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A Market Design Perspective**

Marzena Rostek and Ji Hee Yoon
University of Wisconsin-Madison

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Private Information in Markets: A Market Design Perspective*

Marzena Rostek[†] and Ji Hee Yoon[‡]

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Abstract

This paper studies the impact of heterogeneity in interdependence of trader values on price inference and welfare. A model of double auction with quasilinear-quadratic utilities is introduced that allows for arbitrary Gaussian information structures. With heterogeneous interdependence, some traders learn more from prices whereas others from private signals; thus, heterogeneity separates informed and uninformed trading. Changes in market structure can improve both informativeness of prices and private signals of a trader and make some traders learn more from prices than others. We characterize conditions on the information structure for price and signal inference to involve no tradeoff.

JEL CLASSIFICATION: D43, D53, G11, G12, L13

KEYWORDS: Decentralized Markets; Price Inference; Networks; Dark Pools; Market Design

In many markets, designers aim to enhance traders' learning about the value of the traded good or asset – typically in auction settings. In other trade settings, such as dark pools, traders are concerned with *minimizing* the informativeness of the market-clearing price about their values and prefer that other market participants not use their private signal as the basis for inference. This paper examines how market design itself can be used to induce the desired informational properties of prices. To this end, we present a model in which information structure is derived from a network that captures how private information about a good or asset is distributed among traders.

The literature on information aggregation in divisible-good markets has focused on trading environments in which price is equally informative to *all* market participants; that is, the model of expectations conditional on price and signal is the same across traders.¹ This corresponds to information structures in which each trader value is correlated with those of the remaining participants in the same way, on average. While admitting heterogeneous interdependence, the maintained symmetry assumption on information structures is admittedly strong in many economic settings, such as networks. This paper

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[†] University of Wisconsin-Madison, 1180 Observatory Drive, Madison, WI 53706, U.S.A.; mrostek@ssc.wisc.edu.

[‡] University of Wisconsin-Madison, 1180 Observatory Drive, Madison, WI 53706, U.S.A.; jyoon43@wisc.edu.

¹ The typical assumption in the information aggregation literature is that the values of the good of each bidder, observed with noise, is affected by a common and idiosyncratic shocks (e.g., Dubey, Geanakoplos, and Shubik (1987); Kyle (1989); Perry and Reny (2007); Vives (2011); Ostrovsky (2013)). A weaker symmetry assumption is that each bidder value is correlated with the remaining bidders in the same way on average (equicommonality, Rostek and Weretka (2012)).

departs from the underlying *symmetry* assumption on information structures. The model accommodates markets with *arbitrary* Gaussian information structures. We show that the heterogeneity alters qualitatively some of the central predictions that hold for markets with symmetric information structures and develop new results. The analysis is cast in a linear-normal setting for a divisible good (i.e., utilities are quasilinear and quadratic in quantity traded; trader values are jointly Gaussian). All traders are Bayesian and strategic.

The goal of this project is to develop market design principles for information sharing in decentralized markets. In markets in which price is equally informative to all traders, changes in market structure that increase the average correlation, have a monotone – and the same across traders – impact on the informativeness of price, signal and, thus, efficiency loss. We provide sufficient conditions for the monotonicity of price and signal informativeness. We develop three sets of implications – for bidding behavior, informational efficiency and welfare. The main economic implications of markets in which some traders learn from prices more than others are as follows:

- In the symmetric information structures, signal and price are always substitutes in a trader’s inference about the asset’s value. In the heterogeneous model, changes in market structure – such as inclusion of new traders, introduction of new shocks, or changes in the network structure – can enhance *both* learning from prices and signals; for the changes in market structure that monotonically increase price inference, signal inference can be non-monotone; changes in the market structure that lower price informativeness for some market participants may improve the price informativeness of other agents. We establish the necessary and sufficient condition – on the information structures and preferences – for the price and signal to be substitutes or complements in inference.²
- Some (changes in) market structures make some traders learn more from prices and others – from signals.³ Thus, *centralized* trading based on a single market clearing price, can isolate informed trading (learning from signals) from uninformed trading (learning from prices).⁴ We develop comparative statics that suggests how to achieve separation in specific information structures (e.g., network topologies).

² A large body of research in the past decade has demonstrated non-trivial effects of changes in the precision of public and private information on equilibrium and welfare effects (e.g., Morris and Shin (2002); Angeletos and Pavan (2007) provide conditions under which public and private signals are strategic complements or substitutes). This literature assumes that the information structure based on the fundamental value assumption and the precisions are common to all traders. Thus, our results on the impact of the heterogeneity in inference are complementary.

³ Institutional investors increasingly choose to participate in liquidity (dark) pools to achieve privacy in trade execution and avoid exposure of their orders to front running. In less than a decade, liquidity pool trading in the U.S. has grown by more than 50%, and more than doubled in Europe.

⁴ “many traders who prefer limited-display venues do so because they believe that they obtain better executions in these venues. Many of these traders are institutional traders who prefer to trade only with other similar institutional or retail traders and specifically not with hedge funds that they widely believe to be better informed. These institutional traders would rather not trade than trade on the wrong side of the market with a well-informed trader. Any moves to reduce trader access to limited display venues may very well have the unintended consequence of increasing transaction costs for large institutional traders, many of whom predominately manage money for retail mutual funds and pension beneficiaries” Angel, Harris and Spatt (2013, p.29).

- Whether participation of traders should be encouraged is nontrivial. The model suggests ways for the market designer to induce certain statistical dependence or determine which information structures should promote and which should restrict entry.⁵ Apart from affecting information aggregation per se, encouraging participation of bidders whose values are affected by *distinct* shocks (e.g., to endowments, liquidity or preferences), a designer may improve the informativeness of private signals as well as of prices for all traders.
- In a heterogeneous model, some agents in the market have greater impact on the inference of market participants about values. In this sense, some agents are “informationally large” and some are “informationally small;” this happens endogenously and depends on the information structure and preferences (risk aversions). One can identify agents with the greatest impact on inference. Unlike large, competitive markets (or the symmetric model) agents whose values are more correlated with the market need not learn more from prices. Furthermore, agents who learn more from prices are not necessarily informationally large.
- Changes in market structure can change price impact of some or all traders in a non-monotone way.

We consider a number of empirically common network structures, such as dark pools, single-dealer markets, multiple-dealer markets, hub-and-spoke networks, and core-periphery networks.⁶ We show that certain statistic can be identified such that, based on the behavior they exhibit, information structures can be categorized according to the information sharing properties.

As a modeling contribution, to the best of our knowledge, we are the first to characterize asymmetric equilibria⁷ for markets with any number of traders in one-sided or double auctions for divisible goods.

1 Model

Consider a market of a divisible good with $I \geq 2$ traders. The market is a double auction in the linear-normal setting. Trader i has a quasilinear and quadratic utility function

$$U_i(q_i) = \theta_i q_i - \frac{\mu_i}{2} q_i^2, \quad (1)$$

where q_i is the obtained quantity of the good auctioned and $\mu_i > 0$. Each trader is uncertain about how much the good is worth. Trader uncertainty is captured by the randomness of the intercepts of marginal utility functions $\{\theta_i\}_{i \in I}$, referred to as *values*. Randomness in θ_i is interpreted as arising from shocks to preferences, endowment, or other shocks that shift the marginal utility of a trader in a market where traders value goods differently, and can derive from a network structure, such that a link between two traders in the network represents the non-zero correlation between them.

⁵ E.g., participation in dark pools is typically restricted.

⁶ The growing literature on the microstructure of decentralized markets reports that core-periphery and hub-and-spoke architectures are dominant market structures for homogeneous assets (e.g., both in the U.S. federal funds market of overnight unsecured loans and European banking systems; Bech and Atalay (2010), Afonso, Kovner, and Schoar (2012); Craig and Peter (2010)).

⁷ Indeed, that price is equally informative across bidders permits a symmetric equilibrium.

Information Structure. Prior to trading, each trader i observes a noisy signal about his true value θ_i , $s_i = \theta_i + \varepsilon_i$. The information structure is Gaussian: Random vector $\{\theta_i, \varepsilon_i\}_{i \in I}$ is jointly normally distributed; noise ε_i is mean-zero i.i.d. with variance σ_ε^2 , and expectation $E(\theta_i)$ and variance σ_θ^2 of θ_i are the same for all i . The variance ratio $\sigma^2 \equiv \sigma_\varepsilon^2 / \sigma_\theta^2$ measures the relative importance of noise in the signal. The $I \times I$ variance-covariance matrix of the joint distribution of values $\{\theta_i\}_{i \in I}$, normalized by variance σ_θ^2 , specifies the matrix of correlations,

$$\mathcal{C} \equiv \begin{pmatrix} 1 & \rho_{1,2} & \dots & \rho_{1,I} \\ \rho_{2,1} & 1 & \dots & \rho_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1} & \rho_{I,2} & \dots & 1 \end{pmatrix}.$$

In the analysis, as a benchmark, we use the information structures on which the literature on price inference has focused:

- the *fundamental value* model: each agent i 's value θ_i is determined by a common and idiosyncratic shock, $\theta_i = \theta + \tilde{\theta}_i$, where $\tilde{\theta}_i$ is i.i.d. This admits the *independent (private) value* model, $\rho_{i,j} = 0$ for all $j \neq i$; the *pure common value* model $\rho_{i,j} = 1$ for all $j \neq i$ (e.g., Kyle (1989)); or, more generally, a family of information structures with $\rho_{i,j} = \rho \in [-1, 1]$ (Vives (2011)).
- the *equicommonal* model: each agent i 's value θ_i is correlated with the market in the same way on average, $\frac{1}{I-1} \sum_{j \neq i} \rho_{i,j} = \bar{\rho}$ for all i (Rostek and Weretka (2012)). Unlike the fundamental value model, this admits heterogeneity in $\{\rho_{i,j}\}_{j \neq i}$ across traders.

Throughout, *symmetric* information structures are understood to mean equicommonal – in the equicommonal model, equilibrium in symmetric bid functions (and symmetric conditional expectations) exists. We do not assume equicommonality. The key new feature of the model is that it permits *arbitrary heterogeneous* interdependencies among values $\{\theta_i\}_{i \in I}$, subject to existence conditions.

Double Auction. We study double auctions based on the standard uniform-price mechanism (e.g., Kyle (1989); Vives (2011)).⁸ Buyers and sellers submit strictly downward-sloping net demand schedules $\{q_i(p)\}_{i \in I}$, with the bid part for negative quantities interpreted as a supply schedule. The market clearing price p^* is one for which aggregate demand equals zero, $\sum_{i \in I} q_i(p^*) = 0$.⁹ Bidder i obtains the quantity determined by his submitted bid evaluated at the equilibrium price, $q_i^* = q_i(p^*)$, for which he pays $q_i^* \cdot p^*$. Bidder payoff is given by $U_i(q_i^*) - q_i^* \cdot p^*$. We study linear¹⁰ Bayesian Nash equilibrium (henceforth, “equilibrium”).

⁸ Clearly, the results extend to a larger class of models, competitive and strategic, including one-sided auctions (with an elastic or inelastic demand) and non-market settings, in which I Bayesian agents each make inference about a random variable θ_i based on the observed signal s_i and a statistic that is a deterministic function of the average signal.

⁹ If multiple or no prices exist, then no trade takes place. The assumption that bids are strictly downward-sloping rules out trivial (no-trade) equilibria.

¹⁰ “Linear” is understood as bids having the functional form of $q_i(p) = \alpha_{0i} + \alpha_{si}s_i + \alpha_{pi}p$; equilibrium is “symmetric linear” if the coefficients α_{0i} , α_{si} , and α_{pi} are the same across bidders.

2 Equilibrium

2.1 Strategies and Conditional Expectations

We conjecture a linear equilibrium: bids have a functional form of $q_i(p) = \alpha_i + \beta_i s_i + g_i p$. Given the bids of traders $j \neq i$, trader i faces a residual supply with a slope λ_i and a stochastic intercept which is a function of other traders' signals $\{s_j\}_{j \neq i}$. The slope λ_i is i 's price impact, which measures a price increase resulting from a marginal increase in the quantity demanded by i . The best response of trader i to the residual supply is given by the first-order (necessary and sufficient) condition: for any p ,

$$E(\theta_i | p, s_i) - \mu_i q_i = p + \lambda_i q_i, \quad (2)$$

Using (2), the equilibrium bid is

$$q_i(p) = \frac{1}{(\mu_i + \lambda_i)} [E(\theta_i | p, s_i) - p]. \quad (3)$$

Aggregating, for each i , the bids submitted by $j \neq i$, gives the residual supply for i , the slope of which defines i 's price impact. We obtain the following fixed point for the price impacts

$$\lambda_i = - \left(\sum_{j \neq i} (\partial q_j(p) / \partial p) \right)^{-1} = \left(\sum_{j \neq i} \frac{1 - c_{pj}}{\mu_j + \lambda_j} \right)^{-1} = \left(\sum_{j \neq i} \gamma_j (1 - c_{pj}) \right)^{-1}, \quad i \in I, \quad (4)$$

where $\gamma_i \equiv (\mu_i + \lambda_i)^{-1}$. Given an affine information structure, the conditional expectation is linear $E(\theta_i | s_i, p) = c_{\theta i} E(\theta_i) + c_{s i} s_i + c_{p i} p$. By the first-order condition (2) and market clearing, the equilibrium price is characterized by

$$p^* = \left(\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i} \right)^{-1} \sum_{i \in I} \frac{1}{\mu_i + \lambda_i} [c_{\theta i} E(\theta_i) + c_{s i} s_i]. \quad (5)$$

Unlike the equicommonal model, price impact matters λ_i for inference and results about information aggregation; price impact introduces a source of heterogeneity separate from the heterogeneity in risk aversion. Define $\Gamma_i \equiv \gamma_i c_{s i}$. The equilibrium inference coefficients (derived in the Appendix) in matrix form and in terms of $\bar{\rho}_i$ are

$$\begin{aligned} c_{s i} &= \frac{\Gamma^T (\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma - [\mathcal{C} \Gamma]_i [(\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma]_i}{(1 + \sigma^2) \Gamma^T (\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma]_i^2} \\ &= \frac{1}{1 + \sigma^2} - \frac{\sigma^2}{1 + \sigma^2} \frac{\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I - 1} \bar{\rho}_i \Gamma_i^2}{(1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) - \left(\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I - 1} \bar{\rho}_i \Gamma_i^2 \right)} \end{aligned} \quad (6)$$

$$\begin{aligned} c_{p i} &= \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id}) \Gamma]_i}{(1 + \sigma^2) \Gamma^T (\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma]_i^2} \left(\sum_{j \in I} \gamma_j (1 - c_{pj}) \right) \\ &= \frac{\frac{\sigma^2}{I - 1} \left(\frac{1}{\lambda_i} + \frac{1}{\mu_i + \lambda_i} \right) \Gamma_i \bar{\rho}_i}{(1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) - \left(\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I - 1} \bar{\rho}_i \Gamma_i^2 \right) + \frac{\sigma^2}{(I - 1)(\mu_i + \lambda_i)} \Gamma_i \bar{\rho}_i}, \end{aligned} \quad (7)$$

where $\Gamma \equiv \{\Gamma_i\}_{i \in I} = \{\gamma_i c_{si}\}_{i \in I} \in \mathbb{R}^I$, and $[\cdot]_i$ and $[\cdot]_{ij}$ denote the i th element of a vector and the (i, j) -element of a matrix. The explicit formula for c_{pi} is obtained using $\sum_{j \in I} \gamma_j (1 - c_{pj}) = \frac{1}{\lambda_i} + \gamma_i (1 - c_{pi})$. As we will show, the following is the counterpart of the equicommonality statistic

$$\bar{\rho}_i \equiv \frac{1}{\Gamma_i} \left(\frac{1}{I-1} [(C - \mathbf{Id})\Gamma]_i \right) = \frac{1}{c_{si}\gamma_i} \left(\frac{1}{I-1} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right),$$

which is interpreted as capturing interdependence with the system rather than a weighted correlation. In the symmetric model, $\bar{\rho}_i = \frac{1}{I-1} \sum_{j \neq i} \rho_{ij}$ is constant across agents $i \in I$.¹¹

Example 1 summarizes the properties of models with symmetric information structures.

EXAMPLE 1 (Symmetric Information Structures). *In the equicommonal model, the symmetric equilibrium¹² bid of trader i is*

$$q_i(p) = \frac{\gamma - c_p}{1 - c_p} \frac{c_\theta}{\mu} E(\theta_i) + \frac{\gamma - c_p}{1 - c_p} \frac{c_s}{\mu} s_i - \frac{\gamma - c_p}{\mu} p, \quad (8)$$

where the inference coefficients in the conditional expectation $E(\theta_i | s_i, p)$ are given by

$$c_s = \frac{1 - \bar{\rho}}{1 - \bar{\rho} + \sigma^2}, \quad (9)$$

$$c_p = \frac{(2 - \gamma) \bar{\rho}}{1 - \gamma + \bar{\rho}} \frac{\sigma^2}{1 - \bar{\rho} + \sigma^2}, \quad (10)$$

$$c_\theta = 1 - c_s - c_p. \quad (11)$$

The inference coefficients c_s is decreasing and c_p is increasing in the value of $\bar{\rho}$.¹³ Price impact is given by $\lambda_i = -\frac{(\partial q_i(p)/\partial p)^{-1}}{I-1} = \frac{(1-\gamma)\mu}{\gamma-c_p}$, for all i .

In more general (asymmetric) information structures, equilibrium bids depend on the information structure $\mathcal{C}$ not just through commonality; the dependence is a function of the information structure, risk aversion μ_i and, hence, price impact λ_i .

2.2 Existence

Proposition 1 demonstrates the existence of a linear Bayesian Nash equilibrium in double auctions under quite general conditions, which are assumed in the sequel.¹⁴

¹¹ We show that the upper existence bound on $\bar{\rho}_i$ tends to $1 + \sigma^2$, as $I \rightarrow \infty$. Equal to a weighted sum of correlations $\{\rho_{ij}\}_{j \neq i}$ with weights $\{c_{sj}\gamma_j\}_{j \neq i}$, $\bar{\rho}_i$ can be interpreted as the weighted commonality.

¹² This is the equilibrium from Vives (2011; the fundamental value model) and Rostek Weretka (2012; the equicommonal model). Technically, the equicommonality assumption relaxes the assumption of $\rho_{i,j} = \rho$ for all $i \neq j$ predominant in the information aggregation literature, in order to accommodate heterogeneous interdependence while still preserving tractability.

¹³ $\frac{\partial c_p}{\partial \bar{\rho}} = (2 - \gamma) \sigma^2 \frac{(1 - \gamma)(1 + \sigma^2) + \bar{\rho}^2}{(1 - \gamma + \bar{\rho})(1 - \bar{\rho} + \sigma^2)}$

¹⁴ Condition (i) can be stated in terms of the *equally weighted* average of correlations, $\frac{1}{I-1} \sum_{j \neq i} \rho_{ij}$. Condition (ii) can be stated in terms of the *non-equally weighted* average, $\bar{\rho}_i$. The weights in $\bar{\rho}_i$, $\{\Gamma_j\}_{j \neq i} = \{(c_{sj}/(\mu_j + \lambda_j))\}_{j \neq i}$, are endogenous, but are independent of i .

PROPOSITION 1 (Existence of Equilibrium). *Assume $\sum_{i,j \in I} \rho_{ij} \geq 0$. There exist bounds $\{\bar{\rho}_i^-(I, \mathcal{C}), \bar{\rho}_i^+(I, \mathcal{C})\}_i$ such that, in a double auction characterized by $(I, \{\bar{\rho}_i\}_i)$, a linear Bayesian Nash equilibrium exists if, and only if, $\bar{\rho}_i^-(I, \mathcal{C}) < \bar{\rho}_i < \bar{\rho}_i^+(I, \mathcal{C})$.*

Apart from the joint distribution of $\{\theta_i\}_i$ to be well defined ($\sum_{i,j \in I} \rho_{ij} \geq 0$), equilibrium existence entails the condition that bids be downward-sloping, equivalent to the *negation* of the following inequalities:

$$-\frac{\sigma^2}{(\mu_i + \lambda_i)(I-1)} < \frac{1 + \sigma^2}{I-1} \left(\frac{(1 + \sigma^2) \text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\bar{\rho}_i \Gamma_i} - \Gamma_i \right) + \left(\frac{(1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\bar{\rho}_i \Gamma_i} - \bar{\rho}_i \Gamma_i \right) < \frac{\sigma^2}{\lambda_i(I-1)}. \quad (12)$$

Remarks:

1. Unlike the symmetric model, (1) the condition that the demand be downward-sloping yields an upper *and a lower* bound, and the lower bound can be binding, depending on the model. Thus, apart from the upper bound on the (now weighted) commonality, the lower bound is the maximum of *two* bounds; (2) best-response bids of some bidders might involve *upward*-sloping demand schedules;¹⁵ and (3) the bid slope condition involves weighted correlations; (4) the upper bound on $\bar{\rho}_i$ goes to $1 + \sigma^2 > 1$ as $I \rightarrow \infty$; as in the equicommonal model, the lower bound goes to 0.
2. As $I \rightarrow \infty$, $\Gamma_i = \frac{c_{si}}{\mu_i + \lambda_i} \approx \frac{c_{si}}{\mu_i} \in (0, \frac{1}{\mu_i})$, and $\lambda_i \sim \mathcal{O}((I-1)^{-1})$; in the limit, inequality (12) becomes¹⁶

$$0 < \lim_{I \rightarrow \infty} \left(\frac{(1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\bar{\rho}_i \Gamma_i} - \bar{\rho}_i \Gamma_i \right) < \lim_{I \rightarrow \infty} \frac{\sigma^2}{\lambda_i(I-1)}.$$

The bounds are monotone increasing as $I \rightarrow \infty$ (can show analytically). Then, we can find the limit of bounds of $\bar{\rho}_i$ when $I \rightarrow \infty$ as $0 < \bar{\rho}_{i,\infty}^- < \bar{\rho}_i < \bar{\rho}_{i,\infty}^+$; $\bar{\rho}_{i,\infty}^-$ is the positive solution of quadratic equation

$$\bar{\rho}_i^2 \Gamma_i^2 = (1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} - \bar{\rho}_i \left(\lim_{I \rightarrow \infty} \frac{\sigma^2}{\lambda_i(I-1)} \right),$$

$$\bar{\rho}_{i,\infty}^+ \text{ is } \bar{\rho}_i^2 \Gamma_i^2 = (1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}.$$

3. We characterize existence conditions explicitly for all models studied in this paper (Appendix).

3 Informativeness of Price and Signal

We study information aggregation in the standard privately revealing sense, which compares the informativeness of equilibrium price and the total available information, which corresponds to the profile of all bidders' signals, $s \equiv \{s_i\}_{i \in I}$. The equilibrium price is *privately revealing* if, for any bidder i ,

¹⁵ Vives (2011) demonstrates existence of equilibria with upward-sloping demands in a market with I strategic traders and an exogenous downward-sloping demand schedule.

¹⁶ As $I \rightarrow \infty$, the lower bound is increasing to zero, the limit of the upper bound is positive, and the first term of middle part goes to zero.

the conditional c.d.f.'s of the posterior of θ_i satisfy $F(\theta_i|s_i, p^*) = F(\theta_i|s)$ for every state s , given the corresponding equilibrium price $p^* = p^*(s)$.

LEMMA 1 (Information Aggregation). *In a finite double auction, the equilibrium price is privately revealing if, and only if, $\rho_{i,j} = \bar{\rho}$ for all $j \neq i$.*

As is the case generically in the equicommonal class of information structures, in the non-equicommonal model, equilibrium price is *not* fully revealing in the non-equicommonal information structures. Nevertheless, other predictions that hold in the symmetric (fundamental value or, more generally, equicommonal) model do not carry over, in general, in the heterogeneous model.

To quantify contributions of the signal s_i and price p^* to learning, let us define the index of *price informativeness*

$$\psi_{p,i}^+ \equiv \frac{\text{Var}(\theta_i|s_i) - \text{Var}(\theta_i|s_i, p^*)}{\text{Var}(\theta_i) - \text{Var}(\theta_i|s)},$$

and *signal informativeness*

$$\psi_{s,i}^+ \equiv \frac{\text{Var}(\theta_i) - \text{Var}(\theta_i|s_i)}{\text{Var}(\theta_i) - \text{Var}(\theta_i|s)},$$

relative to the maximal possible reduction in variance $\text{Var}(\theta_i) - \text{Var}(\theta_i|s)$, given the total information in the market $\{s_i\}_i$; $\psi_{p,i}^+ = 1$ if, and only if, price aggregates information (i.e., the fundamental value model); in which case, $\psi_{s,i}^+ = 0$. Then, *informational loss* $\psi_i^- \equiv 1 - (\psi_{s,i}^+ + \psi_{p,i}^+) = \frac{\text{Var}(\theta_i|s_i, p^*) - \text{Var}(\theta_i|s)}{\text{Var}(\theta_i) - \text{Var}(\theta_i|s)}$; $\psi_{s,i}^+, \psi_{p,i}^+, \psi_i^- \in [0, 1]$. Note that the informativeness of the signal $\psi_{s,i}^+$ is evaluated with respect to $\text{Var}(\theta_i|s)$ to capture that the signal may be picking information *available in the market* (i.e., for a fixed set of traders and the joint distribution of private information) that price does not. Since $\text{Cov}(\theta_i, s_i) = \sigma_\theta^2$, $\text{Cov}(\theta_i, s_j) = \sigma_\theta^2 \rho_{ij}$, and $\text{Cov}(s_i, s_j) = \sigma_\theta^2 \rho_{ij}$ for any $i \neq j$,

$$\text{Var}(\theta_i|s) = \sigma_\theta^2 - \sigma_\theta^2 \mathcal{C}_i (\mathcal{C} + \sigma^2 \mathbf{Id})^{-1} \mathcal{C}_i^T,$$

where $\mathcal{C}$ is the correlation matrix and $\mathcal{C}_i$ is the i th row of $\mathcal{C}$. Here, $\mathcal{C}_i (\mathcal{C} + \sigma^2 \mathbf{Id})^{-1} \mathcal{C}_i^T$ is the (i, i) -element of the matrix $\mathcal{C} (\mathcal{C} + \sigma^2 \mathbf{Id})^{-1} \mathcal{C}$. The inference indices in matrix form and in terms of $\bar{\rho}_i$ are

$$\begin{aligned} \psi_{s,i}^+ &= \frac{1}{1 + \sigma^2} \frac{1}{[\mathcal{C} (\mathcal{C} + \sigma^2 \mathbf{Id})^{-1} \mathcal{C}]_{ii}} = \frac{1}{1 + \sigma^2} \frac{1}{1 - \sigma^2 + \sigma^4 [(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1}]_{ii}} \\ \psi_{p,i}^+ &= \sigma^2 \psi_{s,i}^+ \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id}) \Gamma]_i^2}{(1 + \sigma^2) \Gamma^T (\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma]_i^2} \\ &= \sigma^2 \psi_{s,i}^+ \frac{\sigma^2 \bar{\rho}_i^2}{(1 + \sigma^2) \left(\frac{(1 + \sigma^2)}{I-1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \left(\frac{(1 + \sigma^2)}{I-1} + \bar{\rho}_i \right)}. \\ \psi_{\text{Loss},i}^- &= \sigma^4 \psi_{s,i}^+ \left((1 + \sigma^2) [(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1}]_{ii} - 1 - \frac{[(\mathcal{C} - \mathbf{Id}) \Gamma]_i^2}{(1 + \sigma^2) \Gamma^T (\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma]_i^2} \right). \end{aligned}$$

A couple of properties are noteworthy. First, signal informativeness $\psi_{s,i}^+$ is determined by the primitives and not endogenous parameters (in particular, price impact), but is not affected by the risk-aversion μ_i . In turn, price informativeness $\psi_{p,i}^+$ depends on the endogenous variables $\{c_{sj}, \lambda_j\}_{j \in I}$. Furthermore, for any agent i , $\psi_{s,i}^+$ depends, in general, on *all* correlations in the market, including $\{\rho_{j,k}\}_{j,k \neq i}$.¹⁷

¹⁷ It is useful to observe further that $\Gamma_i, \bar{\rho}_i, \text{Avg}\{\Gamma_j^2\}_{j \neq i}, \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}$ are sufficient for all the variables $c_{si}, c_{pi}, \psi_{s,i}^+$,

4 Inference in the Heterogeneous Model

One implication of the symmetry (i.e., equicommonality) is that the private signal and the equilibrium price are always substitutes in a trader's conditional expectations about the asset's value: c_s and c_p are, respectively, monotone decreasing and increasing in $\bar{\rho}$, while ψ^+ is monotone in $|\bar{\rho}|$. In the heterogeneous model, $c_{s,i}$ and $c_{p,i}$ can *both* increase for all agents as the per-link correlation increases in absolute value, or with other changes in market structure. Likewise, signal and price indices ψ_{si}^+ and ψ_{ip}^+ do not necessarily exhibit a tradeoff. While in the symmetric model, the determinants of information aggregation (c_p or, there, equivalently, ψ^+) also fully characterize individual equilibrium bids, in the heterogeneous model, inference coefficients in bids $c_{s,i}$ and $c_{p,i}$ can adjust in the same direction, even if ψ_{si}^+ and ψ_{ip}^+ exhibit a tradeoff. In particular, by observing changes in bidding behavior, an econometrician cannot directly infer how price informativeness changed. Example 2 illustrates.

EXAMPLE 2 (Learning from Signal vs Price: No tradeoff for a Trader). *In the heterogeneous model, changes in market structure may improve learning from both prices and signals.*

- *In the Group Model, c_s and c_p can both increase (for all agents) as the cross-group correlation increases (Fig 5 and 7). Likewise, ψ_{si}^+ and ψ_{pi}^+ can both increase (for all agents) as the cross-group correlation increases (Fig 5 and 7).*
- *In the Multiple-dealer model, as the number of inter-dealer links, n_L , increases, ψ_{pi}^+ is always increasing for both groups, and $|c_p|$ and $|c_s|$ are increasing for both groups in region 2 (Fig 15).¹⁸ Further examples and, additionally, separation of bidding behavior c_s, c_p and price and signal informativeness ψ_{pi}^+, ψ_{si}^+ can be seen in the following models:*
- *In the Dealer Model, inference coefficients c_s and c_p can both increase (for all agents), as the number of equally correlated links increases (Fig 12). Price informativeness $\psi_{p,i}^+$ and the information loss ψ_{Loss}^- are monotonically increasing in $|\rho|$, while the signal informativeness $\psi_{s,i}^+$ is decreasing in $|\rho|$ (see also Fig 10).*
- *In the Dealer Model, for group S , for the changes in market structure that monotonically increase $c_{p,i}$ or $\psi_{p,i}^+$, the behavior of equilibrium $c_{s,i}$ or $\psi_{s,i}^+$ can be non-monotone (Fig 10).*
- *In the Group Model: For group L , ψ_L^+ increases and c_{pL} decreases at some level of $\bar{\rho}$ (Fig 7). A trader can put a positive weight on the price c_{pi} even if $\bar{\rho}_i < 0$; e.g., he is correlated negatively with all other traders.*

In the symmetric model, private signals and price are equally informative to all traders. Markets with heterogeneous information structures can enhance informed trading (learning from signals) for

$\psi_{p,i}^+$ and $\psi_{Loss,i}^-$. All the variables except $\psi_{s,i}^+$ depend on the (weighted) $\bar{\rho}_i$, where the matrix form is useful. When $\bar{\rho}_i = \bar{\rho}$, for all i , the formulas from the symmetric model (Example 1) obtain (see the Appendix).

¹⁸ Even though $|c_p|$ and $|c_s|$ are both increasing in n_L , c_p is decreasing in n_L , and c_s is increasing in n_L . In the numerical experiments, c_θ is also increasing in n_L . Here, the behavior of inference coefficients does not match that of indices: as n_L increases, $|c_p|$, c_{si} , ψ_{pi}^+ increase but ψ_{si}^+ decreases.

Figure 1: **Learning from Signal vs Price.** Behavior of bidding coefficients c_s, c_p, c_θ , price impact, and signal and price informativeness ψ_s^+, ψ_p^+ , as the number of links between dealers in the multiple dealer model.

some traders and uninformed trading (learning from price) for others.¹⁹ This is illustrated in Example 3.

EXAMPLE 3 (Informed and Uninformed Trading). *Changes in market structure can increase $c_{p,i}$ or improve price informativeness ψ_{pi}^+ for some traders and lower it for others.*

- In the Group, for group S , but not for L , and all Tree Models including the Dealer Model, price and signal informativeness ψ_{si}^+ and ψ_{pi}^+ change in the same direction with the considered changes in market structure.²⁰ (Fig 5 and 7.)

*A change in market structure can make some traders learn more from prices and others – from signals.*²¹

- In Group Model, for a relatively low within-group correlation ρ_L of group L , $c_{pS} \approx 1$ while $c_{pL} \approx 0$.
- In the Single Dealer model, if the number of customers for the central dealer is large, $c_{pD} \approx 1$ and $c_{pC} \approx 0$.

¹⁹ In the symmetric model, changes in market structure affect the bidding behavior and efficiency to the extent they affect the average correlation statistic. Hence, price informativeness is always monotone in the average correlation, the absolute value of which is the sufficient for the information aggregation for all traders in the market. It follows that changes in the market structure that lower average correlation lower price informativeness of *all* agents. Likewise, the efficiency loss, while in general idiosyncratic (so long as traders do not face an identical residual market) is monotone decreasing for *all* agents in increases in the common sufficient statistic. In the heterogeneous model, changes in market structure that increase a trader's sufficient statistic – now, heterogeneous across traders – can have a non-monotone impact on the sufficient statistics, and hence price informativeness, of other agents.

²⁰ In fact, L is the only exception in all models in the examples we consider.

²¹ In the symmetric model, as $\bar{\rho} \rightarrow 1$, $\psi_p^+ \rightarrow 1$ and $c_{si} \rightarrow 0$, for all traders.

Figure 2: **Decomposition of Informativeness** (a) Group model with small $I = 8$ and $G_S = 3$, and (b) multiple-dealer model with $I = 12$, $G_D = 4$, $n_C = 2$.

		c_s	c_p	c_θ	ψ_s^+	ψ_p^+	ψ_{Loss}^-
Dark Pool model	group L	0.7049	0.2397	0.0554	0.9734	0.0264	0.0001
	group S	0.2745	0.9075	-0.1820	0.8123	0.1303	0.0574
Single-dealer model	customer C	0.7951	0.0357	0.1692	0.9905	0.0002	0.0093
	dealer D	0.5109	0.9633	-0.4742	0.9371	0.0629	0

Table 1: **Informed and Uninformed Traders** For the darkpool model, $I = 40$, $G_S = 15$, $\rho_S = 1$, $\rho_L = 0.4$, $\rho = 0.6$. For the single-dealer model, $I = 8$, $\rho = 0.34$.

- *The network matters: Price informativeness can be non-monotone for all agents as the per-link correlation increases, the ranking of which trader learns more from signal can switch between trader groups.*

In the heterogeneous model, agents whose values are more correlated with the market do not necessarily learn more from prices. This contrasts sharply with the symmetric model (or, large, competitive markets).

EXAMPLE 4 (Agents less Correlated with Market can Learn More from Price). *Unlike large, competitive markets (or the symmetric model) agents whose values are more correlated with the market need not learn more from prices.*

- *In the Star Model, the periphery trader has fewer links but for a small negative per-link correlation ρ can have stronger influence (in the sense of $c_{si}\gamma_i$).*

Figure 3: **Decomposition of Informativeness** (a) Group model with large I , (b) single-dealer, (c) multiple-dealer with equal group size, (d) core-periphery model, and (e) location model.

(a)

(b)

Figure 4: **Informed and Uninformed Traders** (a) Darkpool model with $I = 40, G_S = 15, \rho_S = 1, \rho_L = 0.4$, and (b) single-dealer model with $I = 8$.

- In the Group Model, $\rho_S < \rho_L$ but $c_{sS}\gamma_S > c_{sL}\gamma_L$ (also for $\rho_S, \rho_L > 0$).
- For a large enough I , traders with a relatively higher average correlation have higher price informativeness $\psi_{p,i}^+$ and $c_{p,i}$ (Fig 6). But not finite markets in general (e.g., Fig 5 and 7). However, the ranking of $\{c_{s,i}\}_i$ is ambiguous; that is, changes in market structure that increase per-link correlation can switch the ranking of $\{c_{s,i}\}_i$ across traders. (Fig 10)
- Price informativeness can be non-monotone for all agents as the minimal – across traders – average correlation increases (holding all other correlations fixed).

In the heterogeneous model, one can ask: Who are the traders from whom others learn the most? In the price informativeness index, $\psi_{p,i}^+$, the weights $c_{sj}\gamma_j$ on correlation ρ_{ij} are same for all trades $i \in I$. Using the fact that weights $\gamma_j c_{sj}$ on ρ_{ij} is congruent for every agent $i \in I$, we can thus identify the *most (and the least) influential trader* for price inference in the market. In the heterogeneous model, agents who learn more from prices are not necessarily those from whom others learn the most (in the sense of weights $\gamma_j c_{sj}$). Being more correlated with the market increases own c_{pi} and increases others' price impact, hence decreasing c_{pi} . With agent-specific inference coefficients, either effect may dominate.

EXAMPLE 5 (Informational Size). *The Group Model.*

In the symmetric model, the efficiency loss is idiosyncratic, even though ψ_p^+ is not. Nevertheless, changes in the market structure that lower price informativeness ψ_p^+ increase the loss $\psi_{Loss,i}^+$. In the heterogeneous model, this need *not* be the case: as $\psi_{p,i}^+$ decreases, $\psi_{s,i}^+$ can increase so that $\psi_{Loss,i}^+$ decreases.

EXAMPLE 6 (Informational Loss). *As the per-link correlation increases in the Location Model, the informational loss increases (Fig 8new).*

In summary, in the symmetric model, signal and price are always substitutes in inference; inference coefficients c_s and c_p are monotone in the average correlation $\bar{\rho}$; $\psi_{s,i}^+$, ψ_p^+ , $\psi_{Loss,i}^+$ are monotone in its absolute value, $|\bar{\rho}|$; all traders learn equally from price and signal (in the sense of $c_{p,i}$, $c_{s,i}$, ψ_p^+ , and $\psi_{s,i}^+$; welfare is monotone in the heterogeneity in $\{\rho_{i,j}\}_{j \neq i}$, holding $\bar{\rho}$ fixed. We show that none of these predictions holds in general in the heterogeneous model. In general, the joint behavior of signal and price informativeness indices (monotonicity (for a given trader) and the ranking (across traders)), c_{si} , c_{pi} and $\psi_{s,i}^+$, ψ_p^+ , $\psi_{Loss,i}^+$ can be essentially *arbitrary*, as the market structure changes (e.g., Fig 1 from 9/11). We next characterize the conditions for the inference coefficients c_s and c_p as well as informativeness indices $\psi_{s,i}^+$, $\psi_{p,i}^+$, $\psi_{Loss,i}^+$ to be monotone in the general model.

5 Inference Tradeoffs

5.1 Signal and Price Informativeness

To establish in which models a trade-off between the informativeness of signal and price is absent (or present, like in the symmetric model), we first develop the conditions on the information structures for which the informativeness of signal and price ψ_{si}^+ and ψ_{ip}^+ and inference coefficients c_{si} , c_{pi} are monotone.

PROPOSITION 2 (Monotonicity of Signal Informativeness). *The signal informativeness of trader i $\psi_{s,i}^+$ increases if, and only if, $[(C + \sigma^2 \mathbf{Id})^{-1}]_{ii}$ decreases;*

$$[(C + \sigma^2 \mathbf{Id})^{-1}]_{ii} = ((1 + \sigma^2) - C_{i,-i}(C_{-i,-i} + \sigma^2 \mathbf{Id})^{-2} C_{i,-i}^T)^{-1}$$

Notice that $[(C + \sigma^2 \mathbf{Id})^{-1}]_{ii} > (1 + \sigma^2)^{-1}$ for any positive definite C with $C_i \neq \mathbf{e}_i$, where $\mathbf{e}_i$ is a vector that puts 1 on the i^{th} and 0 elsewhere, and $\psi_{s,i}^+ \leq 1$. Therefore, the signal informativeness of a trader must be strictly positive in any information network, unless he has no link to any other trader.²²

Since $\psi_{p,i}^+$ is proportional to $\psi_{s,i}^+$, we will characterize the sufficient conditions for no tradeoff in the price and signal informativeness of trader i when $\psi_{s,i}^+$ and $\psi_{p,i}^+/\psi_{s,i}^+$ are comonotone.²³

PROPOSITION 3 (Monotonicity of Price Informativeness). *The price-to-signal informativeness ratio of trader i , $\psi_{p,i}^+/\psi_{s,i}^+$, increases if, and only if,*

$$\sigma^2 \bar{\rho}_i \left(2(I-1) \left(\frac{(1+\sigma^2)}{I-1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \right) > 0.$$

As a general observation, the behavior of $\psi_{s,i}^+$ corresponds to that of $C_{i,-i}$.²⁴ In particular, when $C_{i,-i} \sim 0$, then $\psi_{s,i}^+$ is maximized. In turn, the behavior of $\psi_{p,i}^+$ corresponds to that of the weighted commonality $\bar{\rho}_i = \Gamma_i^{-1} \left(\frac{1}{I-1} \sum_{j \neq i} \Gamma_j \rho_{ij} \right)$, which affects the informativeness ratio

$$\frac{\psi_{p,i}^+}{\sigma^2 \psi_{s,i}^+} = \frac{\sigma^2 \bar{\rho}_i^2}{(1 + \sigma^2) \left(\frac{(1+\sigma^2)}{I-1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \left(\frac{(1+\sigma^2)}{I-1} + \bar{\rho}_i \right)}$$

in the general model analogously to how the equally-weighted commonality $\bar{\rho} = \frac{1}{I-1} \sum_{j \neq i} \rho_{ij}$ does in the symmetric model.^{25,26}

The next result characterizes the conditions, in the general Gaussian model, when no tradeoff arises

²² In numerical experiments for Group Models, $\psi_{s,L}^+$ and $\psi_{s,S}^+$ are maximized when the cross-group correlation ρ is zero, for any I and $R = G_L/G_S$.

²³ Clearly, an absence of a tradeoff may involve an increase in $\psi_{p,i}^+/\psi_{s,i}^+$, and a decrease in $\psi_{s,i}^+$ and $\psi_{p,i}^+$. The behavior (e.g., argmin) of the ratio – but not $\psi_{p,i}^+$ – matches that of c_p . (The argmin does not coincide for $\psi_{s,i}^+$ and c_s). In the formula for the price informativeness $\psi_{p,i}^+$, the numerator is a weighted sum of correlations $\{\rho_{ij}\}_{j \neq i}$. The weights on ρ_{ij} are different for each $j \neq i$, except in the symmetric model.

$\psi_{p,i}^+ / (\sigma^2 \psi_{s,i}^+)$ is also the price informativeness used in Rostek and Weretka (2012).

²⁴ $\psi_{s,i}^+$ increases if, and only if, $(C_{i,-i}(C_{-i,-i} + \sigma^2 \mathbf{Id})^{-2} C_{i,-i}^T)$ decreases. The term $(C_{i,-i}(C_{-i,-i} + \sigma^2 \mathbf{Id})^{-2} C_{i,-i}^T)$ is a quadratic function of $C_{i,-i}$, which represents the information network from trader i 's perspective.

²⁵ In the equicommonal model, $\Gamma_i \equiv c_s/(\mu + \lambda)$ and $\bar{\rho}_i \equiv \frac{1}{I-1} \sum_{j \neq i} \rho_{ij} = \bar{\rho}$ are constant across i ; hence, $\frac{\psi_{p,i}^+}{\sigma^2 \psi_{s,i}^+} = \frac{\sigma^2 \bar{\rho}^2}{(1+\sigma^2) \left(\frac{(1+\sigma^2)}{I-1} + \bar{\rho} \right) - \bar{\rho} \left(\frac{(1+\sigma^2)}{I-1} + \bar{\rho} \right)} = \frac{\sigma^2 \bar{\rho}^2}{\left[\frac{(1+\sigma^2)}{I-1} + \bar{\rho} \right] [(1+\sigma^2) - \bar{\rho}]}$.

²⁶ The effect of $\bar{\rho}_i$ on $\psi_{p,i}^+/\psi_{s,i}^+$ differs from the effect of $\bar{\rho}$ on the price informativeness (ψ_p^+/ψ_s^+) instead of ψ_p^+ in Rostek and Weretka (2012). In the symmetric model, a change any trader's commonality must be accompanied by identical changes in the commonalities of all other traders. In the heterogeneous model, the changes in $\bar{\rho}_i$ and $\{\bar{\rho}_j\}_{j \neq i}$ can be separated in the formula for $(\psi_{p,i}^+/\psi_{s,i}^+)$. If all statistics $\bar{\rho}_i$ and $\{\bar{\rho}_j\}_{j \neq i}$ (or, $\text{Avg}\{\bar{\rho}_j\}_{j \neq i}$) change simultaneously, monotonic behavior of the price informativeness might result.

between price and signal informativeness,²⁷ when viewed as a function of a variable x that indexes a change in the market structure, such as the market size I , per-link correlation ρ .

PROPOSITION 4 (Price and Signal Informativeness: Tradeoffs). $\psi_{p,i}^+/\psi_{s,i}^+(x)$ and $\psi_{s,i}^+(x)$ exhibit a tradeoff if $\mathcal{C}_{i,-i} = 0$ if, and only if, $\bar{\rho}_i = 0$.

5.2 Inference Coefficients

In the symmetric model, the conditions that characterize the behavior of signal and price informativeness also characterize the signal and price inference coefficients in the conditional expectations. As Example 2 demonstrates, in the heterogeneous model, bidding behavior may respond to signal and price changes differently than the respective informativeness indices. Therefore, we establish the conditions for the monotonicity of c_{si}, c_{pi} of trader i as a function of own $\bar{\rho}_i$ and for price and signal to be substitutes or complements in a trader's inference.

We consider a direct effect of $\bar{\rho}_i$ on the inference coefficients by holding fixed $\{\Gamma_j\}_{j \in I}$, which are endogenously determined by $\bar{\rho}_i$ and affect the coefficient indirectly.

PROPOSITION 5 (Inference Coefficients: Monotonicity). Fix $\{\Gamma_j\}_{j \in I}$. Price inference coefficient c_{pi} is always increasing in $\bar{\rho}_i$. The signal inference coefficient $c_{s,i}$ is increasing if, and only if,

$$\left(\bar{\rho}_i + \frac{1 + \sigma^2}{2(I-1)}\right) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I-1} \text{Avg}\{\Gamma_j^2\}_{j \neq i}\right) < 0.$$

Hence, if the average of the commonalities of other market participants is large enough,²⁸ $c_{s,i}$ is maximized when $\bar{\rho}_i = -\frac{1+\sigma^2}{2(I-1)} < 0$, and is decreasing when $\bar{\rho}_i > -\frac{1+\sigma^2}{2(I-1)}$.

PROPOSITION 6 (Price and Signal Inference Coefficients). When $\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} > -\frac{1+\sigma^2}{I-1} \text{Avg}\{\Gamma_j^2\}_{j \neq i}$, inference coefficients $|c_{p,i}|$ and $c_{s,i}$ are both decreasing in weighted commonality $\bar{\rho}_i$ if, and only if, $-\frac{1+\sigma^2}{2(I-1)} < \bar{\rho}_i < 0$. When $\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} < -\frac{1+\sigma^2}{I-1} \text{Avg}\{\Gamma_j^2\}_{j \neq i}$, inference coefficients $|c_{p,i}|$ and $c_{s,i}$ are both decreasing in weighted commonality $\bar{\rho}_i$ if, and only if, $\bar{\rho}_i < -\frac{1+\sigma^2}{2(I-1)}$ or $\bar{\rho}_i > 0$.

Notice that this condition for no trade-off in inference for bidder i does not depend on the endogenous weight $\{\Gamma_j\}_{j \in I}$ or commonalities of bidders other than i , $\{\bar{\rho}_j\}_{j \neq i}$.

6 Examples

6.1 Group Model

There are two groups; the larger group (L) with G_L agents and the smaller group (S) with G_S agents. Total number of agents is denoted by $I = G_L + G_S$, and $G_S \leq G_L \leq I/2$. Agents in each group

²⁷ This explains why $\psi_{p,i}^+$ and $\psi_{s,i}^+$ always tradeoff in Dealer Models. There, by construction, $\mathcal{C}_{i,-i} = 0$ if, and only if, $\bar{\rho}_i = 0$; thus, no values of ρ exists such that both the ratio $\psi_{p,i}^+/\psi_{s,i}^+$ and $\psi_{s,i}^+$ increase. In the Group Model, there is a range of parameter ρ for which there is no tradeoff between $\psi_{p,i}^+$ {ratio?} and $\psi_{s,i}^+$, which both increase.

Let us observe that $\psi_{s,i}^+$ is determined only by $\mathcal{C}$ but $\psi_{p,i}^+$ is affected by $\{c_{si}\gamma_i\}_{i \in I}$. Thus, signal informativeness $\psi_{s,i}^+$ treats each correlation ρ_{ij} equally on links when $(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1}$. However, price informativeness $\psi_{p,i}^+$ depends on weighted commonality $\bar{\rho}_i$ and weights $\Gamma = \{c_{si}\gamma_i\}$.

²⁸ The condition involving the average is the same as in no trade-off between $\psi_{s,i}^+$ and $\psi_{p,i}^+$.

$k \in \{L, S\}$ have common risk-aversion μ_k , and their values are perfectly correlated (i.e., $\rho_{ij} = 1$ if agents i and j are in the same group). Cross-group correlation ρ can be any number in $[-1, 1]$. The commonality function $\bar{\rho}_k$ for each group $k \in \{L, S\}$ is $\bar{\rho}_k(G_k, I) = \frac{1}{I-1} (G_k - 1 + (I - G_k)\rho)$. Furthermore, the fixed-point problem for $\{c_{pk}, c_{sk}, \lambda_k\}_{k \in \{L, S\}}$ can be written as the following system of equations.

$$\begin{aligned}\lambda_k &= ((G_k - 1)\gamma_k(1 - c_{pk}) + G_l\gamma_l(1 - c_{pl}))^{-1}, \quad k \neq l \in \{L, S\} \\ c_{pk} &= \frac{\sigma^2 ((G_k - 1)\gamma_k c_{sk} + G_l\gamma_l c_{sl}\rho) (G_k\gamma_k(1 - c_{pk}) + G_l\gamma_l(1 - c_{pl}))}{(1 + \sigma^2)H - (G_k\gamma_k c_{sk} + G_l\gamma_l c_{sl}\rho + \gamma_k c_{sk}\sigma^2)^2} \\ c_{sk} &= \frac{H - (G_k\gamma_k c_{sk} + G_l\gamma_l c_{sl}\rho) (G_k\gamma_k c_{sk} + G_l\gamma_l c_{sl}\rho + \gamma_j c_{sk}\sigma^2)}{(1 + \sigma^2)H - (G_k\gamma_k c_{sk} + G_l\gamma_l c_{sl}\rho + \gamma_j c_{sk}\sigma^2)^2}\end{aligned}$$

where $\gamma_k = \frac{1}{\mu_k + \lambda_k}$ for $k \in \{L, S\}$, and

$$H = (G_L^2 + G_L\sigma^2)\gamma_L^2 c_{sL}^2 + (G_S^2 + G_S\sigma^2)\gamma_S^2 c_{sS}^2 + 2G_L G_S \gamma_L \gamma_S c_{sL} c_{sS} \rho.$$

[Insert Figure 5, 6 and 7 here]

6.2 Dark Pool

As an extension of the group model, the correlation within groups can be any value between -1 and 1 . We denote the correlation between any two agents in group L by ρ_L , and the correlation between any two agents in group S by ρ_S . The Group Model is a special case with $\rho_L = \rho_S = 1$.

[Insert Figure 8 and 9 here]

In Fig 4, the commonality of group L is larger than the commonality of group S if and only if $\rho < 0.04$, for given parameter values. In Fig 5, it is the case if and only if $\rho_L > 0.8$. In both figures, we can see that whether an endogenous parameter of group L is larger than the one of group S is determined by the commonalities of two groups. The group with larger commonality has smaller c_S, λ_S but larger c_p, ψ_p^+ .

6.3 Dealer Model

EXAMPLE 7 (Single Central Dealer). *There are two groups of agent; the D group has only one agent, i.e. $G_D = 1$, while the C group has $G_C \geq 2$ agents. We call the agent in group D (central) dealer. An agent in group C , who is called customer, can only have a link to the dealer, and there is no link between agents within group C . The number of links which the central dealer has is same as the number of agents G_C in group C . The strength of links (correlation of valuation between two agents in each link) is equally $\rho \in [-1, 1]$.*

If $G_C = 5$, the network graph and the corresponding covariance matrix looks as follows.

The commonality functions are calculated as $\bar{\rho}_C = \frac{\rho}{I-1}$ for customers in group C and $\bar{\rho}_D = \rho$ for the central dealer in group D . Furthermore, the fixed point problem for $\{c_{pk}, c_{sk}, \lambda_k\}_{k \in \{C, D\}}$ is written

by the following system of equations.

$$\begin{aligned} \lambda_C &= (\gamma_D(1 - c_{pD}) + (G_C - 1)\gamma_C(1 - c_{pC}))^{-1} \\ \lambda_D &= (G_C\gamma_C(1 - c_{pC}))^{-1} \\ c_{pC} &= \sigma^2 \frac{\gamma_D c_{sD} \rho (\gamma_D(1 - c_{pD}) + G_C\gamma_C(1 - c_{pC}))}{(1 + \sigma^2)H - ((1 + \sigma^2)\gamma_C c_{sC} + \gamma_D c_{sD} \rho)^2} \\ c_{pD} &= \sigma^2 \frac{G_C\gamma_C c_{sC} \rho (\gamma_D(1 - c_{pD}) + G_C\gamma_C(1 - c_{pC}))}{(1 + \sigma^2)H - ((1 + \sigma^2)\gamma_D c_{sD} + G_C\gamma_C c_{sC} \rho)^2} \\ c_{sC} &= \frac{H - (\gamma_C c_{sC} + \gamma_D c_{sD} \rho) ((1 + \sigma^2)\gamma_C c_{sC} + \gamma_D c_{sD} \rho)}{(1 + \sigma^2)H - ((1 + \sigma^2)\gamma_C c_{sC} + \gamma_D c_{sD} \rho)^2} \\ c_{sD} &= \frac{H - (\gamma_D c_{sD} + G_C\gamma_C c_{sC} \rho) ((1 + \sigma^2)\gamma_D c_{sD} + G_C\gamma_C c_{sC} \rho)}{(1 + \sigma^2)H - ((1 + \sigma^2)\gamma_D c_{sD} + G_C\gamma_C c_{sC} \rho)^2} \end{aligned}$$

where $\gamma_k = (\mu_k + \lambda_k)^{-1}$ for $k \in \{C, D\}$, and

$$H = (1 + \sigma^2)\gamma_D^2 c_{sD}^2 + (1 + \sigma^2)G_C\gamma_C^2 c_{sC}^2 + 2G_C\gamma_C\gamma_D c_{sC} c_{sD} \rho.$$

[Insert Figure 10, 11 and 12 here]

EXAMPLE 8 (Groups with Equal Sizes). *There are two groups of $I/2$ agents each. An agent in group C (customers) can only have links to agents in group D (dealers), and there is no link between agents within group C . An agent in group C has only one link to an agent in group D . An agent in group D has n_D within-group links in addition to a link to an agent in group C . The strength of links (correlation of valuation between two agents in each link) is equally $\rho \in [-1, 1]$. The network is symmetric for agents in each group. By construction, agents in group D have more links than agents in group C have.*

For example, there are eight agents, half of which is in group D and the other half of which is in group C . When $n_D = 2$, suppose that the agents in group D have even indices and the agents in group C have odd indices. The network and the covariance matrix look as follows. The figure in below uses wrong indications. L and S should be changed.

[Insert Figure 13, 14, 15 and 16 here]

EXAMPLE 9 (Multiple Customers per Dealer). *There are two groups of agents; An agent in group C (customers) can only have a unique link to an agent in group D (dealers), and there is no link between agents within group C. An agent in group D has n_D within-group links in addition to $n_C \geq 1$ links to n_C agents in group C. The strength of links (correlation of valuation between two agents in each link) is equally $\rho \in [-1, 1]$. The network is symmetric for agents in each group. By the construction, the number of agents in group D, which is denoted as G_D is less than the number of agents in group C, G_C , and $G_C = n_C \cdot G_D$ holds.*

The network in Example 8 is the special case of this model with $n_C = 1$, where the number of customers and of dealers are same. As an example, there are four dealers, each of whom has two customers and also has two other links within group D, i.e. $G_D = 4, n_C = 2$, and $n_D = 2$. Then, the total number of agents in group C is $G_C = 8$. The network looks as follows.

$$\begin{pmatrix} 1 & \rho & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho & 1 & 0 & \rho & 0 & 0 & 0 & \rho \\ 0 & 0 & 1 & \rho & 0 & 0 & 0 & 0 \\ 0 & \rho & \rho & 1 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \rho & 0 & 0 \\ 0 & 0 & 0 & \rho & \rho & 1 & 0 & \rho \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \rho \\ 0 & \rho & 0 & 0 & 0 & \rho & \rho & 1 \end{pmatrix}$$

[Insert Figure 17, 18, 19, and 20 here]

6.4 Tree Models

EXAMPLE 10 (Core-Periphery Model). *There are three types of agents; a central dealer (D-group), intermediate dealers (I-group), and customers (C-group). The central dealer is linked to G_I intermediate dealers, and thus, the number of links which the central dealer has is G_I . Each intermediate dealer in group I has n_C customers. Each customer in group C has only one link to her dealer.*

The total number of customers is $G_C = n_C G_I$, and the total number of agents is $I = 1 + G_I + G_C$. The following example is when $G_I = 4, n_C = 2$.

[Insert Figure 21, 22 and 23 here]

EXAMPLE 11 (Location Model). *Suppose that the information structure follows the binomial tree which has tree links only for the first generation, and after that, each agent has two children. Then the peripheral agents has two links, and the other agents has tree links. This model can be used to explore the effect of agents' location in the tree, while the Core-Periphery Model is more appropriate to explore the effect of number of links. The number of generation is N , and each of generation is denoted by $i = 1, \dots, N$, from the center to the peripheral.*

The number of links for the generation i is 3 if $i < N$, or 1 if $i = N$. The number of agents in the generation i is $3 \cdot 2^{i-2}$ if $i > 1$, or 1 if $i = 1$. Hence, the total number of agents are $I = 3 \cdot 2^{N-1} - 2$. Remark that the case with $N = 3$ is equivalent to the core-periphery model with $G_I = 3, n_C = 2$. The following structure is when $N = 4$.

[Insert Figure 24 and 25 here]

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Appendix

A. Equilibrium

Derivation of linear equilibrium in the general model: Given an affine information structure, the conditional expectation is linear, $E(\theta_i|p, s_i) = E(\theta_i) + c_{si}(s_i - E(s_i)) + c_{pi}(p - E(p)) = E(\theta_i)(1 - c_{si} - c_{pi}) + c_{si}s_i + c_{pi}p$, using $E_i(\theta_i) = E(\theta_i)$ for all i . Hence, $c_{\theta i} = 1 - c_{si} - c_{pi}$ and the equilibrium bid

is

$$q_i(p) = \frac{1}{(\mu_i + \lambda_i)} [c_{\theta i} E(\theta_i) + c_{si} s_i - (1 - c_{pi})p]$$

Aggregating bids $q_i(p)$, by market clearing, the equilibrium price is

$$p^* = \left(\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i} \right)^{-1} \sum_{i \in I} \frac{1}{\mu_i + \lambda_i} [c_{\theta i} E(\theta_i) + c_{si} s_i]. \quad (13)$$

That is, agents with larger $\frac{c_{si}}{\mu_i + \lambda_i}$ impact price more. The equilibrium price impact of agent i is

$$\lambda_i = - \frac{1}{\sum_{j \neq i} (\partial q_j(p) / \partial p)} = \frac{1}{\sum_{j \neq i} \frac{1 - c_{pj}}{\mu_j + \lambda_j}}.$$

Using (13), random vector (θ_i, s_i, p^*) is jointly normally distributed,

$$\begin{pmatrix} \theta_i \\ s_i \\ p^* \end{pmatrix} = \mathcal{N} \left[\begin{pmatrix} E(\theta_i) \\ E(s_i) \\ E(p) \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov(\theta_i, p^*) \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & cov(s_i, p^*) \\ cov(p^*, \theta_i) & cov(p^*, s_i) & Var(p^*) \end{pmatrix} \right]. \quad (14)$$

$$\begin{aligned} & \mu_\theta + \Sigma_{\theta, s} \Sigma_{s, s}^{-1} (s - \mu_s) \\ &= \mu_\theta + \begin{pmatrix} \sigma_\theta^2 & cov(\theta_i, p^*) \end{pmatrix} \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & cov(s_i, p^*) \\ cov(p^*, s_i) & Var(p^*) \end{pmatrix}^{-1} (s - \mu_s) \\ E(p^*) &= \left(\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i} \right)^{-1} \sum_{i \in I} \frac{1}{\mu_i + \lambda_i} [c_{\theta i} m_i + c_{si} E(s_i)] = \frac{\left[\sum_{i \in I} \frac{(c_{\theta i} + c_{si}) m_i}{\mu_i + \lambda_i} \right]}{\left[\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i} \right]}. \end{aligned}$$

Let $G \equiv \sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i}$, $\gamma_i \equiv \frac{1}{\mu_i + \lambda_i}$ and $\bar{\rho}_i \equiv \frac{1}{I-1} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij}$. The covariances in (14) are given by

$$cov(\theta_i, p^*) = \left(\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i} \right)^{-1} \left(\gamma_i c_{si} + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) \sigma_\theta^2 = \frac{1}{G} \left(\gamma_i c_{si} + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) \sigma_\theta^2 \quad (15)$$

$$\left(v.s. \frac{1}{I} \frac{c_s}{1 - c_p} (1 + (I - 1) \bar{\rho}) \sigma_\theta^2 \text{ in the equicommonal model} \right) \quad (16)$$

$$cov(s_i, p^*) = \frac{1}{G} \left(\gamma_i c_{si} (1 + \sigma^2) + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) \sigma_\theta^2 \quad (17)$$

$$\left(v.s. \frac{1}{I} \frac{c_s}{1 - c_p} ((1 + \sigma^2 + (I - 1) \bar{\rho})) \sigma_\theta^2 \text{ in the equicommonal model} \right) \quad (18)$$

and

$$Var(p^*) = \left(\frac{1}{\left[\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i} \right]} \right)^2 \left(\sum_i \frac{c_{si}^2 (1 + \sigma^2)}{(\mu_i + \lambda_i)^2} + \sum_i \frac{c_{si}}{\mu_i + \lambda_i} \sum_{j \neq i} \frac{c_{sj}}{\mu_j + \lambda_j} \rho_{ij} \right) \sigma_\theta^2 \quad (19)$$

$$= \frac{1}{G^2} \left((1 + \sigma^2) \sum_i \gamma_i^2 c_{si}^2 + \sum_i \gamma_i c_{si} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) \sigma_\theta^2 \quad (20)$$

$$\left(v.s. \frac{1}{I} \left(\frac{c_s}{1 - c_p} \right)^2 (1 + \sigma^2 + (I - 1) \bar{\rho}) \sigma_\theta^2 \text{ in the equicommonal model} \right). \quad (21)$$

Compared to the equicommonal information structures, the covariances in (14) depend on $\frac{c_{si}}{\mu_i + \lambda_i}$, $\sum_{i \in I} \frac{1 - c_{pi}}{\mu_i + \lambda_i}$ and $\frac{1}{I-1} \sum_{j \neq i} \frac{c_{sj}}{c_{si}} \frac{\mu_i + \lambda_i}{\mu_j + \lambda_j} \rho_{ij}$ (rather than c_s , $1 - c_p$, and $\bar{\rho}$, respectively).

Inference coefficients: Applying the projection theorem and the method of undetermined coefficients yields the inference coefficients c_{si} and c_{pi} .

$$\begin{aligned} c_{si} &= \frac{\sigma_\theta^2 \text{Var}(p^*) - \text{cov}(\theta_i, p^*) \text{cov}(p^*, s_i)}{(\sigma_\theta^2 + \sigma_\varepsilon^2) \text{Var}(p^*) - (\text{cov}(s_i, p^*))^2} \\ c_{pi} &= \frac{\text{cov}(\theta_i, p^*) (\sigma_\theta^2 + \sigma_\varepsilon^2) - \sigma_\theta^2 \text{cov}(s_i, p^*)}{(\sigma_\theta^2 + \sigma_\varepsilon^2) \text{Var}(p^*) - (\text{cov}(s_i, p^*))^2} \end{aligned}$$

$$\begin{aligned} c_{si} &= \frac{\left((1 + \sigma^2) \sum_i \gamma_i^2 c_{si}^2 + \sum_i \gamma_i c_{si} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) - \left(\gamma_i c_{si} + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) \left(\gamma_i c_{si} (1 + \sigma^2) + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)}{(1 + \sigma^2) \left((1 + \sigma^2) \sum_i \gamma_i^2 c_{si}^2 + \sum_i \gamma_i c_{si} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) - \left(\gamma_i c_{si} (1 + \sigma^2) + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)^2} \\ c_{pi} &= \frac{\sigma^2 \left(\sum_{i \in I} \gamma_i (1 - c_{pi}) \right) \left(\sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)}{(1 + \sigma^2) \left((1 + \sigma^2) \sum_i \gamma_i^2 c_{si}^2 + \sum_i \gamma_i c_{si} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) - \left(\gamma_i c_{si} (1 + \sigma^2) + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)^2} \end{aligned}$$

Let $H \equiv \left((1 + \sigma^2) \sum_i \gamma_i^2 c_{si}^2 + \sum_i \gamma_i c_{si} \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right) = \sigma^2 \sum_{j \in I} \gamma_j^2 c_{sj}^2 + \sum_{i, j \in I} \gamma_i \gamma_j c_{si} c_{sj} \rho_{ij} = \Gamma^T (\mathcal{C} + \sigma^2 \mathbf{Id}) \Gamma$, independent of i .

$$\begin{aligned} c_{si} &= \frac{H - \left(\sum_{j \in I} \gamma_j c_{sj} \rho_{ij} \right) \left(\sigma^2 \gamma_i c_{si} + \sum_{j \in I} \gamma_j c_{sj} \rho_{ij} \right)}{(1 + \sigma^2) H - \left(\sigma^2 \gamma_i c_{si} + \sum_{j \in I} \gamma_j c_{sj} \rho_{ij} \right)^2} = \frac{H - [\mathcal{C}\Gamma]_i [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i}{(1 + \sigma^2) H - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} \\ &= \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} - \bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I-1} \left(\text{Avg}\{\Gamma_j^2\}_{j \neq i} - \bar{\rho}_i \Gamma_i^2 \right)}{(1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} - \bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I-1} \left((1 + \sigma^2) \text{Avg}\{\Gamma_j^2\}_{j \neq i} - \bar{\rho}_i \Gamma_i^2 \right)} \\ &= \frac{1}{1 + \sigma^2} - \frac{\sigma^2}{1 + \sigma^2} \frac{\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I-1} \bar{\rho}_i \Gamma_i^2}{(1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I-1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) - \left(\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I-1} \bar{\rho}_i \Gamma_i^2 \right)}, \\ c_{pi} &= \frac{\sigma^2 \left(\sum_{j \in I} \gamma_j (1 - c_{pj}) \right) \left(\sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)}{(1 + \sigma^2) H - \left(\sigma^2 \gamma_i c_{si} + \sum_{j \in I} \gamma_j c_{sj} \rho_{ij} \right)^2} = \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i \left(\sum_{j \in I} \gamma_j (1 - c_{pj}) \right)}{(1 + \sigma^2) H - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} \\ &= \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i \left(\frac{1}{\lambda_i} + \gamma_i \right)}{(1 + \sigma^2) H - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2 + \sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i \gamma_i} = \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i \left(\frac{1}{\lambda_i} + \gamma_i \right)}{(1 + \sigma^2) H - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2 + \sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i \gamma_i} \\ &= \frac{\frac{\sigma^2}{I-1} \left(\frac{1}{\lambda_i} + \frac{1}{\mu_i + \lambda_i} \right) \Gamma_i \bar{\rho}_i}{(1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I-1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) - \left(\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I-1} \bar{\rho}_i \Gamma_i^2 \right) + \frac{\sigma^2}{(I-1)(\mu_i + \lambda_i)} \Gamma_i \bar{\rho}_i}. \end{aligned}$$

Inference indices: Inference indices in terms of H , in matrix form and in terms of $\bar{\rho}_i$ are

$$\psi_{s,i}^+ = \frac{1}{1 + \sigma^2} \frac{1}{[\mathcal{C}(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1} \mathcal{C}]_{ii}} = \frac{1}{1 + \sigma^2} \frac{1}{1 - \sigma^2 + \sigma^4 [(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1}]_{ii}}$$

$$\begin{aligned}
\frac{\psi_{p,i}^+}{\sigma^2 \psi_{s,i}^+} &= \frac{\sigma^2 (\sum_{j \neq i} c_{sj} \gamma_j \rho_{ij})^2}{(1 + \sigma^2)H - \left((1 + \sigma^2) \gamma_i c_{si} + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)^2} = \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i^2}{(1 + \sigma^2)H - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} \\
&= \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i^2}{(1 + \sigma^2)\Gamma^T(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} \\
&= \frac{\sigma^2 \bar{\rho}_i^2}{(1 + \sigma^2) \left(\frac{(1+\sigma^2)}{I-1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \left(\frac{(1+\sigma^2)}{I-1} + \bar{\rho}_i \right)}
\end{aligned}$$

$$\psi_{Loss,i}^- = 1 - \psi_{p,i}^+ - \psi_{s,i}^+ = \sigma^4 \psi_{s,i}^+ \left((1 + \sigma^2)[(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1}]_{ii} - 1 - \frac{[(\mathcal{C} - \mathbf{Id})\Gamma]_i^2}{(1 + \sigma^2)H - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} \right).$$

$$\begin{aligned}
\psi_{p,i}^+ &= \frac{\sigma^4 \psi_{s,i}^+ (\sum_{j \neq i} c_{sj} \gamma_j \rho_{ij})^2}{(1 + \sigma^2)H - \left((1 + \sigma^2) \gamma_i c_{si} + \sum_{j \neq i} \gamma_j c_{sj} \rho_{ij} \right)^2} = \sigma^2 \psi_{s,i}^+ \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i^2}{(1 + \sigma^2)\Gamma^T(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} = \\
&\quad \frac{\left(1 - \frac{1}{1+\sigma^2}\right)}{[\mathcal{C}(\mathcal{C} + \sigma^2 \mathbf{Id})^{-1}\mathcal{C}]_{ii}} \cdot \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i}{(1 + \sigma^2)(\sigma^2 \Gamma^T \Gamma + \Gamma^T \mathcal{C} \Gamma) - [\sigma^2 \Gamma + \mathcal{C} \Gamma]_i} \\
&= \psi_{s,i}^+ \cdot \frac{\sigma^4 [(\mathcal{C} - \mathbf{Id})\Gamma]_i}{(1 + \sigma^2)(\sigma^2 \Gamma^T \Gamma + \Gamma^T \mathcal{C} \Gamma) - [\sigma^2 \Gamma + \mathcal{C} \Gamma]_i} \\
&= \sigma^2 \psi_{s,i}^+ \frac{\sigma^2 \bar{\rho}_i^2}{(1 + \sigma^2) \left(\frac{(1+\sigma^2)}{I-1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \left(\frac{(1+\sigma^2)}{I-1} + \bar{\rho}_i \right)}
\end{aligned}$$

where $\Gamma = (c_{si} \gamma_i)_{i \in I} \in \mathbb{R}^I$. Here, $[v]_i$ is the i th element of vector v , and $[A]_{ij}$ is the (i, j) -element of matrix A . The presence of the trade-off or no trade-off between ψ_s^+ and ψ_p^+ is determined by the second-term in the last formula.

B. Equilibrium Existence

Proof of Proposition 1 (Existence of Equilibrium; *heuristic*) (*Only if*) The profile of bids (8), $i \in I$, constitutes an equilibrium with strictly downward-sloping bids only if

(i) $\text{Var}(\frac{1}{I} \sum_i \theta_i) \geq 0$, so that the joint distribution of $\{\theta_i\}_i$ is well defined; equivalently, $\sum_{i,j \in I} \rho_{ij} \geq 0$. Note that $\sum_{j \neq i} \rho_{ij} \geq -1$, $i \in I$, holds for any random vector $\{\theta_i\}_{i \in I}$. This implicitly defines a lower bound $\bar{\rho}_i^-(I, \mathcal{C})$.

(ii) for demand schedules to be downward sloping,

$$\infty > -(1 - c_{pi}) / \left(\mu_i + \left(\sum_{j \neq i} \frac{1 - c_{pj}}{\mu_j + \lambda_j} \right)^{-1} \right) > 0;$$

or $c_{pi} \leq 1$ for any $i \in I$. Using the explicit formula (7) for c_{pi} given $\{c_{sj}, \gamma_j\}_{j \in I}$, this condition is equivalent to

$$\begin{aligned}
c_{pi} > 1 &\Leftrightarrow -\frac{\sigma^2}{\mu_i + \lambda_i} < \frac{(1 + \sigma^2)\Gamma^T(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2}{[(\mathcal{C} - \mathbf{Id})\Gamma]_i} < \frac{\sigma^2}{\lambda_i} \\
\Leftrightarrow -\frac{\sigma^2}{\mu_i + \lambda_i} &< \frac{(1 + \sigma^2)^2(\Gamma^T \Gamma - \Gamma_i^2) + (1 + \sigma^2)(I - 1)(\Gamma^T \text{diag}(\bar{\rho})\Gamma - 2\bar{\rho}_i \Gamma_i^2) - (I - 1)^2 \bar{\rho}_i^2 \Gamma_i^2}{(I - 1)\bar{\rho}_i \Gamma_i} < \frac{\sigma^2}{\lambda_i},
\end{aligned}$$

where $\bar{\rho} = \{\bar{\rho}_i\}_{i \in I} \in \mathbb{R}^I$. Term $(\Gamma^T \Gamma - \Gamma_i^2) = \sum_{j \neq i} c_{sj}^2 \gamma_j^2$ represents the sum of squared weights in $\bar{\rho}_i$ and term $(\Gamma^T \text{diag}(\bar{\rho}) \Gamma - \bar{\rho}_i \Gamma_i^2)$ is equivalent to $\sum_{j \neq i} \bar{\rho}_j \Gamma_j^2 = \sum_{j \neq i} (c_{sj} \gamma_j)^2 \bar{\rho}_j$. Therefore,

$$\begin{aligned} & \frac{(1 + \sigma^2)^2 (\Gamma^T \Gamma - \Gamma_i^2) + (1 + \sigma^2)(I - 1)(\Gamma^T \text{diag}(\bar{\rho}) \Gamma - 2\bar{\rho}_i \Gamma_i^2) - (I - 1)^2 \bar{\rho}_i^2 \Gamma_i^2}{(I - 1) \bar{\rho}_i \Gamma_i} \\ &= \frac{(1 + \sigma^2)^2 \text{Avg}\{\Gamma_j^2\}_{j \neq i} + (1 + \sigma^2)(I - 1) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} - (I - 1) \bar{\rho}_i^2 \Gamma_i^2 - (1 + \sigma^2) \bar{\rho}_i \Gamma_i^2}{\bar{\rho}_i \Gamma_i} \\ &= \frac{(1 + \sigma^2) \left((1 + \sigma^2) \text{Avg}\{\Gamma_j^2\}_{j \neq i} - \bar{\rho}_i \Gamma_i^2 \right) + (I - 1) \left((1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} - \bar{\rho}_i^2 \Gamma_i^2 \right)}{\bar{\rho}_i \Gamma_i}. \end{aligned}$$

Hence, condition (ii) for existence is equivalent to the *negation* of following inequalities:

$$-\frac{\sigma^2}{(\mu_i + \lambda_i)(I - 1)} < \frac{1 + \sigma^2}{I - 1} \left(\frac{(1 + \sigma^2) \text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\bar{\rho}_i \Gamma_i} - \Gamma_i \right) + \left(\frac{(1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\bar{\rho}_i \Gamma_i} - \bar{\rho}_i \Gamma_i \right) < \frac{\sigma^2}{\lambda_i(I - 1)}.$$

This implicitly defines two upper bounds: $\bar{\rho}_i^+(I, \mathcal{C})$. The desired upper bound is then $\bar{\rho}_i^+(I, \mathcal{C}) \equiv \max\{\bar{\rho}_i^+(I, \mathcal{C})\}$, whereas the lower bound is $\bar{\rho}_i^-(I, \mathcal{C}) \equiv \max\{\bar{\rho}_i^-(I, \mathcal{C}), \max\{\bar{\rho}_i^+(I, \mathcal{C})\}\}$.

(If) For any $\{\bar{\rho}_i\}_i$ such that $\bar{\rho}_i^-(I, \mathcal{C}) < \bar{\rho}_i < \bar{\rho}_i^+(I, \mathcal{C})$, $i \in I$, the first-order condition (2) is necessary and *sufficient* for optimality of the bid (8) (for any price) for each i , given that bidders $j \neq i$ submit bids (8). It follows that the bids(8) constitute a linear Bayesian Nash equilibrium. Q.E.D.

Existence conditions (i) and (ii) in specific models:

Group Model: The arguments are stated in terms of the cross-group correlation ρ , not in terms of the weighted commonalities, to facilitate comparison with the Figures. For any trader in either group L or S , condition (i) is

$$G_L^2 + G_S^2 + 2G_L G_S \rho \geq 0 \quad \Leftrightarrow \quad \rho \geq -1 \geq -\frac{G_L^2 + G_S^2}{2G_L + G_S},$$

which holds for any $\rho \in [-1, 1]$ and for any $G_L \geq G_S > 0$. Condition (ii) for traders in group L is that the quadratic function of ρ ,

$$-G_S^2 \gamma_S^2 c_{sS}^2 \left(\rho - \sigma^2 \frac{(G_L - 1) \gamma_L c_{sL}}{G_S \gamma_S c_{sS}} \right)^2 + (1 + \sigma^2) \left(\sigma^2 G_L (G_L - 1) \gamma_L^2 c_{sL}^2 + G_S (G_S + \sigma^2) \gamma_S^2 c_{sS}^2 \right),$$

should not be between two linear functions, $\frac{\sigma^2}{\lambda_L} ((G_L - 1) + G_S \rho)$ and $-\frac{\sigma^2}{\mu_L + \lambda_L} ((G_L - 1) + G_S \rho)$. Condition (ii) for group L is $\rho < \rho_L^+$, where ρ_L^+ is the positive solution of

$$\begin{aligned} 0 &= -G_S^2 \gamma_S^2 c_{sS}^2 \left(\rho - \sigma^2 \frac{(G_L - 1) \gamma_L c_{sL} - \frac{1}{2} \gamma_L}{G_S \gamma_S c_{sS}} \right)^2 + \frac{\sigma^4}{4} \gamma_L^2 \\ &\quad + (1 + \sigma^2) \left(\sigma^2 (G_L \gamma_L c_{sL} - \gamma_L) \gamma_L c_{sL} + G_S (G_S + \sigma^2) \gamma_S^2 c_{sS}^2 \right). \end{aligned}$$

Condition (ii) for group S is symmetric with condition (ii) for group L .²⁹

Single-Dealer Model: Condition (i) for the dealer and customers is

$$I + 2(I - 1)\rho \geq 0 \quad \Leftrightarrow \quad \rho \geq -\frac{I}{2(I - 1)}.$$

²⁹{CHECK: There might be a bound on the $\rho_{S,L}$ as a function of the within-group correlation (now 1).}

The lower bound $-\frac{I}{2(I-1)}$ is increasing in I and converges to $-1/2$ as $I \rightarrow \infty$. Condition (ii) for the central dealer is that the quadratic function of ρ , $\gamma_C c_{sC}((1 + \sigma^2)^2 - (I - 1)\rho^2)$ should not be between two linear functions $\frac{\sigma^2}{\lambda_D}\rho$ and $-\sigma^2\gamma_D\rho$. This condition is equivalent to $\rho \in (\rho_D^-, \rho_D^+)$, where $\rho_D^+ > 0$ and $\rho_D^- < 0$ satisfy

$$\begin{aligned} 0 &= (I - 1)\gamma_C c_{sC}(\rho_D^+)^2 - \gamma_C c_{sC}(1 + \sigma^2)^2 + \frac{\sigma^2}{\lambda_D}\rho_D^+ \\ 0 &= (I - 1)\gamma_C c_{sC}(\rho_D^-)^2 - \gamma_C c_{sC}(1 + \sigma^2)^2 - \sigma^2\gamma_D\rho_D^-. \end{aligned}$$

C. Inference

Proof of Lemma 1 (Information Aggregation) (*If*) Proved in Vives (2011). (*Only if; heuristic – complete by showing genericity*) For the class of equicommonal models, this part is proved in Rostek and Weretka (2012, Proposition 3). Consider a model in which the equicommonality assumption is not satisfied $\bar{\rho}_i \neq \bar{\rho}_j$ for some $i \neq j$. From the Projection Theorem, for every trader, there exists a statistic that is sufficient for the information contained in the signals of others. That statistic equals an weighted average signal with weights that are functions of correlations and, thus, generically, the statistic differs from the unweighted average $(1/I)\sum_i s_i$. The equilibrium price is a deterministic function of the unweighted average. *Q.E.D.*

Proof of Proposition 2 (Monotonicity of Signal Informativeness) Since

$$\begin{aligned} \mathcal{C}(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}\mathcal{C} &= (\mathcal{C} + \sigma^2\mathbf{Id})(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}\mathcal{C} - \sigma^2(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}\mathcal{C} = \mathcal{C} - \sigma^2(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}\mathcal{C} \\ &= \mathcal{C} - \sigma^2(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}(\mathcal{C} + \sigma^2\mathbf{Id}) + \sigma^4\sigma^2(\mathcal{C} + \sigma^2\mathbf{Id})^{-1} \\ &= \mathcal{C} - \sigma^2\mathbf{Id} + \sigma^4(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}, \end{aligned}$$

we can write $\psi_{s,i}^+$ as

$$\psi_{s,i}^+ = \frac{1}{1 + \sigma^2} \frac{1}{[\mathcal{C}(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}\mathcal{C}]_{ii}} = \frac{1}{1 - \sigma^4 + \sigma^4(1 + \sigma^2)[(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}]_{ii}}.$$

Hence, the signal informativeness of trader i increases if, and only if,

$$[(\mathcal{C} + \sigma^2\mathbf{Id})^{-1}]_{ii} = ((1 + \sigma^2) - \mathcal{C}_{i,-i}(\mathcal{C}_{-i,-i} + \sigma^2\mathbf{Id})^{-2}\mathcal{C}_{i,-i}^T)^{-1}$$

decreases. *Q.E.D.*

Proof of Proposition 3 (Monotonicity of Price Informativeness) We have

$$\begin{aligned}
\frac{\psi_{p,i}^+}{\sigma^2 \psi_{s,i}^+} &= \frac{\sigma^2 [(\mathcal{C} - \mathbf{Id})\Gamma]_i^2}{(1 + \sigma^2)\Gamma^T(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma - [(\mathcal{C} + \sigma^2 \mathbf{Id})\Gamma]_i^2} \\
&= \frac{\sigma^2 (I - 1)^2 \bar{\rho}_i^2 \Gamma_i^2}{(1 + \sigma^2)^2 (\Gamma^T \Gamma - \Gamma_i^2) + (1 + \sigma^2)(I - 1)(\Gamma^T \text{diag}(\bar{\rho})\Gamma - 2\bar{\rho}_i \Gamma_i^2) - (I - 1)^2 \bar{\rho}_i^2 \Gamma_i^2} \\
&= \frac{\sigma^2 \bar{\rho}_i^2 \Gamma_i^2}{\frac{(1 + \sigma^2)^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} + (1 + \sigma^2) \text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} - \frac{(1 + \sigma^2)}{I - 1} \bar{\rho}_i \Gamma_i^2 - \bar{\rho}_i^2 \Gamma_i^2} \\
&= \frac{\sigma^2 \bar{\rho}_i^2}{\frac{(1 + \sigma^2)^2}{I - 1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + (1 + \sigma^2) \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} - \frac{(1 + \sigma^2)}{I - 1} \bar{\rho}_i - \bar{\rho}_i^2} \\
&= \frac{\sigma^2 \bar{\rho}_i^2}{(1 + \sigma^2) \left(\frac{(1 + \sigma^2)}{I - 1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \left(\frac{(1 + \sigma^2)}{I - 1} + \bar{\rho}_i \right)}.
\end{aligned}$$

Taking the partial derivative of $(\psi_{p,i}^+/\psi_{s,i}^+)$ with respect to $\bar{\rho}_i$ proves the assertion,

$$\frac{\partial}{\partial \bar{\rho}_i} \left(\frac{\psi_{p,i}^+}{\sigma^2 \psi_{s,i}^+} \right) = \frac{\sigma^2 \bar{\rho}_i \left(2(1 + \sigma^2) \left(\frac{(1 + \sigma^2)}{I - 1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \frac{(1 + \sigma^2)}{I - 1} \bar{\rho}_i \right)}{\left[(1 + \sigma^2) \left(\frac{(1 + \sigma^2)}{I - 1} \frac{\text{Avg}\{\Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} + \frac{\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i}}{\Gamma_i^2} \right) - \bar{\rho}_i \left(\frac{(1 + \sigma^2)}{I - 1} + \bar{\rho}_i \right) \right]^2}.$$

Q.E.D.

Proof of Proposition 4 (Price and Signal Informativeness: Tradeoffs) The following geometric interpretation summarizes the idea of the proof. We first show that, when viewed as a function of a variable x that indexes a change in the market structure, the ratio $\psi_{p,i}^+/\psi_{s,i}^+(x)$ is concave (and minimized at $\bar{\rho}_i = 0$), whereas $\psi_{s,i}^+(x)$ is convex. Then, the sufficient condition for the existence of the tradeoff for all values of x can be characterized by $\arg \min \{\psi_{s,i}^+(x)\} = \arg \max \{\psi_{p,i}^+/\psi_{s,i}^+(x)\}$. Otherwise, there are values of x for which price and signal informativeness do not tradeoff. *Q.E.D.*

Proof of Proposition 5 (Inference Coefficients: Monotonicity) Fix $\{\Gamma_j\}_{j \in I}$. The result follows by direct computation, taking the derivatives with respect to $\bar{\rho}_i$,

$$\begin{aligned}
\frac{\partial c_{si}}{\partial \bar{\rho}_i} \Big|_{\{\Gamma_j\}} &= \frac{-\sigma^2 \left(2\bar{\rho}_i \Gamma_i^2 + \frac{1 + \sigma^2}{I - 1} \Gamma_i^2 \right) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right)}{\left((1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) - \left(\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I - 1} \bar{\rho}_i \Gamma_i^2 \right) \right)^2}, \\
\frac{\partial c_{pi}}{\partial \bar{\rho}_i} \Big|_{\{\Gamma_j\}} &= \frac{\frac{\sigma^2 \Gamma_i}{I - 1} \left(\frac{1}{\lambda_i} + \frac{1}{\mu_i + \lambda_i} \right) \left((1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) + \bar{\rho}_i^2 \Gamma_i^2 \right)}{\left((1 + \sigma^2) \left(\text{Avg}\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} + \frac{1 + \sigma^2}{I - 1} \text{Avg}\{\Gamma_j^2\}_{j \neq i} \right) - \left(\bar{\rho}_i^2 \Gamma_i^2 + \frac{1 + \sigma^2}{I - 1} \bar{\rho}_i \Gamma_i^2 \right) + \frac{\sigma^2}{(I - 1)(\mu_i + \lambda_i)} \Gamma_i \bar{\rho}_i \right)^2}.
\end{aligned}$$

Coefficient c_{pi} is always increasing in $\bar{\rho}_i$.

Q.E.D.

Proof of Proposition 6 (Inference Coefficients: Tradeoffs) Fix $\{\Gamma_j\}_{j \in I}$. When viewed as a function of a variable x that indexes a change in the market structure, signal and price inference coefficients exhibit a tradeoff as variable x changes if, and only if, $\arg \min c_{si}(x) \neq \arg \min c_{pi}(x)$. Using the formulas for c_{si} and c_{pi} in terms of weighted commonalities $\{\bar{\rho}_j\}_{j \neq i}$, $c_{pi} = 0$ if, and only if, $\bar{\rho}_i = 0$, ignoring the indirect

effect through weights $\{\Gamma_j\}_{j \in I}$. Hence, $|c_{p,i}(x)|$ is minimized at zero for $\bar{\rho}_i = 0$ and otherwise increasing in $\bar{\rho}_i$.

As $\bar{\rho}_i$ varies (but $\{\Gamma_j\}_{j \in I}$ is fixed), if the average of the commonalities of traders $j \neq i$ is large, $Avg\{\bar{\rho}_j \Gamma_j^2\}_{j \neq i} > -\frac{1+\sigma^2}{I-1} Avg\{\Gamma_j^2\}_{j \neq i}$, $c_{s,i}(x)$ is maximized at $\bar{\rho}_i = -\frac{1+\sigma^2}{2(I-1)} < 0$, and is decreasing when $\bar{\rho}_i > -\frac{1+\sigma^2}{2(I-1)}$. *Q.E.D.*

Models	dependent variable	c_s		$ c_p $		ψ_p^+		λ	
		$S(D)$	$L(C)$	$S(D)$	$L(C)$	$S(D)$	$L(C)$	$S(D)$	$L(C)$
Group	ρ	$+-$	$+-$	$-+$	$-+$	$-+$	$-+$	$-+$	$-+$
	$R = G_L/G_S$	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$+$
Single Dealer	ρ	$+-$	$-$	$-+$	$-+$	$-+$	$-+$	$+$	$+$
	I (region 1)	$-$	$-$	$+$	$-$	$+$	$-$	$-$	$-$
	I (region 2)	$-$	$-$	$+$	$-$	$+$	$-$	$-$	$-$
	I (region 3)	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$-$
Multiple Dealers $n_C = 1$	ρ	$+-$	$-$	$-+$	$-+$	$-+$	$-+$	$+$	$+$
	n_D (region 1)	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$
	n_D (region 2)	$+$	$\cdot$	$+$	$+$	$+$	$+$	$-$	$-$
	n_D (region 3)	$-$	$+$	$+$	$-$	$+$	0	$+$	$+$
	I (region 1)	$-$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
	I (region 2)	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
	I (region 3)	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$-$
Multiple Dealers $n_C \geq 2$	ρ	$+-$	$-$	$-+$	$-+$	$-+$	$-+$	$+$	$+$
	n_C (region 1)	$-$	$-$	$+$	$-$	$+$	$-$	$-$	$-$
	n_C (region 2)	$-$	$-$	$+$	$-$	$+$	$-$	$-$	$-$
	n_C (region 3)	$+-$	$+$	$+$	$-$	$+$	$-$	$-$	$-$
	I (region 1)	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
	I (region 2)	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
	I (region 3)	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$-$

Table 2: **Various Models: (Non-)Monotonicity or Non-Monotonicity of $c_s, |c_p|, \psi_p^+$, and λ ; n_D** is the number of links per dealer within a dealer group, and n_C is the number of customers per dealer.

Models	dependent variable	c_s			$ c_p $			ψ_p^+			λ		
		D	I	C	D	I	C	D	I	C	D	I	C
Tree	ρ	$+-$	$+-$	$-$	$-+$	$-+$	$-+$	$-+$	$-+$	$-+$	$+$	$+$	$+$
	n_C (region 1)	$+$	$-$	$-$	$-$	$+$	$-$	$-$	$+$	$-$	$-$	$-$	$-$
	n_C (region 2)	$-$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$	$-$	$-$	$-$
	n_C (region 3)	$-$	$-$	$-$	$-$	$+$	$-$	$-$	$+$	0	$-$	$-$	$-$
	n_C (region 4)	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$+$	$-$	$-$	$-$	$-$
	I (region 1)	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$
	I (region 2)	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
	I (region 3)	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
	I (region 4)	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$

Table 3: **Tree Model: (Non) Monotonicity of $c_s, |c_p|, \psi_p^+$, and λ .**

Figure 5: **Group Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the cross-group correlation ρ .** The solid line is for group S and the dashed line is for group L , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = \sigma_\varepsilon/\sigma_\theta = 0.5$, $\mu = 2$, and $G_S = 15$, $G_L = 25$, $I = 40$.

Figure 6: **Group Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the ratio of group sizes $R = G_L/G_S$.** The solid line is for group S and the dashed line is for group L , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = \sigma_\varepsilon/\sigma_\theta = 0.5$, $\mu = 2$, and cross-group correlation $\rho = -0.5$.

Figure 7: **Group Model with small I :** The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the cross-group correlation ρ . The solid line is for group S and the dashed line is for group L , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = \sigma_\varepsilon/\sigma_\theta = 0.5$, $\mu = 2$, and $G_S = 3, G_L = 5, I = 8$.

Figure 8: **Dark Pool:** The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the cross-group correlation ρ . The solid line is for group S and the dashed line is for group L , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = \sigma_\varepsilon/\sigma_\theta = 0.5$, $\mu = 2$, $G_S = 15, G_L = 25, I = 40$, and the correlation $\rho_L = 0.6, \rho_S = 1$.

Figure 9: **Dark Pool: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the correlation ρ_L within group L .** The solid line is for group S and the dashed line is for group L , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = \sigma_\varepsilon/\sigma_\theta = 0.5$, $\mu = 2$, $G_S = 15$, $G_L = 25$, $I = 40$, and the correlation $\rho = 0.5$ between groups, and $\rho_S = 1$ within group S .

Figure 10: **Single Dealer: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the per-link correlation ρ .** The solid line is for group D (dealer) and the dashed line is for group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = 0.5$, $\mu = 2$, and $I = 8$ (i.e. there are seven agents and a central dealer).

Figure 11: **Single Dealer: Thresholds of $c_{sD}(\rho)$ and $c_{sC}(\rho)$ as a function of the number of customers G_C .** The black line corresponds to the left threshold, separating $c_{sD} < c_{sC}$ from $c_{cD} > c_{sC}$. The red line corresponds to the right crossing point, separating $c_{sD} > c_{sC}$ from $c_{cD} < c_{sC}$. The dashed line corresponds to the lowest correlation for which an equilibrium exists.

(a)

(b)

(c)

Figure 12: **Single Dealer:** The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the number of traders I . The solid line is for group D (dealer) and the dashed line is for group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = 0.5$, $\mu = 2$, and $\rho = -0.2$ in (a), $\rho = -0.03$ in (b), and $\rho = 0.10$ in (c).

Figure 13: **Multiple Dealer with $n_C = 1$: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the per-link correlation ρ .** The solid line is for group D (dealer) and the dashed line is for group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $I = 32$, $\sigma = 0.5$, $\mu = 2$, and $n_D = 2$.

Figure 14: **Multiple Dealer with $n_C = 1$: Thresholds of $c_{sD}(\rho)$ and $c_{sC}(\rho)$ as a function of n_D in (a) and as a function of I in (b).** The black line corresponds to the left threshold, separating $c_{sD} < c_{sC}$ from $c_{sD} > c_{sC}$. The red line corresponds to the right threshold, separating $c_{sD} > c_{sC}$ from $c_{sD} < c_{sC}$. The dashed line corresponds to the lowest correlation for which an equilibrium exists.

(a)

(b)

(c)

Figure 15: **Multiple Dealer with $n_C = 1$: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the number of links n_D within a group D .** The solid line is for group D (dealer) and the dashed line is for group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $I = 32$, $\sigma = 0.5$, $\mu = 2$, and $\rho = -0.17$ in (a), $\rho = -0.03$ in (b), and $\rho = 0.10$ in (c).

(a)

(b)

(c)

Figure 16: **Multiple Dealer with $n_C = 1$: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the total number of traders I .** The solid line is for group D (dealer) and the dashed line is for group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $n_D = 2$, $\sigma = 0.5$, $\mu = 2$, and $\rho = -0.40$ in (a), $\rho = -0.20$ in (b), and $\rho = 0.10$ in (c).

Figure 17: **Multiple Dealer with $n_C > 1$: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i as a function of the per-link correlation ρ .** The solid line is for group D (dealer) and the dashed line is for group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $I = 12$, $G_D = 4$, $\sigma = 0.5$, $\mu = 2$, and $n_C = 2$, $n_D = 2$.

Figure 18: **Multiple Dealer with $n_C > 1$: Thresholds of $c_{sD}(\rho)$ and $c_{sC}(\rho)$ as a function of n_C in (a) and as a function of I in (b).** The black line corresponds to the left threshold, separating $c_{sD} < c_{sC}$ from $c_{cD} > c_{sC}$. The red line corresponds to the right threshold, separating $c_{sD} > c_{sC}$ from $c_{cD} < c_{sC}$. The dashed line corresponds to the lowest correlation which an equilibrium exists.

(a)

(b)

(c)

Figure 19: **Multiple Dealer with $n_C > 1$: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the number of customers of a dealer n_C .** The solid line is for Group D (dealer) and the dashed line is for Group C (customer), in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $G_D = 4$, $n_D = 2$, $\sigma = 0.5$, $\mu = 2$, and $\rho = -0.18$ in (a), $\rho = -0.03$ in (b), and $\rho = 0.10$ in (c).

(a)

(b)

(c)

Figure 20: **Multiple Dealer with $n_C > 1$: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the total number of traders I .** The solid line is for Group D (dealer) and the dashed line is for Group C (customer) in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $n_C = 2$, $n_D = 2$, $\sigma = 0.5$, $\mu = 2$, and $\rho = -0.28$ in (a), $\rho = -0.10$ in (b), and $\rho = 0.20$ in (c).

Figure 21: **Core-Periphery Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the per-link correlation ρ .** The solid line is for the central dealer D , the blue dashed line is for intermediate dealers I , and the red dashed line is for customers C in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $n_C = 3$, $G_I = 6$, $\sigma = 0.5$, $\mu = 2$.

(a)

(b)

(c)

(d)

Figure 22: **Core-Periphery Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the number of customers per intermediate dealer.** The solid line is for the central dealer D , the blue dashed line is for intermediate dealers I , and the red dashed line is for customers C , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $G_I = 8, \sigma = 0.5, \mu = 2$, and $\rho = -0.22$ in (a), $\rho = 0.14$ in (b), $\rho = -0.04$ in (c), and $\rho = 0.10$ in (d).

(a)

(b)

(c)

Figure 23: **Core-Periphery Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the total number of traders I .** The solid line is for the central dealer D , the blue dashed line is for intermediate dealers I , and the red dashed line is for customers C , in each figure. The dotted lines indicate that the nominal value is negative. The exogenous parameters are $\sigma = 0.5$, $\mu = 2$, $R := G_d/n_C = 2$, and $\rho = -0.20$ in (a), $\rho = -0.02$ in (b), and $\rho = 0.10$ in (c).

(a)

(b)

Figure 24: **Location Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the per-link correlation ρ .** The exogenous parameters are $\sigma = 0.5$, $\mu = 2$, and $N = 4$ in (a) and $N = 10$ in (b).

(a)

(b)

(c)

Figure 25: **Location Model: The schedule of $c_{s,i}$, $|c_{p,i}|$, $|c_{\theta,i}|$, $\psi_{p,i}^+$, $\psi_{s,i}^+$, and λ_i against the number of generations.** The solid line is for the central dealer (the first generation), the dashed line is for the $(N - 1)$ th generation, and the dotted line is for the last generation. The exogenous parameters are $\sigma = 0.5$, $\mu = 2$, and $\rho = -0.40$ in (a), $\rho = -0.04$ in (b), and $\rho = 0.20$ in (c).