Auctions for Online Display Advertising Exchanges: Approximations and Design

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Joint work with Santiago Balseiro and Omar Besbes

June 7, 2013
NET Institute Conference
James and Heat Assume New Role as Underdogs

By HOWARD BECK
Published: June 12, 2012

OKLAHOMA CITY — In a moment of introspection and thoughtfulness, LeBron James was short-circuited by a short circuit. A slight crackle, then silence as the podium microphone sputtered in mid-sentence.

Web-page

$11 billions in the US!
How should publishers manage this new market?
Challenges/Contributions

1) Model for advertisers’ bidding behavior

Fulfill campaign... subject to budget.
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Dynamic game of incomplete information:
Traditional game theory is intractable and implausible for practical instances.
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New tractable and behaviorally appealing equilibrium concept
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New tractable and behaviorally appealing equilibrium concept

2) Study publishers’ revenue maximizing problem

Monetize inventory…

1. Allocation
2. Reserve price
3. Information disclosure
Challenges/Contributions

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1. Allocation
2. Reserve price
3. Information disclosure

Tool for back-testing and revenue optimization
Advertisers bid Poisson($\eta$)

Publisher

One slot

Max. profit

Alternative Channel

Opp. cost ($c$)

An advertiser’s type $\theta$ is stochastic

Campaign length ($s_\theta$)

Poisson($\lambda$)

2nd price auction with reserve ($r$)

bids

Advertisers

max utility $\theta$
s.t. budget($b_\theta$)
Valuation Model

• How do advertisers value users?

Targeting criteria + User Information = Value
Valuation Model

• How do advertisers value users?

• Two-stage independent private value model

1. Participate in auction with probability $\alpha_\theta$

2. Conditional private value drawn from $V_\theta \sim F_{v_\theta}(\cdot)$
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[Diagram: Targeting criteria + User Information = Value]

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Solution Concept

Large # of advertisers

Mean Field Approx.

Compete against a stationary distribution of maximum competing bid
Iyer et al. (2012), Gummadi et al. (2012)

Large # of auctions

Fluid Approx.

Satisfy budget constraint in expectation & restrict to state-independent strategies
Gallego and van Ryzin (1994)
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**Theorem:** Optimal best response bidding strategy is

\[ \beta^F_{\theta}(v) = \frac{1}{1 + \mu_\theta} v \]

where \( \mu_\theta \geq 0 \) is the Lagrange multiplier of the budget constraint
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Fluid Mean Field Equilibrium

\[ D \sim \max \left( \{ \beta_{\theta}(V_{\theta}) \}_{\text{matching bidders}}, r \right) \]

Bid functions \( \hat{\beta} \) (multipliers \( \{ \mu_{\theta} \}_{\theta} \))

Best-response

Bid landscape \( D \sim F_d(\cdot; \hat{\beta}) \)
Fluid Mean Field Equilibrium

Theorem
FMFE always exists and is unique (under sufficient conditions).

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Steady-state

Bid landscape \( D \sim F_d(\cdot; \hat{\beta}) \)

Best-response
Fluid Mean Field Equilibrium

**Theorem**

FMFE always exists and is unique (under sufficient conditions).

**Diagram**

- Bid functions $\hat{\beta}$ (multipliers $\{\mu_\theta\}_\theta$)
- Steady-state: $D \sim \max \left( \{\beta_\theta(V_\theta)\}_{\text{matching bidders}}, r \right)$
- Best-response
- Bid landscape $D \sim F_d(\cdot; \hat{\beta})$

**Note**

FMFE is tractable and behaviorally appealing
Consider a sequence of markets with increasing number of advertisers and number of auctions and constant number of bidders per auction. Then,
FMFE as an Approximation

Consider a sequence of markets with increasing
– number of advertisers and number of auctions
and constant
– number of bidders per auction
– ratio of budget to number of auctions participated
then,

Profits under best response strategy, given others play FMFE

Profits under FMFE strategy, given others play FMFE

→ 1
FMFE as an Approximation

Consider a sequence of markets with increasing
- number of advertisers and number of auctions
and constant
- number of bidders per auction
- ratio of budget to number of auctions participated
then,

\[
\frac{\text{Profits under best response strategy, given others play FMFE}}{\text{Profits under FMFE strategy, given others play FMFE}} \to 1
\]

Moreover, numerical experiments show that FMFE strategy can be optimal best response even with small number of competitors.
Auction Design: Revenue Optimizing Reserve Price (Homogeneous Case)

Given auction design decisions, advertisers respond bidding according to FMFE strategies.
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Given auction design decisions, advertisers respond bidding according to FMFE strategies.

**Theorem**

The optimal reserve price is

$$\max\{r_c^*, \bar{r}\}$$

where

$r_c^*$: optimal reserve static SPA.

$\bar{r}$: greatest price at which bidders deplete budgets.
Auction Design: Revenue Optimizing Reserve Price
(Homogeneous Case)

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**Theorem**

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where

- \( r_c^* \): optimal reserve static SPA.
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- Always price higher than in absence of budget.
- Advertisers always bid truthfully at optimal reserve price.
Publisher’s Problem: Back-Testing

Practitioners typically

• use historical bids
• ignore budget constraints
• assume advertisers won’t react to auction changes
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Back-test on actual AdX data from 1 publisher (Heterogeneous bidders)
Publisher’s Problem: Back-Testing

Optimal reserve considering the strategic response of budget-constrained advertisers

![Graph showing optimal reserve price](image)
Publisher’s Problem: Back-Testing

Optimal reserve considering the strategic response of budget-constrained advertisers

16% profit gain
Conclusions

1. FMFE: New approach to analyze competition between budget-constrained advertisers.

2. Quantify the tradeoffs in the publisher’s revenue maximization problem in AdX. Insight into:
   - Optimal reserve price
   - Optimal allocation between contracts and exchange
   - Extent of information disclosure
Conclusions

1. FMFE: New approach to analyze competition between budget-constrained advertisers.

2. Quantify the tradeoffs in the publisher’s revenue maximization problem in AdX. Insight into:
   – Optimal reserve price
   – Optimal allocation between contracts and exchange
   – Extent of information disclosure

Further Questions:
- Study “throttling” vs bid shading
- Optimal Mechanism Design
- Empirical analysis of bidding behavior and auction design
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The Advertiser’s Problem

**STEP 1. Mean Field Approximation (no Fluid)**

- Assume a stationary maximum competing bid $D \sim F_d(\cdot)$
- Find a strategy $\beta_{\theta} : (\text{budget left, time left, valuation}) \rightarrow \text{bid}$

$$\max_{\beta_{\theta}} \mathbb{E} \left[ \sum_{i=1}^{M} \text{utility}_i(\beta_{\theta}; F_d) \right]$$

s.t. $\sum_{i=1}^{M} \text{payment}_i(\beta_{\theta}; F_d) \leq b_{\theta}$ (a.s.)

Advertisers need to solve a dynamic program to determine their strategies...
The Advertiser’s Problem

**STEP 2. Fluid Mean Field Approximation**

- Assume a stationary maximum competing bid $D \sim F_d(\cdot)$
- Satisfy budget in expectation.
- Find a state-independent strategy $\beta_\theta: \text{valuation} \rightarrow \text{bid}$

\[
\max_{\beta_\theta} \mathbb{E}\left[ \sum_{i=1}^{M} \text{utility}_i(\beta_\theta; F_d) \right]
\]

s.t. \[
\mathbb{E}\left[ \sum_{i=1}^{M} \text{payment}_i(\beta_\theta; F_d) \right] \leq b_\theta
\]

Solution to fluid problem provides near-optimal strategy in real system!
Auction Design Problem

\[
\max_{r, \eta, \iota} \ E\left[ \text{revenue} \right] - \ E\left[ \text{opportunity cost} \right]
\]

s.t.

\[
\mu = \text{FMFE}(r, \eta, \iota)
\]

where:

- \( r \) : reserve price
- \( \eta \) : allocation to the exchange
- \( \iota \) : information disclosure
An example...

2 types
Budgets:
\( b_\theta = (\$1; \$8) \)
Heterogeneous Advertisers

Type 1: low-budget

Type 2: high-budget

Optimal reserve

$r^*$ (optimal reserve)

$\eta$ (allocation)
Disclosure of Information: Homogenous Case

The tradeoff

- Less info ($\iota$) → Higher values ($V$)
  - Thinner markets ($\alpha$)
- More info ($\iota$)

Theorem

Suppose $\eta$ fixed. When publisher reacts to thinner markets by setting the optimal reserve price, then disclosing more information improves profits.