $x^* = \frac{1}{2}$ and this result implies equal market share for the two CPs. In stage 1, the ISP chooses p to maximize its profit given the participation constraints for consumers and content providers:

$$\max_p \pi_{ISP} = p\lambda,$$

$$s.t.$$

$$U_L\left(\frac{1}{2}\right) \geq 0, U_H\left(\frac{1}{2}\right) \geq 0.$$

From the constraints, $U_H\left(\frac{1}{2}\right) \geq 0$ and $U_L\left(\frac{1}{2}\right) \geq 0$, we can obtain the upper bound of p ,

$\frac{2V(\lambda) - t}{2\lambda} \geq p$. Therefore, when the constraint is binding, the highest value of the objective

function is $\pi_{ISP} = \frac{2V(\lambda) - t}{2}$ where $2V(\lambda) - t \geq 0$.

Case 2. The ISP allows CP L to subsidize its consumers' data usage.

In this case, the timeline of the game is given as follows: in stage 1, the ISP announces p , which is the usage-based price for both consumers and CP L ; in stage 2, L decides the amount of its subsidization, $\lambda_{s,L} \in [0, \lambda]$; finally, in stage 3, the end consumers choose either L or H . We solve this game-theoretical model using backward induction.

In stage 3, The marginal consumer is given by: $U_L(x^*) = U_H(x^*)$ where $U_L(x) = V(\lambda) - p(\lambda - \lambda_{s,L}) - tx$ and $U_H(x) = V(\lambda) - p\lambda - t(1 - x)$. Since L subsidizes consumers' data usage by $\lambda_{s,L}$, the ISP does not charge p for the quantity of the data-packet $\lambda_{s,L}$. In contrast with case 1, we find that the utility of a consumer increases by $p\lambda_{s,L}$ in case 2. The marginal consumer is given by $x^* = \frac{1}{2} + \frac{p\lambda_{s,L}}{2t}$; Because the marginal consumer is located at $x^* \in [0, 1]$, the following inequality conditions should be satisfied: $0 \leq x^* = \frac{1}{2} + \frac{p\lambda_{s,L}}{2t} \leq 1$. From the constraint, two results can be shown as follows: (1) the upper bound of p is $p \leq \frac{t}{\lambda_{s,L}}$; (2) the market share of CP L is $x^* \geq \frac{1}{2}$. If we compare x^* with the market share of CP L in Case 1, the market share of CP L increases by $\frac{p\lambda_{s,L}}{2t}$ with the subsidization.

In stage 2, given p , CP L decides the subsidized amount $\lambda_{s,L}$ to maximize its profit: $\pi_L = r_L \lambda x^* - p \lambda_{s,L} x^*$. From the first order condition of the profit function, $\frac{\partial \pi_L}{\partial \lambda_{s,L}} = 0$, we can obtain the optimal the subsidized amount of CP L , $\lambda_{s,L}^* = \frac{r_L \lambda - t}{2p}$.

From this result, we can derive the optimal value of $\lambda_{s,L}^*$ based on $\lambda_{s,L} \in [0, \lambda]$:

$$\lambda_{s,L}^* = \begin{cases} 0, & \text{if } r_L < \frac{t}{\lambda} \\ \frac{r_L \lambda - t}{2p}, & \text{if } \frac{t}{\lambda} \leq r_L < \frac{2p+t}{\lambda} \\ \lambda, & \text{if } r_L \geq \frac{2p+t}{\lambda} \end{cases}$$

In stage 1, the ISP decides p with the participation constraints and the formulation is as follows:

$$\begin{aligned} \max_p \quad & \pi_{ISP} = p\lambda \\ \text{s.t.} \quad & \\ & U_L(x^*) \geq 0, x^* \in [0, 1] \\ & U_H(x^*) \geq 0, x^* \in [0, 1] \\ & \pi_L(\lambda_{s,L}^*) \geq 0 \end{aligned}$$

From the marginal consumer's participation constraint $U_L(x^*) \geq 0$ and $U_H(x^*) \geq 0$, which is demonstrated as $V(\lambda) - p(\lambda - \lambda_{s,L}^*) - tx^* \geq 0$, we can obtain the upper bound of p as follows:

if $r_L < \frac{t}{\lambda}$, $\frac{2V(\lambda)-t}{2\lambda} \geq p$; if $\frac{t}{\lambda} \leq r_L < \frac{2p+t}{\lambda}$, $V(\lambda) + \frac{r_L \lambda - 3t}{4\lambda} \geq p$; if $r_L \geq \frac{2p+t}{\lambda}$, $\frac{2V(\lambda)-t}{\lambda} \geq p$. The constraint $\pi_L(\lambda_{s,L}^*) \geq 0$ is always satisfied.

Therefore, the equilibrium is given by the following three cases: (1) If $r_L < \frac{t}{\lambda}$, $x^* = \frac{1}{2}$, $p^* =$

$\frac{2V(\lambda)-t}{2\lambda}$, and $\lambda_{s,L}^* = 0$ with a constraint $2V(\lambda) - t \geq 0$;

(2) If $\frac{t}{\lambda} \leq r_L < \frac{2p+t}{\lambda}$, $x^* = \frac{1}{2} + \frac{r_L \lambda - t}{4t}$, $p^* = V(\lambda) + \frac{r_L \lambda - 3t}{4\lambda}$, and $\lambda_{s,L}^* = \frac{r_L \lambda - t}{2p}$ with constraints

$V(\lambda) + \frac{r_L \lambda - 3t}{4\lambda} \geq 0$ and $r_L \lambda < 3t$;

(3) If $r_L \geq \frac{2p+t}{\lambda}$, $x^* = \frac{1}{2} + \frac{p\lambda}{2t} = \frac{V(\lambda)}{t}$, $p^* = \frac{2V(\lambda)-t}{\lambda}$, and $\lambda_{s,L}^* = \lambda$ with constraints $2V(\lambda) - t \geq 0$ and $V(\lambda) < t$.

Case 3. The ISP allows CP H to subsidize its consumers' data usage by $\lambda_{s,H}$.

The results of Case 2 and Case 3 are symmetric. Therefore, we can obtain the following three possible results that yield the highest objective values:

(1) If $r_H < \frac{t}{\lambda}$, $x^* = \frac{1}{2}$, $p^* = \frac{2V(\lambda)-t}{2\lambda}$, and $\lambda_{s,H}^* = 0$ with a constraint $2V(\lambda) - t \geq 0$;

(2) If $\frac{t}{\lambda} \leq r_H < \frac{2p+t}{\lambda}$, $x^* = \frac{1}{2} + \frac{r_H\lambda-t}{4t}$, $p^* = V(\lambda) + \frac{r_H\lambda-3t}{4\lambda}$, and $\lambda_{s,H}^* = \frac{r_H\lambda-t}{2p}$ with constraints $V(\lambda) + \frac{r_H\lambda-3t}{4\lambda} \geq 0$ and $r_H\lambda < 3t$;

(3) If $r_H \geq \frac{2p+t}{\lambda}$, $x^* = \frac{1}{2} + \frac{p\lambda}{2t} = \frac{V(\lambda)}{t}$, $p^* = \frac{2V(\lambda)-t}{\lambda}$, and $\lambda_{s,H}^* = \lambda$ with constraints $2V(\lambda) - t \geq 0$ and $V(\lambda) < t$.

Case 4. The ISP allows both CP L and CP H to subsidize their consumers' data usage by $\lambda_{s,L}$ and $\lambda_{s,H}$, respectively.

In this case, the timeline of the game is given as follows: in stage 1, the ISP announces p , which is the usage-based price for consumers and two CPs; in stage 2, L and H decide the amount of their subsidization, $\lambda_{s,L} \in [0, \lambda]$ and $\lambda_{s,H} \in [0, \lambda]$ respectively; finally, in stage 3, the end consumers choose either L or H . We solve this game-theoretical model using backward induction.

In stage 3, the marginal consumer is given by $U_L(x^*) = U_H(x^*)$ where $U_L(x) = V(\lambda) - p(\lambda - \lambda_{s,L}) - tx$ and $U_H(x) = V(\lambda) - p(\lambda - \lambda_{s,H}) - t(1 - x)$. Since L and H subsidize consumers' data usage by $\lambda_{s,L}$ and $\lambda_{s,H}$, the ISP does not charge p for the quantity of the data-packet $\lambda_{s,L}$ and $\lambda_{s,H}$. Accordingly, we find that a consumer's utility increases by $p\lambda_{s,L}$ and $p\lambda_{s,H}$ comparing Case 4 with Case 1. From the above equation, we can derive the market share of content providers: the market share for content provider L is given by $x^* = \frac{1}{2} + \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t}$; the market share for content provider H is given by $(1 - x^*) = \frac{1}{2} - \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t}$. Because the marginal consumer is located at $x^* \in [0, 1]$, the following inequality condition should be satisfied: $0 \leq x^* = \frac{1}{2} + \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t} \leq 1$. From the constraint, we obtain the following results: the upper bound of p , $p \leq \frac{t}{(\lambda_{s,L} - \lambda_{s,H})}$ if $\lambda_{s,L} \geq \lambda_{s,H}$ and $p \leq \frac{t}{(\lambda_{s,H} - \lambda_{s,L})}$ if $\lambda_{s,H} \geq \lambda_{s,L}$;

In stage 2, given p , CP L and CP H decide the amount of subsidization $\lambda_{s,L}$ and $\lambda_{s,H}$ to maximize their profit: $\pi_L = r_L \lambda x^* - p\lambda_{s,L} x^*$ and $\pi_H = r_H \lambda x^* - p\lambda_{s,H} x^*$. From the first order conditions of the profit functions, $\frac{\partial \pi_L}{\partial \lambda_{s,L}^*} = 0$ and $\frac{\partial \pi_H}{\partial \lambda_{s,H}^*} = 0$, we obtain the following reaction functions,

$$\lambda_{s,L}^*(\lambda_{s,H}) = \frac{-t + p\lambda_{s,H} + r_L \lambda}{2p} \text{ and } \lambda_{s,H}^*(\lambda_{s,L}) = \frac{-t + p\lambda_{s,L} + r_H \lambda}{2p}.$$

Therefore, the reaction functions $\lambda_{s,L}^*(\lambda_{s,H})$ and $\lambda_{s,H}^*(\lambda_{s,L})$ are given by:

$$\lambda_{s,L}^*(\lambda_{s,H}) = \begin{cases} 0, & \text{if } \lambda_{s,H} \leq \frac{t-r_L\lambda}{p} \\ \frac{-t+p\lambda_{s,H}+r_L\lambda}{2p}, & \text{if } \frac{t-r_L\lambda}{p} < \lambda_{s,H} \leq \frac{t-r_L\lambda}{p} + 2\lambda, \\ \lambda, & \text{if } \lambda_{s,H} > \frac{t-r_L\lambda}{p} + 2\lambda \end{cases}$$

$$\lambda_{s,H}^*(\lambda_{s,L}) = \begin{cases} 0, & \text{if } \lambda_{s,L} \leq \frac{t-r_H\lambda}{p} \\ \frac{-t+p\lambda_{s,L}+r_H\lambda}{2p}, & \text{if } \frac{t-r_H\lambda}{p} < \lambda_{s,L} \leq \frac{t-r_H\lambda}{p} + 2\lambda. \\ \lambda, & \text{if } \lambda_{s,L} > \frac{t-r_H\lambda}{p} + 2\lambda \end{cases}$$

In stage 1, the ISP chooses p given the participation constraints:

$$\begin{aligned} \max_p \pi_{ISP} &= p\lambda \\ \text{s.t.} \\ U_L(x^*) &\geq 0, U_H(x^*) \geq 0 \\ 0 \leq x^* &= \frac{1}{2} + \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t} \leq 1 \\ \pi_L(\lambda_{s,L}^*) &\geq 0, \pi_H(\lambda_{s,H}^*) \geq 0. \end{aligned}$$

The optimal p is derived as $p^* = \frac{2V(\lambda)-t}{2\lambda-(\lambda_{s,L}+\lambda_{s,H})}$.

Combining $\lambda_{s,L}^*(\lambda_{s,H})$ and $\lambda_{s,H}^*(\lambda_{s,L})$ with $p^* = \frac{2V(\lambda)-t}{2\lambda-(\lambda_{s,L}^*+\lambda_{s,H}^*)}$, we can obtain five equilibria

as follows:

- (1) If $\lambda_{s,L} \leq \frac{t-r_H\lambda}{p}$ and $\lambda_{s,H} \leq \frac{t-r_L\lambda}{p}$, $\lambda_{s,L}^* = 0$, $\lambda_{s,H}^* = 0$, and $p^* = \frac{2V(\lambda)-t}{2\lambda}$.
- (2) If $\frac{t-r_H\lambda}{p} < \lambda_{s,L} \leq \frac{t-r_H\lambda}{p} + 2\lambda$ and $\lambda_{s,H} \leq \frac{t-r_L\lambda}{p}$, $\lambda_{s,L}^* = 0$, $\lambda_{s,H}^* = \frac{2(t\lambda-r_H\lambda^2)}{3t-4V(\lambda)-r_H\lambda}$, and $p^* = \frac{4V(\lambda)+r_H\lambda-3t}{4\lambda}$.
- (3) If $\lambda_{s,L} > \frac{t-r_H\lambda}{p} + 2\lambda$ and $\lambda_{s,H} \leq \frac{t-r_L\lambda}{p}$, $\lambda_{s,L}^* = 0$, $\lambda_{s,H}^* = \lambda$, and $p^* = \frac{2V(\lambda)-t}{\lambda}$.

$$\begin{aligned}
(4) \quad & \text{If } \frac{t-r_H\lambda}{p} < \lambda_{s,L} \leq \frac{t-r_H\lambda}{p} + 2\lambda \text{ and } \frac{t-r_L\lambda}{p} < \lambda_{s,H} \leq \frac{t-r_L\lambda}{p} + 2\lambda, \lambda_{s,L}^* = \\
& \frac{2\lambda(-3t+(r_H+2r_L)\lambda)}{3(-3t+2V(\lambda)+(r_H+r_L)\lambda)}, \lambda_{s,H}^* = \frac{2\lambda(-3t+(2r_H+r_L)\lambda)}{3(-3t+2V(\lambda)+(r_H+r_L)\lambda)}, \text{ and } p^* = \frac{-3t+2V(\lambda)+(r_H+r_L)\lambda}{2\lambda}. \\
(5) \quad & \text{If } \lambda_{s,L} > \frac{t-r_H\lambda}{p} + 2\lambda \text{ and } \frac{t-r_L\lambda}{p} < \lambda_{s,H} \leq \frac{t-r_L\lambda}{p} + 2\lambda, \lambda_{s,L}^* = \frac{\lambda(-2t+2V(\lambda)+r_L\lambda)}{-3t+4V(\lambda)+r_L\lambda}, \\
& \lambda_{s,H}^* = \lambda, \text{ and } p^* = \frac{4V(\lambda)-3t+r_L\lambda}{\lambda}.
\end{aligned}$$

Appendix B. Extended Problem

Case 1 (Benchmark Case). The ISP allows no CPs to subsidize their consumers' data usage.

The results of Case 1 in the extended problem are same as the results of Case 1 in the baseline problem.

Case 2. The ISP allows CP L to subsidize the data usage of CP L 's consumers.

In this case, the timeline of the game is given as follows: in stage 1, the ISP announces c , which is the usage-based price for CP L ; in stage 2, L decides the amount of its subsidization, $\lambda_{s,L} \in [0, \lambda]$; finally, in stage 3, the end consumers choose either L or H . We solve this game-theoretical model using backward induction.

The stage 3 in the extended problem is same as the stage 3 in our baseline problem. The market share for content provider L is $x^* = \frac{1}{2} + \frac{p\lambda_{s,L}}{2t}$. The market share for content provider H is

$(1 - x^*) = \frac{1}{2} - \frac{p\lambda_{s,L}}{2t}$. In stage 2, with the assumption, c is given, CP L decides the subsidized amount $\lambda_{s,L}$ to maximize its profit, which is demonstrated with the following equation: $\pi_L =$

$r_L \lambda x^* - c \lambda_{s,L} x^*$. From the first order condition of the profit function, $\frac{\partial \pi_L}{\partial \lambda_{s,L}} = 0$, we can find the

optimal the subsidized amount of CP L , $\lambda_{s,L}^* = \frac{-ct + pr_L \lambda}{2cp}$.

We can derive the optimal value of $\lambda_{s,L}^*$ based on $\lambda_{s,L} \in [0, \lambda]$

$$\lambda_{s,L}^* = \begin{cases} 0, & \text{if } \lambda_{s,L}^* < 0 \\ \frac{-ct + pr_L \lambda}{2cp}, & \text{if } 0 \leq \lambda_{s,L}^* < \lambda \\ \lambda, & \text{if } \lambda_{s,L}^* \geq \lambda \end{cases}$$

In stage 1, the ISP chooses c given the participation constraints:

$$\max_c \pi_{ISP} = p\lambda - p\lambda_{s,L}x^* + c\lambda_{s,L}x^*$$

s.t.

$$U_L(x^*) \geq 0, x^* \in [0, 1]$$

$$U_H(x^*) \geq 0, x^* \in [0, 1]$$

$$\pi_L(\lambda_{s,L}^*) \geq 0.$$

We can obtain the optimal c^* for CP L as follows: $c^* =$

$$\frac{3^{1/3} p^2 r_L^2 \lambda^2 \left[-9p^3 r_L^2 t^4 \lambda^2 + \sqrt{3} \sqrt{p^6 r_L^4 t^6 \lambda^4 (27t^2 + r_L^2 \lambda^2)} \right]^{2/3}}{t^2} \cdot \frac{1}{3^{2/3} \left[-9p^3 r_L^2 t^4 \lambda^2 + \sqrt{3} \sqrt{p^6 r_L^4 t^6 \lambda^4 (27t^2 + r_L^2 \lambda^2)} \right]^{1/3}}. \text{ Substituting the optimal value } c^*, \text{ we find the}$$

equilibrium subsidized amount $\lambda_{s,L}^* = \frac{-c^*t + pr_L \lambda}{2c^*p}$, and equilibrium market share $x^* = \frac{1}{2} + \frac{p\lambda_{s,L}^*}{2t}$

and the constraint $p \leq \frac{t}{\lambda_{s,L}^*}$ from $0 \leq x^* \leq 1$

We summarize these results as follows:

If CP L makes an exclusive contract with ISP to subsidize the data usage of L 's consumers,

1) ISP will choose the usage based price c for content provider L :

$$c^* = \frac{3^{1/3} p^2 r_L^2 \lambda^2 \left[-9p^3 r_L^2 t^4 \lambda^2 + \sqrt{3} \sqrt{p^6 r_L^4 t^6 \lambda^4 (27t^2 + r_L^2 \lambda^2)} \right]^{2/3}}{3^{2/3} \left[-9p^3 r_L^2 t^4 \lambda^2 + \sqrt{3} \sqrt{p^6 r_L^4 t^6 \lambda^4 (27t^2 + r_L^2 \lambda^2)} \right]^{1/3}}.$$

2) CP L will subsidize its consumers' usage by $\lambda_{s,L}^*$.

- If $\lambda_{s,L}^* < 0$, $\lambda_{s,L}^* = 0$.
- If $0 < \lambda_{s,L}^* \leq \lambda$, $\lambda_{s,L}^*$ will be subsidized.
- If $\lambda_{s,L}^* > \lambda$, $\lambda_{s,L}^* = \lambda$.

3) The market share of L is $x^* = \frac{1}{2} + \frac{p\lambda_{s,L}^*}{2t}$.

Case 3. ISP allows CP H to subsidize the data usage of CP H 's consumers.

In this case, the timeline of the game is given as follows: in stage 1, the ISP announces c , which is the usage-based price for CP H ; in stage 2, H decides the amount of its subsidization, $\lambda_{s,H} \in [0, \lambda]$; finally, in stage 3, the end consumers choose either L or H . We solve this game-theoretical model using backward induction.

The stage 3 in the extended problem is the same as the stage 3 in the baseline problem. The market share for content provider L is $x^* = \frac{1}{2} - \frac{p\lambda_{s,H}}{2t}$. The market share for content provider H is $(1 - x^*) = \frac{1}{2} + \frac{p\lambda_{s,H}}{2t}$.

In stage 2, given c , CP H chooses the subsidized amount $\lambda_{s,H}$ to maximize its profit: $\pi_H = r_H \lambda x^* - c \lambda_{s,H} x^*$. From the first order condition of the profit function, $\frac{\partial \pi_H}{\partial \lambda_{s,H}} = 0$, we can obtain the optimal subsidized amount of CP H :

$$\lambda_{s,H}^* = \begin{cases} 0, & \text{if } \lambda_{s,H}^* < 0 \\ \frac{-ct + pr_H \lambda}{2cp}, & \text{if } 0 \leq \lambda_{s,H}^* < \lambda. \\ \lambda, & \text{if } \lambda_{s,H}^* \geq \lambda \end{cases}$$

In stage 1, the ISP chooses c given the participation constraints:

$$\max_c \pi_{ISP} = p\lambda - p\lambda_{s,H}(1 - x^*) + c\lambda_{s,H}(1 - x^*)$$

s.t.

$$U_L(x^*) \geq 0, x^* \in [0, 1]$$

$$U_H(x^*) \geq 0, x^* \in [0, 1]$$

$$\pi_H(\lambda_{s,H}^*) \geq 0.$$

We can obtain the optimal c^* for CP H as follows: $c^* =$

$$\frac{3^{1/3}p^2r_H^2\lambda^2 - \left[-9p^3r_H^2t^4\lambda^2 + \sqrt{3} \sqrt{p^6r_H^4t^6\lambda^4(27t^2 + r_H^2\lambda^2)} \right]^{2/3}}{t^2} \cdot 3^{2/3} \left[-9p^3r_H^2t^4\lambda^2 + \sqrt{3} \sqrt{p^6r_H^4t^6\lambda^4(27t^2 + r_H^2\lambda^2)} \right]^{1/3}. \text{ Substituting the optimal value } c^*, \text{ we find the}$$

equilibrium subsidized amount $\lambda_{s,H}^* = \frac{-c^*t + pr_H\lambda}{2c^*p}$, and the equilibrium market share $x^* = \frac{1}{2} -$

$$\frac{p\lambda_{s,H}^*}{2t}.$$

We summarize these results as follows:

If the CP H makes an exclusive contract with ISP to subsidize the data usage of H 's consumers,

1) ISP will choose the usage based price c for content provider H as

$$c^* = \frac{3^{1/3}p^2r_H^2\lambda^2 - \left[-9p^3r_H^2t^4\lambda^2 + \sqrt{3} \sqrt{p^6r_H^4t^6\lambda^4(27t^2 + r_H^2\lambda^2)} \right]^{2/3}}{t^2} \cdot 3^{2/3} \left[-9p^3r_H^2t^4\lambda^2 + \sqrt{3} \sqrt{p^6r_H^4t^6\lambda^4(27t^2 + r_H^2\lambda^2)} \right]^{1/3}.$$

2) Content provider H will subsidize its consumers' usage by $\lambda_{s,H}^*$.

- If $\lambda_{s,H}^* < 0$, $\lambda_{s,H}^* = 0$.
- If $0 < \lambda_{s,H}^* \leq \lambda$, $\lambda_{s,H}^*$ will be subsidized.
- If $\lambda_{s,H}^* > \lambda$, $\lambda_{s,H}^* = \lambda$.

3) The market share of L is $x^* = \frac{1}{2} - \frac{p\lambda_{s,H}}{2t}$.

Case 4. ISP allows both CP L and CL H to subsidize the data usage.

In this case, the timeline of the game is given as follows: In stage 1, the ISP announces c , which is the usage-based price for CP L ; in stage 2, L and H decides the amount of its subsidization, $\lambda_{s,L} \in [0, \lambda]$ and $\lambda_{s,H} \in [0, \lambda]$, respectively; finally, in stage 3, the end consumers choose either L or H . We solve this game-theoretical model using backward induction.

The stage 3 in the extended problem is the same as the stage 3 in the baseline problem. The market share for content provider L is $x^* = \frac{1}{2} + \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t}$, and the market share for content provider H is $(1 - x^*) = \frac{1}{2} - \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t}$.

In stage 2, with the assumption, c is given, CP L and CP H decide the amount of subsidization $\lambda_{s,L}$ and $\lambda_{s,H}$ to maximize their profit, which is demonstrated with the following equations: $\pi_L = r_L \lambda x^* - c \lambda_{s,L} x^*$ and $\pi_H = r_H \lambda x^* - c \lambda_{s,H} x^*$. From the first order conditions of the profit functions, $\frac{\partial \pi_L}{\partial \lambda_{s,L}} = 0$ and $\frac{\partial \pi_H}{\partial \lambda_{s,H}} = 0$, we can obtain the following reaction functions:

$$\lambda_{s,L}^*(\lambda_{s,H}) = \begin{cases} 0, & \text{if } \frac{-ct + cp\lambda_{s,H}^*(\lambda_{s,L}) + pr_L\lambda}{2cp} \leq 0 \\ \frac{-ct + cp\lambda_{s,H}^*(\lambda_{s,L}) + pr_L\lambda}{2cp}, & \text{if } 0 < \frac{-ct + cp\lambda_{s,H}^*(\lambda_{s,L}) + pr_L\lambda}{2cp} \leq \lambda \\ \lambda, & \text{if } \frac{-ct + cp\lambda_{s,H}^*(\lambda_{s,L}) + pr_L\lambda}{2cp} > \lambda \end{cases}$$

$$\lambda_{s,H}^*(\lambda_{s,L}) = \begin{cases} 0, & \text{if } \frac{-ct + cp\lambda_{s,L}^*(\lambda_{s,H}) + pr_H\lambda}{2cp} \leq 0 \\ \frac{-ct + cp\lambda_{s,L}^*(\lambda_{s,H}) + pr_H\lambda}{2cp}, & \text{if } 0 < \frac{-ct + cp\lambda_{s,L}^*(\lambda_{s,H}) + pr_H\lambda}{2cp} \leq \lambda \\ \lambda, & \text{if } \frac{-ct + cp\lambda_{s,L}^*(\lambda_{s,H}) + pr_H\lambda}{2cp} > \lambda \end{cases}$$

In stage 1, the ISP chooses c given the participation constraints:

$$\max_c \pi_{ISP} = p\lambda - p\lambda_{s,L}x^* + c\lambda_{s,L}x^* - p\lambda_{s,H}(1-x^*) + c\lambda_{s,H}(1-x^*)$$

s.t.

$$U_L(x^*) \geq 0, U_H(x^*) \geq 0$$

$$0 \leq x^* = \frac{1}{2} + \frac{p(\lambda_{s,L} - \lambda_{s,H})}{2t} \leq 1$$

$$\pi_L(\lambda_{s,L}^*) \geq 0, \pi_H(\lambda_{s,H}^*) \geq 0.$$

From the participation constraints, we obtain the following three inequalities:

$$(1) -t + V(\lambda) + \frac{1}{2}p(-\lambda_{s,L} + 2\lambda_{s,H} - 2\lambda + \frac{r_L\lambda}{c}) \geq 0,$$

$$(2) \frac{\{t+p[\lambda_{s,L}^*(\lambda_{s,H})-\lambda_{s,H}^*(\lambda_{s,L})]\}\{c[t-p\lambda_{s,H}^*(\lambda_{s,L})]+pr_L\lambda\}}{4pt} \geq 0,$$

$$(3) \frac{\{t+p[-\lambda_{s,L}^*(\lambda_{s,H})+\lambda_{s,H}^*(\lambda_{s,L})]\}\{c[t-p\lambda_{s,L}^*(\lambda_{s,H})]+pr_H\lambda\}}{4pt} \geq 0.$$

We solve the equations of reaction function simultaneously and substitute the equilibrium subsidized amount $\lambda_{s,L}^*$ and $\lambda_{s,H}^*$. From the first order condition of the objective function, we find nine equilibria as follow:

$$(1) \quad \text{If } \frac{-ct+pr_L\lambda}{2cp} \leq 0 \text{ and } \frac{-ct+pr_H\lambda}{2cp} \leq 0, \lambda_{s,L}^* = 0, \lambda_{s,H}^* = 0 \text{ (there is no subsidization).}$$

$$(2) \quad \text{If } \frac{-3ct+p(r_H+2r_L)\lambda}{4cp} \leq 0 \text{ and } 0 < \frac{-ct+pr_H\lambda}{2cp} \leq \lambda, \lambda_{s,L}^* = 0, \lambda_{s,H}^* = \frac{-ct+pr_H\lambda}{2cp}, \text{ and}$$

$$c^* = \frac{3^{1/3}p^2r_H^2\lambda^2 - \frac{[-9p^3r_H^2t^4\lambda^2 + \sqrt{3}\sqrt{p^6r_H^4t^6\lambda^4(27t^2+r_H^2\lambda^2)}]^{2/3}}{t^2}}{3^{2/3}[-9p^3r_H^2t^4\lambda^2 + \sqrt{3}\sqrt{p^6r_H^4t^6\lambda^4(27t^2+r_H^2\lambda^2)}]^{1/3}}$$

$$(3) \quad \text{If } \frac{-ct+cp\lambda+pr_L\lambda}{2cp} \leq 0 \text{ and } \frac{-ct+pr_H\lambda}{2cp} > \lambda, \lambda_{s,L}^* = 0, \lambda_{s,H}^* = \lambda, \text{ and } c^* = \frac{pr_H\lambda}{2\lambda p+t}$$

$$(4) \quad \text{If } 0 < \frac{-ct+pr_L\lambda}{2cp} \leq \lambda \text{ and } \frac{-3ct+p(2r_H+r_L)\lambda}{4cp} \leq 0, \lambda_{s,L}^* = \frac{-ct+pr_L\lambda}{2cp}, \lambda_{s,H}^* = 0, \text{ and}$$

$$c^* = \frac{3^{1/3}p^2r_L^2\lambda^2 - \frac{[-9p^3r_L^2t^4\lambda^2 + \sqrt{3}\sqrt{p^6r_L^4t^6\lambda^4(27t^2+r_L^2\lambda^2)}]^{2/3}}{t^2}}{3^{2/3}[-9p^3r_L^2t^4\lambda^2 + \sqrt{3}\sqrt{p^6r_L^4t^6\lambda^4(27t^2+r_L^2\lambda^2)}]^{1/3}}.$$

$$(5) \quad \text{If } 0 < \frac{-ct+pr_L\lambda+\frac{1}{3}(-3ct-2pr_H\lambda+pr_L\lambda)}{2cp} \leq \lambda \text{ and } 0 < \frac{-ct+pr_H\lambda+\frac{1}{3}(-3ct-pr_H\lambda+2pr_L\lambda)}{2cp} \leq \lambda,$$

$$\lambda_{s,L}^* = -\frac{3ct+pr_H\lambda-2pr_L\lambda}{3cp}, \lambda_{s,H}^* = -\frac{3ct+2pr_H\lambda-pr_L\lambda}{3cp}, \text{ and}$$

$$c^* = (a) - \frac{1}{t^2} \left[(b) + \sqrt{6}\sqrt{(c)} \right]^{2/3} / \left\{ 36^{2/3} \left[(d) + \sqrt{6}\sqrt{(c)} \right] \right\}^{1/3} \text{ where}$$

$$(a) = 6^{1/3}p^2\lambda(9r_Ht - 9r_Lt + r_H^2\lambda + 2r_Hr_L\lambda + r_L^2\lambda),$$

$$(b) = -54p^3r_H^2t^4\lambda^2 - 108p^3r_Hr_Lt^4\lambda^2 - 54p^3r_L^2t^4\lambda^2,$$

$$(c) = p^6t^6\lambda^3[486(r_H + r_L)^4t^2\lambda + (9r_Ht - 9r_Lt + r_H^2\lambda + 2r_Hr_L\lambda + r_L^2\lambda)^3],$$

$$(d) = -54p^3r_H^2t^4\lambda^2 - 108p^3r_Hr_Lt^4\lambda^2 - 54p^3r_L^2t^4\lambda^2.$$

$$(6) \quad \text{If } 0 < \frac{-ct+cp\lambda+pr_L\lambda}{2cp} \leq \lambda \text{ and } \frac{p(2r_H+r_L)\lambda+c(-3t+p\lambda)}{4cp} > \lambda, \lambda_{s,L}^* = \frac{-ct+cp\lambda+pr_L\lambda}{2cp}, \lambda_{s,H}^* =$$

$$\lambda, \text{ and } c^* = \left\{ (a) + (b) + 3^{1/3} \left[(c) + \sqrt{3}\sqrt{(d)(e)} \right]^{2/3} \right\} / \left\{ (f) \left[(c) + \sqrt{3}\sqrt{(d)(e)} \right]^{1/3} \right\} \text{ where}$$

$$(a) = -3^{2/3}p^2r_L^2t^2\lambda^2 + 23^{2/3}p^5r_L\lambda^4,$$

$$(b) = 3^{2/3}p^4r_L\lambda^3(16t + r_L\lambda) + 23^{2/3}p^3r_Lt\lambda^2(-t + 4r_L\lambda),$$

$$(c) = -9p^3r_L^2t^4\lambda^2 + 144p^4r_L^2t^3\lambda^3 - 558p^5r_L^2t^2\lambda^4 - 144p^6r_L^2t\lambda^5 - 9p^7r_L^2\lambda^6,$$

$$(d) = -p^6r_L^3\lambda^4(-t^2 + 8pt\lambda + p^2\lambda^2)^3,$$

$$(e) = 8p^3\lambda^2 + 6pr_L^2\lambda^2 + r_L^3\lambda^2 + 3r_L(9t^2 - 72pt\lambda - 5p^2\lambda^2),$$

$$(f) = 3(-t^2 + 8pt\lambda + p^2\lambda^2).$$

$$(7) \quad \text{If } \frac{-ct+pr_L\lambda}{2cp} > \lambda \text{ and } \frac{-ct+cp\lambda+p r_H\lambda}{2cp} \leq 0, \lambda_{s,L}^* = \lambda, \lambda_{s,H}^* = 0, \text{ and } c^* = \frac{p r_H\lambda}{(t-p\lambda)}.$$

$$(8) \quad \text{If } \frac{p(r_H+2r_L)\lambda+c(-3t+p\lambda)}{4cp} > \lambda \text{ and } 0 < \frac{-ct+cp\lambda+p r_H\lambda}{2cp} \leq \lambda, \lambda_{s,L}^* = \lambda, \lambda_{s,H}^* =$$

$$\frac{-ct+cp\lambda+p r_H\lambda}{2cp}, \text{ and } c^* = \left\{ (a) + (b) + 3^{1/3} \left[(c) + \sqrt{3}\sqrt{(d)(e)} \right]^{2/3} \right\} / \left\{ (f) \left[(c) + \sqrt{3}\sqrt{(d)(e)} \right]^{1/3} \right\} \text{ where}$$

$$(a) = -3^{2/3}p^2r_H^2t^2\lambda^2 + 23^{2/3}p^5r_H\lambda^4,$$

$$(b) = 3^{2/3}p^4r_H\lambda^3(16t + r_H\lambda) + 23^{2/3}p^3r_Ht\lambda^2(-t + 4r_H\lambda),$$

$$(c) = -9p^3r_H^2t^4\lambda^2 + 144p^4r_H^2t^3\lambda^3 - 558p^5r_H^2t^2\lambda^4 - 144p^6r_H^2t\lambda^5 - 9p^7r_H^2\lambda^6,$$

$$(d) = -p^6r_H^3\lambda^4(-t^2 + 8pt\lambda + p^2\lambda^2)^3,$$

$$(e) = 8p^3\lambda^2 + 6pr_H^2\lambda^2 + r_H^3\lambda^2 + 3r_H(9t^2 - 72pt\lambda - 5p^2\lambda^2),$$

$$(f) = 3(-t^2 + 8pt\lambda + p^2\lambda^2).$$

$$(9) \quad \text{If } \frac{-ct+cp\lambda+pr_L\lambda}{2cp} > \lambda \text{ and } \frac{-ct+cp\lambda+pr_H\lambda}{2cp} > \lambda, \lambda_{s,L}^* = \lambda, \lambda_{s,H}^* = \lambda, \text{ and } c^* \rightarrow \infty.$$