Multidimensional Product Design

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Firms design their products to attract the most valuable users.
Insurers covering gym memberships: Health advance or selection tool?

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It sounds like a great idea: private insurance plans in the Medicare Advantage program offering gym memberships, a small investment that could curb health-care costs among seniors. But what some see as innovation, others view as a stealth campaign to lure the healthiest seniors into the private plans while leaving Medicare with the worst risks.
New Facebook Feature Lures Celebrities

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No doubt, major brands such as Coca-Cola (NYSE:KO[1]), Starbucks (Nasdaq:SBUX[2]) and Nike (NYSE:NKE[3]) have found Twitter to be a key part of their branding strategy. And yes, the service has been critical for celebrities — hey, they’re brands, too. In fact, some celebrities make tidy sums promoting products on Twitter.

Now, Facebook wants a piece of the action. To this end, the company has a new feature meant to boost the social media site’s “content subscribing”: account verification. According to a post in Techcrunch[4]:

“Facebook, a service built on real names and real identities, will tomorrow start allowing prominent public figures to verify their accounts and then opt to display a preferred nickname instead of their birth name. Those with verified accounts will gain more prominent placement in Facebook’s ‘People To Subscribe To’ suggestions.”
Current theory

- insufficient heterogeneity
- no equilibrium
- specific functional forms
- only price instruments

We suggest a tractable model with

- multi-dimensional heterogeneity in preferences and values
- flexible preferences (consumption externalities, non-transferable utility)
- multiple (finite) instruments (nonlinear pricing, quality)
- multiple sides, simple competition

Main result: sorting power $\propto \text{Cov}(\text{preferences}, \text{values} | \text{margin})$

- insurance: $\text{Cov}(\text{taste for gym membership, -cost} | \text{margin})$
- soap operas:
  $\text{Cov}(\text{taste for popularity, contribution to popularity} | \text{margin})$
Simple Example

- Monopoly social network chooses uniform price $P$
  - 1 side (simpler)
- Mass 1 of potential users choose whether to join
- Multi-dimensional types $\theta \sim f(\theta)$ determine heterogeneity in
  - preferences: $u = v(K; \theta) - P$
  - values (contribution to popularity): $k(\theta)$
  - Platform popularity $K = \int_{\text{participants}} k(\theta)f(\theta)d\theta$
  - homogeneous cost $c$ per user
- Outside option = zero.
  - participants are $\{\theta : v \geq P\}$
  - marginal users are $\{\theta : v = P\}$ respond to $P$
- Smoothness assumptions
Maximize welfare. Easy setup: choose $P, K$ maximize

$$
\mathcal{L} = \int_{v \geq P} (v - c) f + \lambda \left( \int_{v \geq P} kf - K \right)
$$
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$$
\frac{\partial \mathcal{L}}{\partial K} = \int_{v \geq P} \frac{\partial u}{\partial K} f + \int_{v = P} \frac{\partial u}{\partial K} (v - c + \lambda k) f - \lambda = 0
$$

$$
\lambda = N \mathbb{E} \left[ \frac{\partial u}{\partial K} \mid v \geq P \right] + M \mathbb{E} \left[ \frac{\partial u}{\partial K} (P - c + \lambda k) \mid v = P \right]
$$

$$
\lambda = N \mathbb{E} \left[ \frac{\partial u}{\partial K} \mid v \geq P \right] + \lambda \text{MCov} \left( \frac{\partial u}{\partial K}, k \mid v = P \right) + 0
$$
\[ P = c - \mathbb{E}[k | \nu = P] \mathbb{N} \mathbb{E} \left[ \frac{\partial u}{\partial K} | \nu \geq P \right] \frac{1}{1 - M \text{Cov} \left( \frac{\partial u}{\partial K}, k | \nu = P \right)} \]

externality from additional user
\[ P = c - \mathbb{E} [k \mid v = P] \mathbb{E} \left[ \frac{\partial u}{\partial K} \mid v \geq P \right] \frac{1}{1 - M\text{Cov} \left( \frac{\partial u}{\partial K}, k \mid v = P \right)} \]

Average popularity along the margin
\[ P = c - \mathbb{E}[k \mid v = P] N \mathbb{E} \left[ \frac{\partial u}{\partial K} \mid v \geq P \right] \frac{1}{1 - M \operatorname{Cov} \left( \frac{\partial u}{\partial K}, k \mid v = P \right)} \]

Average popularity along the margin
Direct value of popularity to ALL participants
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Average popularity along the margin
Direct value of popularity to ALL participants
Feedback/Sorting for popularity by popularity.
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Average popularity along the margin
Direct value of popularity to ALL participants
Feedback/Sorting for popularity by popularity.
1 dimensional heterogeneity: Cov=0
Maximizing profit $\Pi = \int_{v \geq P} (P - c) f$ yields

$$P - \frac{N}{M} = c - \mathbb{E}[k \mid v = P] \mathbb{N} \mathbb{E} \left[ \frac{\partial u}{\partial K} \mid v = P \right] \frac{1}{1 - M \text{Cov} \left( \frac{\partial u}{\partial K}, k \mid v = P \right)}$$

Cournot distortion, Spence distortion
Maximizing profit $\Pi = \int_{v \geq P} (P - c) f$ yields

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Cournot distortion, Spence distortion
Multiple Sides

With two sides and cross-side effects, sorting effect is

\[
1 - M^1 M^2 \text{Cov} \left( \frac{\partial u^1}{\partial k^2}, k^1 \mid v^1 = P^1 \right) \text{Cov} \left( \frac{\partial u^2}{\partial k^1}, k^2 \mid v^2 = P^2 \right)
\]
General Model

- Firm chooses vector of instruments $\rho$
  - non-linear pricing, vertical/horizontal quality, ...
- Vector of characteristics $K$, $K^i = \int_{u \geq 0} k^i f$.
  - captures consumption externalities
  - $k^i(\rho, K; \theta)$ allows for intensity of participation
- Preferences $u(\rho, K; \theta)$
- Multiple sides
- Imperfect competition without consumption externalities
Search engines (multi-sided market with non-transferable utility)
  - platform chooses price to advertisers, quality to users
  - user income has value to advertisers
  - Cov(taste for quality, income)

Credit Cards (multi-sided market with non-linear pricing)
  - platforms chooses “points” per transaction
  - consumer choose # transactions, which has values to merchants
  - Cov(taste for “points”, # transactions)

Competition in insurance
  - two symmetric insurers choose prices and coverages
  - Two margins: Exiting and Switching
    - Welfare maximization considers only exiting margin
Conclusion

- Multi-dimensional heterogeneity in preferences and values
- Flexible preferences (consumption externalities, non-transferable utility)
- Multiple (but finite) instruments (nonlinear pricing, quality)
- Multiple sides, simple competition
- Main result: sorting power $\propto \text{Cov}(\text{preferences, values} | \text{margin})$
Thank you!

Comments are very appreciated.