Auctions for Online Display Advertising Exchanges: Approximations and Design

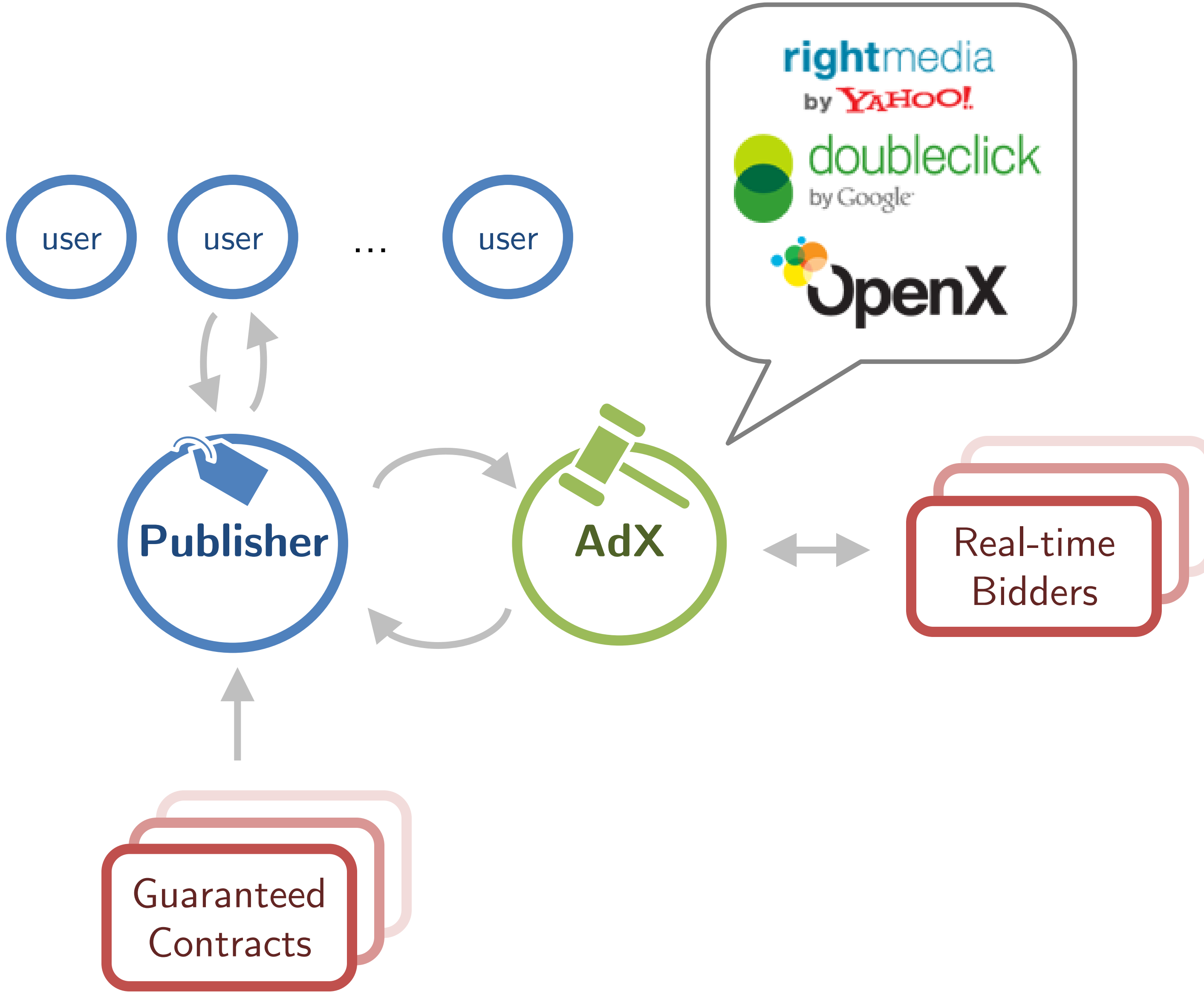
Gabriel Weintraub
Columbia Business School
Joint work with Santiago Balseiro and Omar Besbes

June 7, 2013
NET Institute Conference

Web-page

Display ad

$11 billions in the US!



How should publishers manage this new market?

Challenges/Contributions

1) Model for advertisers' bidding behavior

Fulfill campaign...

...subject to **budget**.

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2) Study publishers' revenue maximizing problem

Monetize inventory...

1. Allocation
2. Reserve price
3. Information disclosure

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Advertisers

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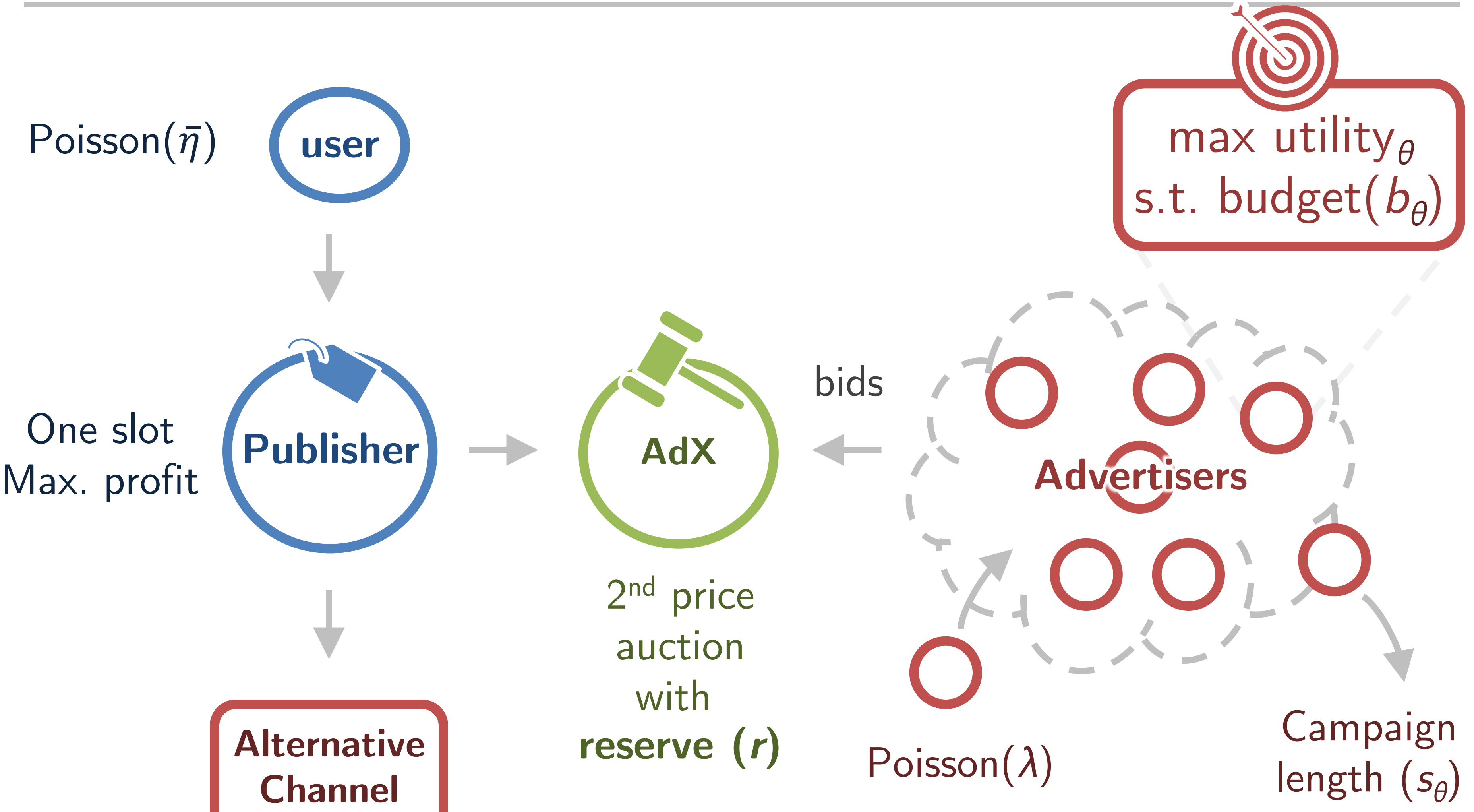
Publisher

Monetize inventory...

1. Allocation
2. Reserve price
3. Information disclosure

→ Tool for back-testing and revenue optimization

Model



An advertiser's type θ is stochastic

Valuation Model

- How do advertisers value users?

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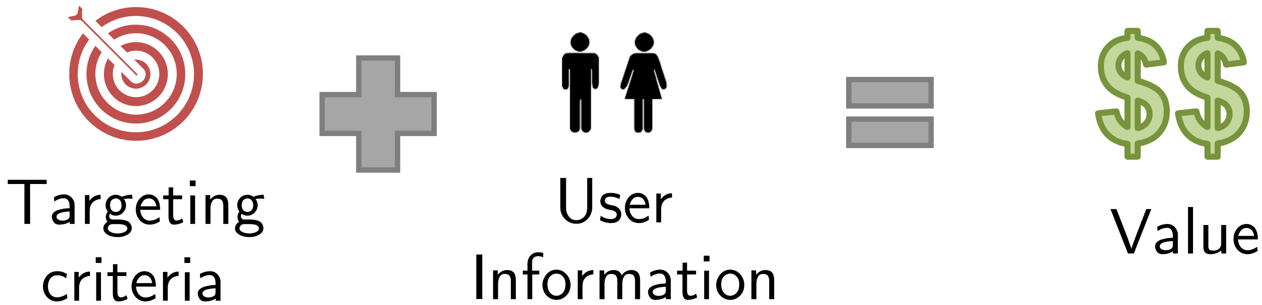

- Two-stage independent private value model

1. Participate in auction with probability α_θ
2. Conditional private value drawn from $V_\theta \sim F_{v_\theta}(\cdot)$

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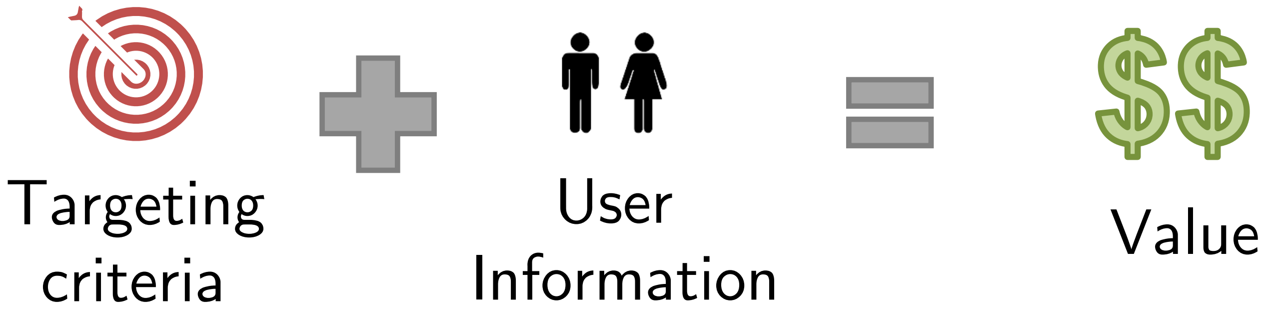

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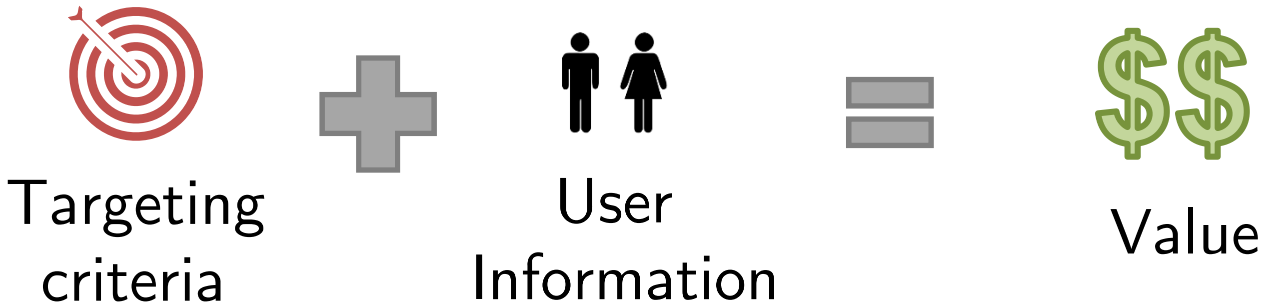

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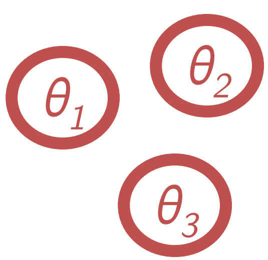

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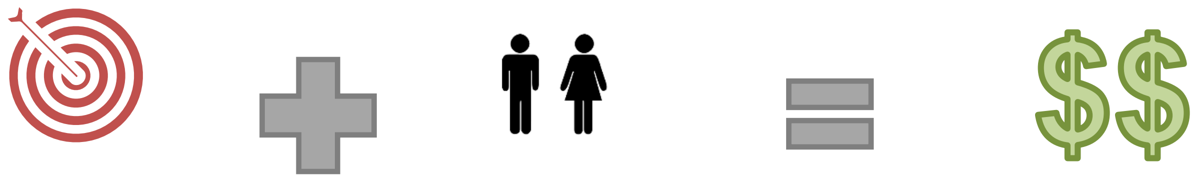

Targeting criteria | User Information | Value

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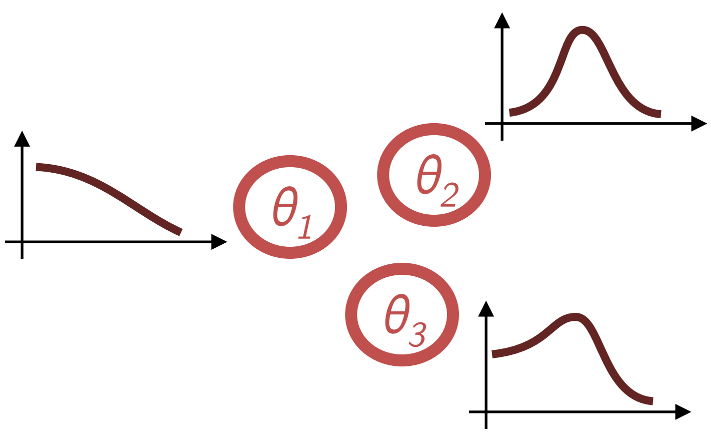

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Solution Concept

Large # of advertisers

Mean Field Approx.

Compete against a stationary distribution of maximum competing bid

Iyer et al. (2012), Gummadi et al. (2012)

Large # of auctions

Fluid Approx.

Satisfy budget constraint in expectation & restrict to state-independent strategies

Gallego and van Ryzin (1994)

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Theorem: Optimal best response bidding strategy is

$$\beta_\theta^F(v) = \frac{1}{1+\mu_\theta} v$$

where $\mu_\theta \geq 0$ is the Lagrange multiplier of the budget constraint

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→ Strategy characterized by single number

where $\mu_\theta \geq 0$ is the Lagrange multiplier of the budget constraint

Fluid Mean Field Equilibrium

Steady-state

$$D \sim \max\left(\{\beta_\theta(V_\theta)\}_{\substack{\text{matching}\\\text{bidders}}}, r\right)$$

Bid functions

$\vec{\beta}$ (multipliers $\{\mu_\theta\}_\theta$)

Bid landscape

$D \sim F_d(\cdot; \vec{\beta})$

Best-response

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FMFE always exists and is unique (under sufficient conditions).

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Theorem

FMFE always exists and is unique (under sufficient conditions).

→ FMFE is tractable and behaviorally appealing

FMFE as an Approximation

Consider a sequence of markets with **increasing**

– number of advertisers and number of auctions

and **constant**

– number of bidders per auction

– ratio of budget to number of auctions participated

then,

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$$\frac{\boxed{\text{Profits under best response strategy, given others play FMFE}}}{\boxed{\text{Profits under FMFE strategy, given others play FMFE}}} \to 1$$

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– ratio of budget to number of auctions participated

then,

Profits under best response strategy, given others play FMFE / Profits under FMFE strategy, given others play FMFE $\rightarrow 1$

Moreover, numerical experiments show that FMFE strategy can be optimal best response even with small number of competitors.

Auction Design: Revenue Optimizing Reserve Price (Homogeneous Case)

Given auction design decisions, advertisers respond bidding according to FMFE strategies.

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Theorem

The optimal reserve price is

$$\max\{r_c^*, \bar{r}\}$$

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r_c^*: optimal reserve static SPA.

$\bar{r}$: greatest price at which bidders deplete budgets.

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- Always price higher than in absence of budget.
- Advertisers always bid truthfully at optimal reserve price.

Publisher's Problem: Back-Testing

Practitioners typically

- use historical bids
- ignore budget constraints
- assume advertisers won't

react to auction changes

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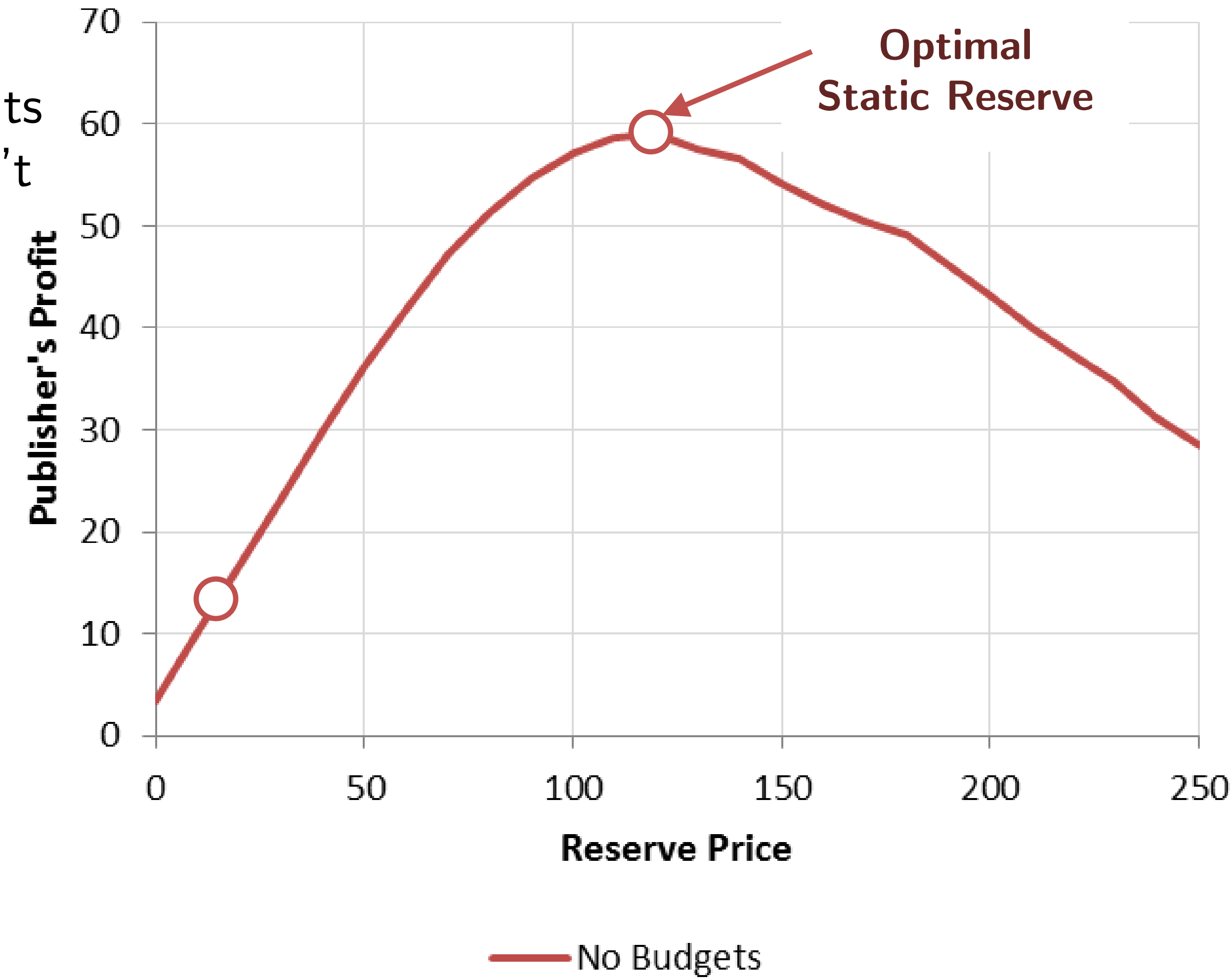
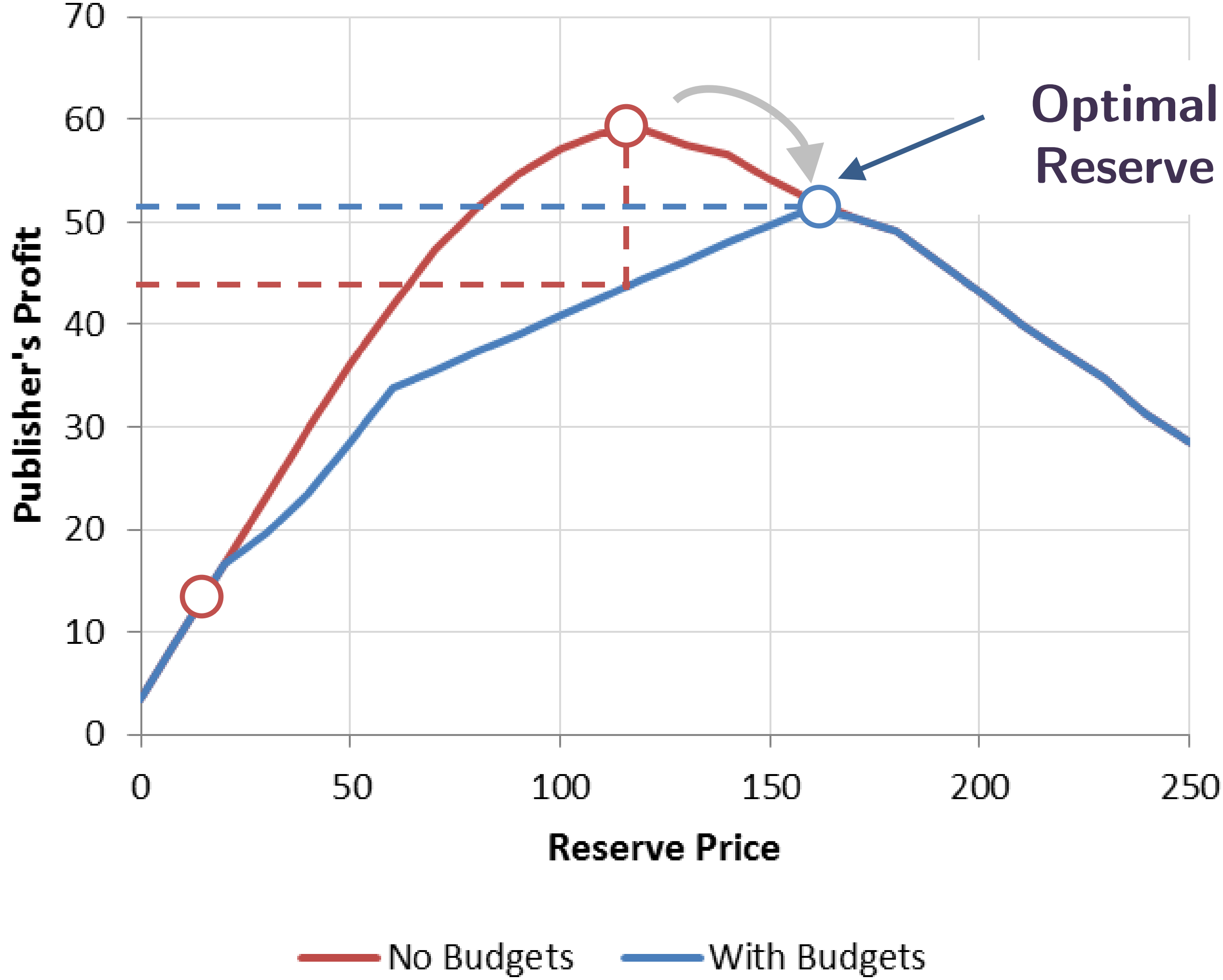
Back-test on actual AdX data from 1 publisher
(Heterogeneous bidders)



Publisher's Problem: Back-Testing

Optimal reserve considering the strategic response of budget-constrained advertisers



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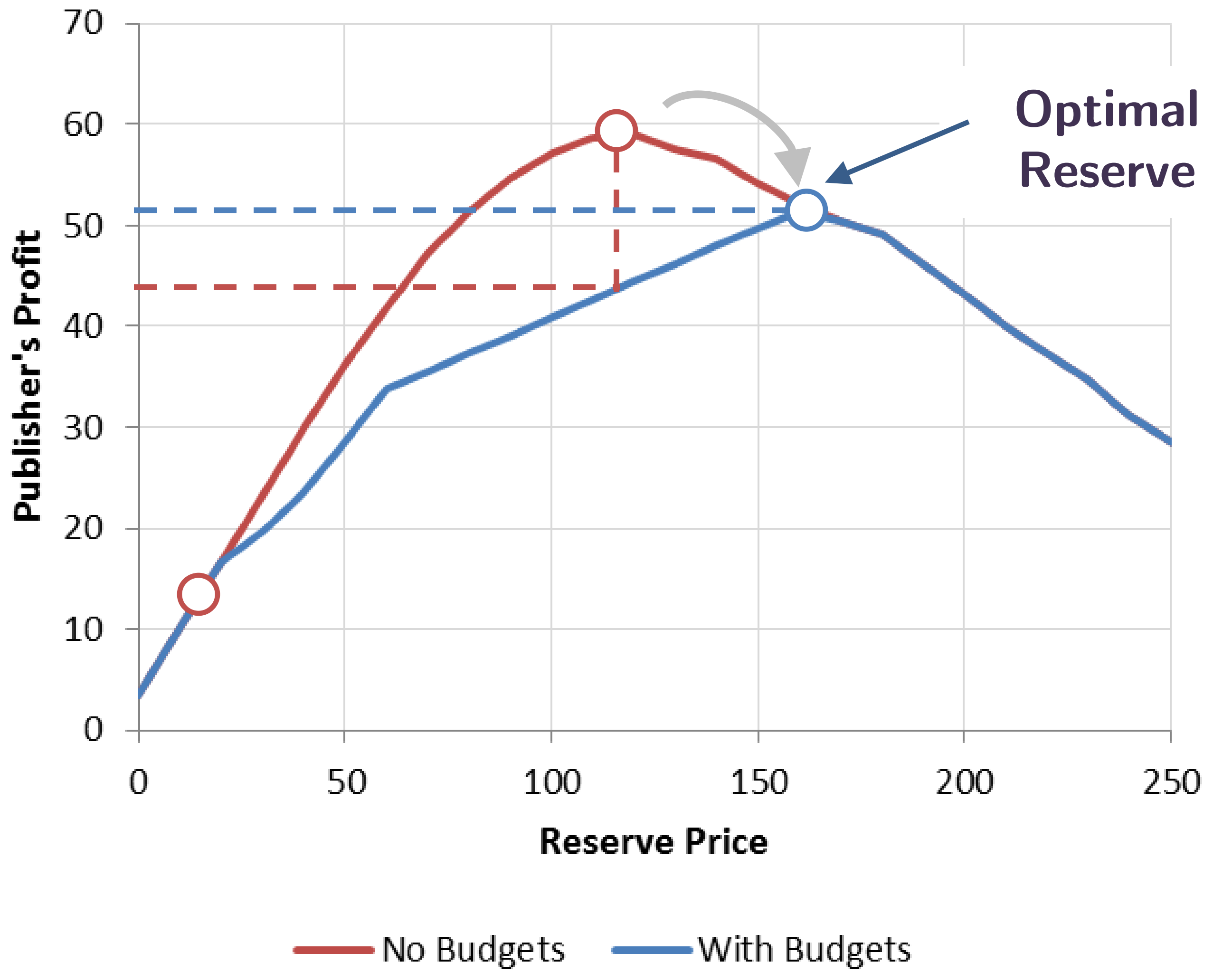
16% profit gain



Conclusions

1. FMFE: New approach to analyze competition between budget-constrained advertisers.
2. Quantify the tradeoffs in the publisher's revenue maximization problem in AdX. Insight into:
   - Optimal reserve price
   - Optimal allocation between contracts and exchange
   - Extent of information disclosure

Conclusions

1. FMFE: New approach to analyze competition between budget-constrained advertisers.
2. Quantify the tradeoffs in the publisher's revenue maximization problem in AdX. Insight into:
   - Optimal reserve price
   - Optimal allocation between contracts and exchange
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Further Questions:

- Study "throttling" vs bid shading
- Optimal Mechanism Design
- Empirical analysis of bidding behavior and auction design

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The Advertiser's Problem

Step 1. Mean Field Approximation (no Fluid)

- Assume a stationary maximum competing bid $D \sim F_d(\cdot)$
- Find a strategy β_θ: $\left(\frac{\text{budget}}{\text{left}}, \frac{\text{time}}{\text{left}}, \text{valuation}\right) \rightarrow \text{bid}$

$$\max_{\beta_\theta} \; \mathbb{E}\left[\sum_{i=1}^{M} \text{utility}_i(\beta_\theta; F_d)\right]$$

$$\text{s.t.} \; \sum_{i=1}^{M} \text{payment}_i(\beta_\theta; F_d) \leq b_\theta \quad \text{(a.s.)}$$

Advertisers need to solve a dynamic program to determine their strategies...

The Advertiser's Problem

Step 2. Fluid Mean Field Approximation

- Assume a stationary maximum competing bid $D \sim F_d(\cdot)$
- Satisfy budget in expectation.
- Find a state-independent strategy β_θ: valuation $\rightarrow$ bid

$$\max_{\beta_\theta} \ \mathbb{E}\left[\sum_{i=1}^{M} \text{utility}_i(\beta_\theta; F_d)\right]$$

$$\text{s.t.} \ \mathbb{E}\left[\sum_{i=1}^{M} \text{payment}_i(\beta_\theta; F_d)\right] \leq b_\theta$$

Solution to fluid problem provides near-optimal strategy in real system!

Auction Design Problem

$$\max_{r,\eta,\iota} \quad \mathbb{E}\Big[\text{revenue}\Big] \quad - \quad \mathbb{E}\Big[\begin{matrix}\text{opportunity}\\ \text{cost}\end{matrix}\Big]$$

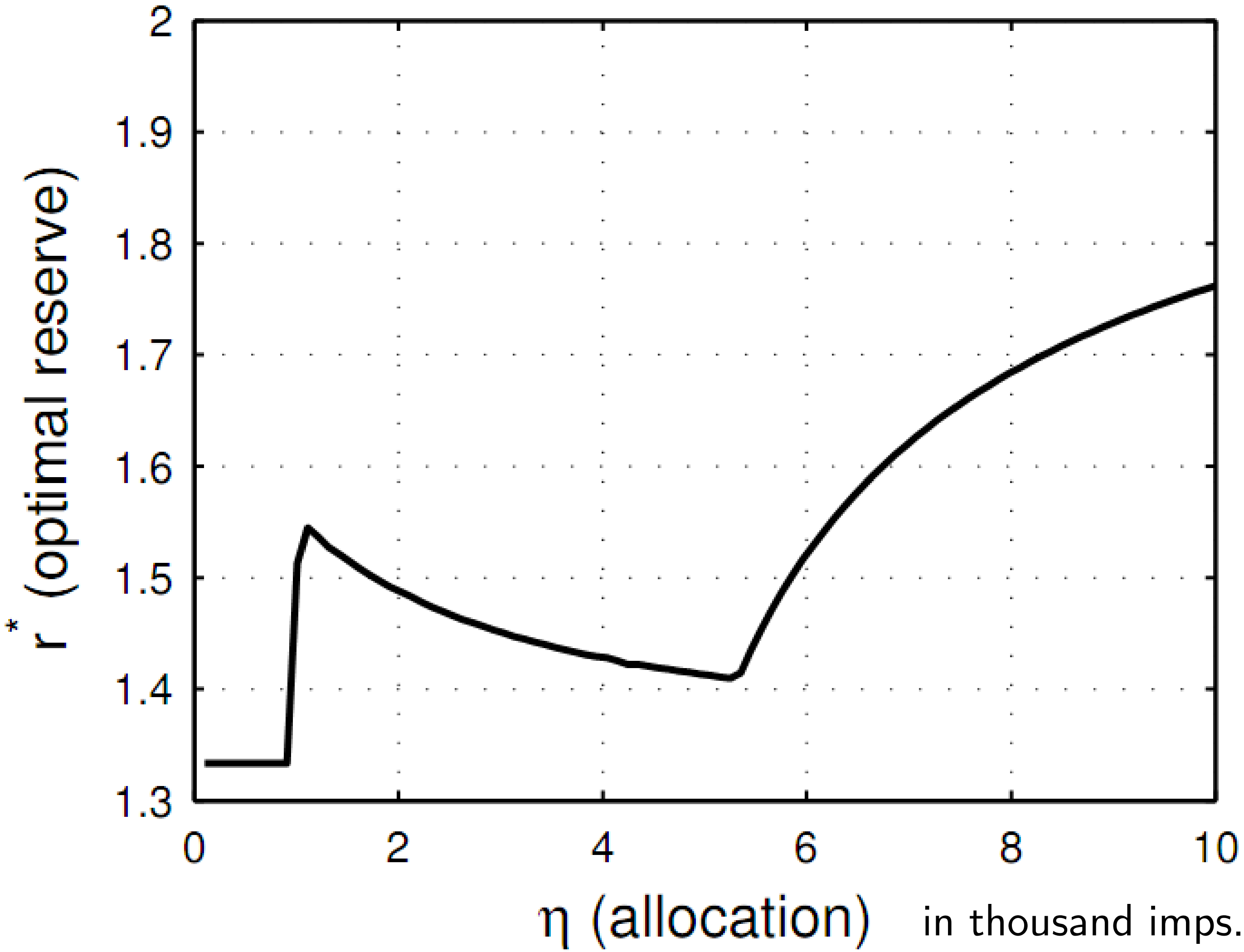
s.t.

$$\mu = \text{FMFE}(r, \eta, \iota)$$

where:

- r : reserve price
- η : allocation to the exchange
- ι : information disclosure

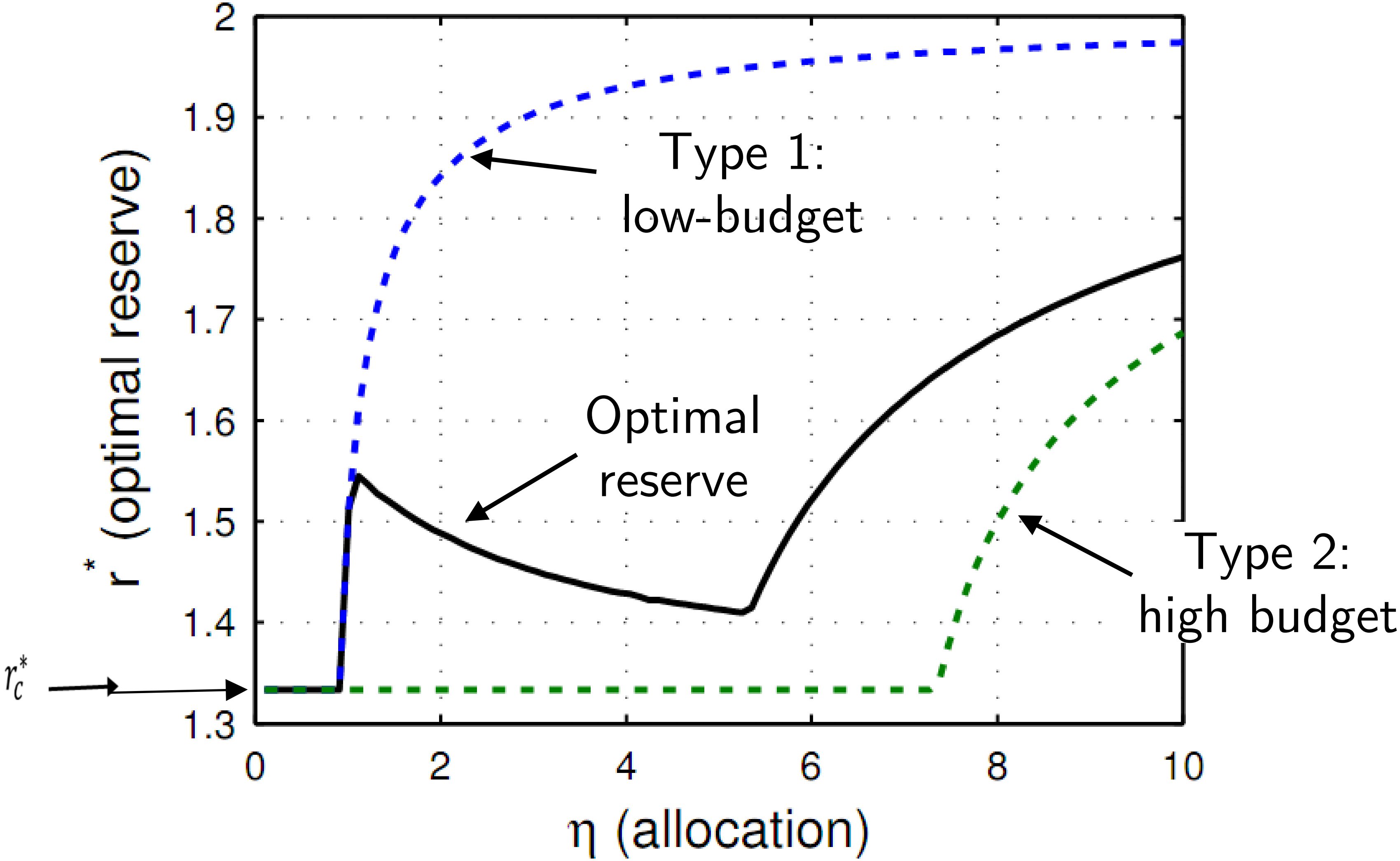
An example...



in thousand imps. per day

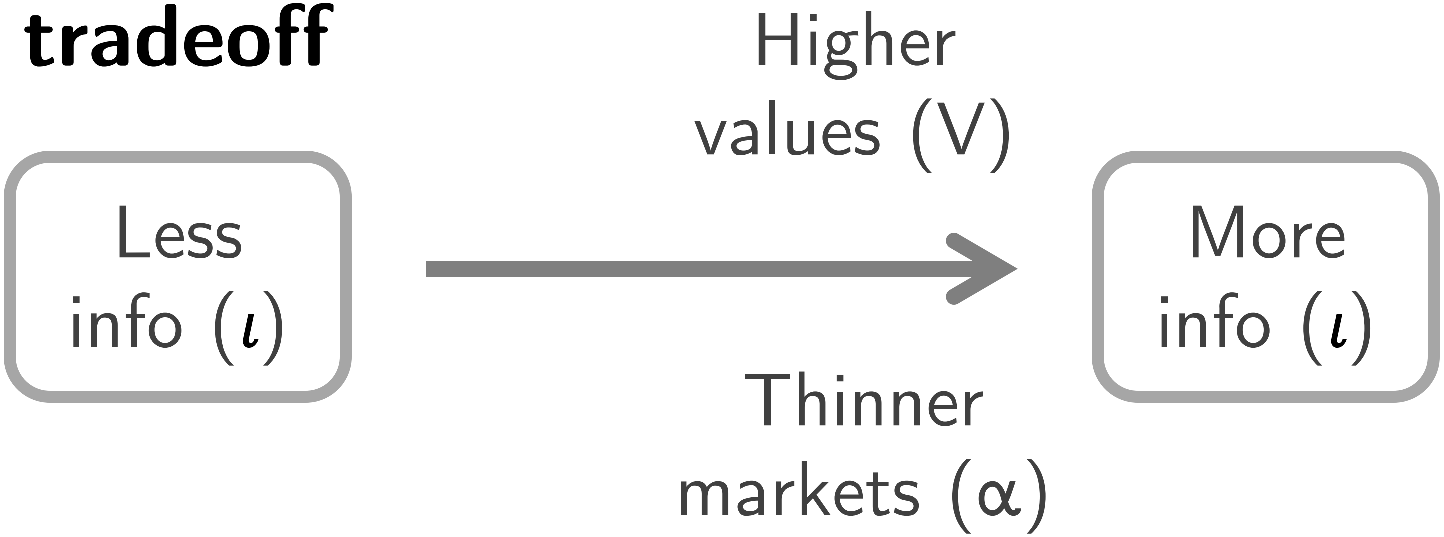
2 types
Budgets:
$b_\theta = (\$1; \$8)$

Heterogeneous Advertisers



Disclosure of Information: Homogenous Case

The tradeoff



Theorem

Suppose η fixed. When publisher reacts to thinner markets by setting the optimal **reserve price**, then disclosing more information improves profits.