Endogenous merger waves in vertically related industries

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* The Networks, Electronic Commerce, and Telecommunications (“NET”) Institute, http://www.NETinst.org, is a non-profit institution devoted to research on network industries, electronic commerce, telecommunications, the Internet, “virtual networks” comprised of computers that share the same technical standard or operating system, and on network issues in general.
Abstract

We study merger waves in vertically related industries where firms can engage in both vertical and horizontal mergers. Even though any individual merger would have been profitable, firms may refrain from merging for fear of negative impacts from other mergers. When they do merge, however, they always merge in waves, which is either vertical or horizontal depending on the relative intensity of double markup and horizontal competitions in the two industries. Finally, merger waves may happen with or without any fundamental change in the underlying economic conditions.

Keywords: merger wave, horizontal mergers, vertical mergers, stable market structure

JEL Code: L13, L42, D43

1 Introduction

It has been well documented that mergers come in waves, clustering in time and by industries (e.g., Andrade et. al, 2001; Mitchell and Mulherin, 1996). Some merger waves are horizontal, consisting mainly of mergers between competing firms in the same industry, while some other waves are vertical, made up mainly of mergers between suppliers and customers across vertically related industries. For example, the first great merger wave in the U.S. (1890s-1904) is horizontal. Consolidation between competitors generated corporate giants in the steel, telephone, oil, mining, railroad, and other major manufacturing and transportation industries. The second great merger wave (1910s-1929), by contrast, is mainly vertical. Ford and General Motors emerged as the major automobile manufacturers through vertical integration, acquiring every business along the supply chain from iron and coal mining, railroads and water transportation, steel mills, body and assembly, all the way to finished vehicles (Lipton, 2006; Martynova and Renneboog, 2008). In the U.S., the cement and ready-mixed industries underwent at least two waves
of vertical integration in the 1960s and 1980s (Hortacsu and Syverson, 2007). More recently, in February 2011 Nokia and Microsoft announced a comprehensive plan to ally vertically in the Smartphone area in response to the success of Apple’s iPhone. Several months later, Google acquired Motorola Mobility in a similar move.

Horizontal mergers may cluster not only in a single industry, but also across several industries that are vertically related. For example, a series of mergers in the US pharmaceutical industry occurred in 2007 to counteract the increased power in the downstream health care industry, where consolidation had been taking place.¹ Furthermore, a wave of integration along one dimension may follow a wave of disintegration along another dimension, i.e., the economy may switch between vertical and horizontal integrations. Semiconductor companies used to be vertically integrated. With rising costs and changing cost structure, economies of scale became more important, and firms started to disintegrate vertically and merge horizontally.²

These examples highlight the need to study horizontal and vertical mergers together, as a merger will alter the incentives of other mergers, and the impacts are likely to be different depending on whether the mergers are vertical or horizontal. Given the empirical regularity of horizontal and vertical merger waves, the following questions immediately emerge: What makes mergers to cluster? What causes a merger wave to occur in the first place? And what determines the wave to be vertical or horizontal? There have been some economic studies on horizontal merger waves (in a single industry) and a few on vertical merger waves, but virtually none in which the two are put together. This research takes a novel approach to study mergers in vertically related industries and is therefore uniquely positioned in addressing the last question. In doing so, the analysis has shed new lights on the answers to the first two questions.

A simple model is hitherto constructed which contains two vertically related industries, each consisting of two firms that may engage in both horizontal and vertical mergers. Although merger decisions are made simultaneously, the equilibrium concept is defined such that a firm considers “responses” from other firms when contemplating a merger. Such consideration is crucial in any model of multiple mergers, where merger incentives are necessarily interdependent. We look for equilibrium market structures that satisfy two conditions. First, a merger wave is an equilibrium phenomenon involving multiple mergers, meaning that firms’ merger incentives must be analyzed together. Second, the merger wave is endogenous, meaning that the original situation, i.e., all firms remaining independent, must also be an equilibrium. If this requirement is not satisfied, the researchers will be hard pressed to explain why a merger wave takes place at a particular time instead of earlier. Therefore, a merger wave in our model represents a change of the market structure from one equilibrium in which no firm merges, to another equilibrium in which many firms merge. Once such a transition between equilibria is established, a comparison between their respective conditions will help answer the question of what may trigger a merger wave.

We find that in any equilibrium, either no firm merges, or all firms merge. Both are driven by a negative impact of a merger on other firms’ payoffs. Fixing the market structure among the remaining firms, a merger between any two firms would have been

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profitable. If other firms also merge, however, the two firms will be hurt so much that they prefer the original situation in which no firm merges. It is this concern that prevents any firm from merging. The negative impact also explains why mergers cluster. A single merger would hurt the other two firms, which would then find merging a dominant strategy. It is therefore impossible to have a single merger in any equilibrium.

While no-merger and multiple mergers can both be equilibria, the conditions for the two equilibria may be identical or different depending on the parameters. This immediately suggests that an endogenous merger wave may be triggered in two ways. Equilibrium may change because the underlying conditions have changed, or because it is a switch between multiple equilibria without any change in the fundamentals. The first case corresponds to an economic shock that changes the cost, demand, regulation or trading opportunities, so the trigger is tangible. By contrast, there is no clear trigger in the second case, or we may say that the trigger is intangible, corresponding to something trivial or totally unrelated to the underlying economic conditions, such as rumors or changes in mood or expectation.

A wave of vertical mergers removes double markup but intensifies downstream competition. A wave of horizontal mergers has the opposite effect. The tradeoff between the two will determine the four firms’ total profit and therefore affect the equilibrium market structure. However, whether a merger wave is vertical or horizontal depends not only on the four firms’ total profit, but also on the distribution of the total profit among the four firms, which in turn depends crucially on the relative intensity of competitions in the upstream and downstream industries. When the competitions are more or less balanced, firms tend to carry out vertical mergers. When the balance is disrupted, for example, when the downstream competition is intensified through price competition and closer substitution between the final products, or when the upstream competition is weakened through increasing marginal cost of input production, firms tend to carry out horizontal mergers.

Researches have long established that mergers tend to occur in waves and usually explained the waves by economic shocks at the economy or industry levels (Andrade et al, 2001; Andrade and Stafford, 2004; Harford, 2005; Jensen, 1993; Jovanovic and Rousseau, 2002; Mitchell and Mulherin, 1996; Shleifer and Vishny, 2003). In the industrial organization literature, there are only a few theoretical researches on merger waves, all focusing on mergers in the same industry. Qiu and Zhou (2007) and Toxvaerd (2008) both explained merger waves by strategic complementarity between horizontal mergers, and both attributed merger waves to economic shocks that change a merger’s profitability. While sharing the same focus on multiple mergers and the attempt to endogenize merger waves, this paper is broader in its setting with both horizontal and vertical mergers, and richer in its finding of a second, intangible trigger of merger waves. In real life, both triggers can be observed: Sometimes a clear trigger can be identified in the form of a sudden or gradual change in the underlying economic conditions such as demand, technology, regulation, or opening to international trade, but sometimes a series of mergers may take place without any obvious change in the fundamentals.

Mergers cluster in this model because a merger reduces other firms’ payoffs. This reason differs from others that have been identified in the literature: For Toxvaerd (2008), firms rush to acquisitions because a merger reduces the availability of potential
acquisition targets. For Qiu and Zhou (2007), mergers occur together because a merger raises other firms’ payoffs and consequently their merger profitability. The negative impact of a merger on other firms’ payoffs in this model is particularly fitting for vertically related industries. Casual observations and industry analysis often indicate that a merger hurts firms in the upstream or downstream industries, where firms usually counter with their own mergers. Furthermore, the negative impact leads to no-merger and merger waves both as equilibria, and therefore is crucial in generating the intangible trigger of endogenous merger waves.

Greenhut and Ohta (1979) have demonstrated that successive duopolists would both merge vertically, but Bonanno and Vickers (1988) and Lin (1988) show that firms may choose vertical disintegration in order to dampen downstream competition. Our model offers a synthesis of the two opposite results, as we show that both market structures can be equilibria even under the same conditions. Like Greenhut and Ohta (1979), we emphasize the role of double markup in driving vertical mergers. Like Bonanno and Vickers (1988) and Lin (1988), we show that firms my refrain from profitable vertical mergers because vertical disintegration generates a vertical externality that mitigates the horizontal externality associated with horizontal competition. Unlike their setting in which the only alternative to no-merger is vertical integration, firms in our model have additional options of horizontal mergers.

Following the equilibrium concept for one-to-one matching games with externalities (Sasaki and Toda, 1996), our simultaneous merger game provides a novel analytical framework for merger studies, which is a modest contribution in its own right. The game has at least two advantages. First, it is able to generate both no-merger and merger wave as equilibria and thereby identify an intangible trigger of endogenous merger waves. Second, it captures the essential interdependence of firms’ merger incentives without having to specify details of the game such as the order of moves, which so often will greatly affect the equilibrium outcome.

The plan of the paper is as follows. Section 2 sets up the baseline model and presents the major findings, which are then generalized in five variation models in Section 3. Section 4 concludes.

2 Model

2.1 Setup

Consider two vertically related industries. The upstream industry consists of two identical firms, A and B, and the downstream industry also consists of two identical firms, 1 and 2. A homogeneous input is produced by the upstream firms at constant marginal cost, c, and sold through arm’s length transaction to the downstream firms, which then transform it into a homogeneous final product at zero extra cost on a one-for-one basis. Firms compete à la Cournot in both industries. Each downstream firm regards the in-

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3 To name just a few examples that have been reported on The Economist: steelmakers and iron ore producers (“Financial prospecting,” November 15, 2007; and “Forging a new world,” April 19, 2008), steelmaking and shipbuilding (“Steely logic,” August 28, 2008), generic drug producers and drug buyers (“All together now,” July 24, 2008).
put price, denoted as $t$, as given and chooses its output facing the demand for the final product, $p = \alpha - Q$, where $Q$ is the total output produced by the downstream firms. The downstream Cournot equilibrium will give rise to $Q$ as a function of $t$, which is then inverted to generate the inverse demand for the input, i.e., $t$ as a function of $Q$. Facing this derived demand, the upstream firms engage in their Cournot competition.\footnote{Such a setting of successive Cournot oligopoly has been widely used. See Abiru et al (1998), Greenhut and Ohta (1976), and Salinger (1988) for examples.}

Firms play a two-stage merger game. In stage one, each firm chooses simultaneously a merger partner. A firm can commit to independence by choosing itself as the partner. A merger can be either horizontal (between two downstream firms or two upstream firms) or vertical (between a downstream firm and an upstream firm), and it takes place if and only if two firms choose each other as the partner. A merged entity is not allowed to participate in any further mergers. The merger decisions result in some market structure, referred to as a configuration, which is publicly announced.

Then the game proceeds to stage two. Given the realized market configuration, all remaining firms, merged or non-merged, compete à la Cournot, choosing their quantities in order to maximize their payoffs. Within a merged entity, the two partners share the merger surplus equally, where the surplus is calculated assuming fixed configuration among the other firms. For example, a merger between firms $A$ and $1$ in $N \equiv \{A,B;1,2\}$ leads to $S \equiv \{B,A1,2\}$, so $\pi^S_A = \pi^N_A + \frac{1}{2}(\pi^S_{A1} - \pi^N_A - \pi^N_{1})$ and $\pi^S_{1} = \pi^N_{1} + \frac{1}{2}(\pi^S_{A1} - \pi^N_A - \pi^N_{1})$, where $A1$ denotes the entity resulting from a merger between firms $A$ and $1$, and $\pi^X_i$ is firm $i$’s payoff in configuration $X$ with $i \in X$.

We look for stable market configurations. A configuration is stable if no deviation by any individual firm or pair of firms is profitable. A deviation is profitable if all deviators are better off (strictly for at least one) for any possible configuration among the other firms.

Our merger game is essentially a one-to-one matching game, so we follow the matching literature (see, for example, Roth and Sotomayor, 1990) to allow both unilateral deviations by individual firms and collective deviations by a pair of firms. Since a merger involves two firms, unilateral deviation alone is not enough to address merger issues. Collective deviations must also be considered, as deviating into a new merger requires the coordination of two firms changing their strategies simultaneously. Furthermore, because a merged entity’s profits depend on other firms’ merger structures, our merger game is essentially a one-to-one matching with externalities. Following Sasaki and Toda (1996), our equilibrium concept therefore requires a deviation to be profitable under all possible configurations among the remaining firms. Such a requirement is obviously sufficient to justify the deviation. Sasaki and Toda (1996) have proved the existence of such an equilibrium in the context of one-to-one matching with externalities, and it works well for our merger game, as the equilibrium set thus generated is non-empty and small.

\subsection*{2.2 Analysis}

The game is solved by backward induction. Suppose that at the beginning of stage two, the realized market configuration contains $v$ vertically integrated firms, $u$ independent
upstream firms, and $d$ independent downstream firms. Because marginal costs are constant and identical within an industry, a horizontal merger has the effect of eliminating one of the merging firms. A vertically integrated firm participates only in the downstream competition, and it differs from an independent downstream firm in that its input is procured at cost $c$ rather than the market price $t$. In the downstream competition, therefore, a vertically integrated firm chooses $q_v$ to maximize $\pi_v \equiv (\alpha - Q - c)q_v$, which leads to the first-order condition $\alpha - q_v - Q = c$. Since $Q = vq_v + dq_d$ (using symmetry), the first-order condition can be rewritten as:

$$\alpha - (v + 1)q_v - dq_d = c.$$ 

Likewise, the first-order condition for an independent downstream firm is $\alpha - q_d - Q = t$, or

$$\alpha - (d + 1)q_d - vq_v = t.$$ 

These two equations lead to

$$q_v = \frac{\alpha - (d + 1)c + dt}{d + v + 1} \quad \text{and} \quad q_d = \frac{\alpha - (v + 1)t + vc}{d + v + 1}.$$ 

The demand for the input for independent upstream firms is therefore

$$Q_i \equiv dq_d = \frac{d[\alpha + vc - (v + 1)t]}{d + v + 1},$$

or

$$t = \frac{\alpha + vc}{v + 1} - \frac{d + v + 1}{d(v + 1)}Q_i,$$

where $Q_i \equiv dq_d \equiv uq_u$ is the total quantity by the independent upstream or downstream firms. Facing this derived demand, an upstream firm will choose $q_u$ to maximize $\pi_u \equiv \left[\frac{\alpha + vc}{v + 1} - \frac{d + v + 1}{d(v + 1)}Q_i - c\right]q_u$. The Cournot equilibrium is then given by

$$q_u = \frac{d(\alpha - c)}{(u + 1)(d + v + 1)}.$$ 

Consequently,

$$qd = \frac{u(\alpha - c)}{(u + 1)(d + v + 1)}, \quad q_v = \frac{[d + (u + 1)(v + 1)](\alpha - c)}{(u + 1)(v + 1)(d + v + 1)},$$

and

$$Q = \frac{[(u + 1)(v + 1)(d + v) - d](\alpha - c)}{(u + 1)(v + 1)(d + v + 1)}.$$ 

As argued by Salinger (1988), a vertically integrated entity will withdraw from the upstream competition—it neither buys the input from other upstream firms nor sells it to other downstream firms. Some researchers have raised concerns about commitment power in Salinger’s argument, but the issue is not serious in this paper because firms compete in quantity rather than in price (Hart and Tirole, 1990). Schrader and Martin (1998) provide further arguments in support of Salinger’s conclusion.
It is clear that all profits are proportional to \((\alpha - c)^2\), so we can normalize \(\alpha - c \equiv 1\) without losing any generality. Then, the profits of the three types of the firms are:

\[
\pi_v(v, u, d) = \frac{[d + (u + 1)(v + 1)]^2}{(u + 1)^2(v + 1)(d + v + 1)^2},
\]

\[
\pi_u(v, u, d) = \frac{d}{(u + 1)^2(v + 1)(d + v + 1)},
\]

\[
\pi_d(v, u, d) = \frac{u^2}{(u + 1)^2(d + v + 1)^2}.
\]

Now move back to stage one. There are six possible market configurations as listed in Table 1. Firms’ payoffs can be calculated using the formula derived above. For example, in \(S_1 \equiv \{A, B; 1, 2\}\), \(v = 0\), \(u = 2\), \(d = 2\). As specified earlier, each merged entity divides the merger surplus/deficit equally between the two partners.

\begin{table}[h]
 \centering
 \begin{tabular}{|c|c|c|c|c|c|}
 \hline
 Configuration & \(\pi_A\) & \(\pi_B\) & \(\pi_1\) & \(\pi_2\) & Total profit \\
 \hline
 \(S_1 \equiv \{A, B; 1, 2\}\) & 74 & 74 & 49.4 & 49.4 & 247 \\
 \(S_2 \equiv \{AB; 12\}\) & 62.5 & 62.5 & 31 & 31 & 188 \\
 \(S_3 \equiv \{A1, B2\}\) & 62.5 & 62.5 & 48.6 & 48.6 & 222 \\
 \(S_4 \equiv \{B; A1, 2\}\) & 99 & 42 & 74.5 & 28 & 243 \\
 \(S_5 \equiv \{AB; 1, 2\}\) & 83 & 83 & 28 & 28 & 222 \\
 \(S_6 \equiv \{A, B; 12\}\) & 55.5 & 55.5 & 55.5 & 55.5 & 222 \\
 \hline
 \end{tabular}
 \end{table}

**Proposition 1.** Only \(S_1\) and \(S_3\) are stable.

*Proof:* \(S_1\) is stable. There are three possible deviations, but none of them are profitable: \(A + B\) is unprofitable when 1 and 2 merge; \(1 + 2\) is unprofitable when \(A\) and \(B\) merge; and \(A + 1\) is unprofitable when \(B\) and 2 merge. Note that \(i + j\) denotes a merger between firms \(i\) and \(j\).

\(S_2\) is not stable because \(A + 1\) is a profitable deviation: If \(B\) and 2 merge, \(A + 1\) will make \(A\) indifferent but 1 strictly better off; if \(B\) and 2 remain independent, both \(A\) and 1 are strictly better off.

\(S_3\) is stable. There are three possible deviations (excluding symmetric ones), but none is profitable: Breaking up \(B2\) is unprofitable when \(A\) and 1 remain merged; \(A + B\) is unprofitable when 1 and 2 merge; \(1 + 2\) is unprofitable when \(A\) and \(B\) merge.

\(S_4\) is not stable because \(B + 2\) is a profitable deviation (whether \(A\) and 1 separate or remain merged).

\(S_5\) is not stable because \(1 + 2\) is a profitable deviation (whether \(A\) and \(B\) separate or remain merged).

\(S_6\) is not stable because \(A + B\) is a profitable deviation (whether 1 and 2 separate or remain merged).

Q.E.D.

\footnote{For ease of comparison, the fractions of the payoffs are multiplied by 1,000 and turned into numerical values.}
Proposition 1 says that the four firms either remain independent or carry out two vertical mergers. To understand the intuition, it is useful to summarize the payoffs in Table 1 into the following three profitability ranking:

*R1:* An exogenous merger, i.e., fixing the configuration among the remaining two firms, is always profitable. This is because a merger internalizes either the horizontal externality (horizontal competition) or the vertical externality (double markup), which benefits the merging partners.

*R2:* A merger always hurts other firms and consequently a breakup always benefits other firms. By eliminating double markup, a vertical merger makes the merged entity more aggressive in the downstream competition, which hurts other firms. A horizontal merger between duopolists will hurt the other industry by reducing quantities supplied or demanded.

*R3:* $S_1$ Pareto dominates $S_3$ which in turn dominates $S_2$. Firms face horizontal externality in $S_3$ and therefore tend to produce too much from the viewpoint of their joint profits. They face vertical externality in $S_2$ and tend to produce too little. Both externalities are present in $S_1$, but their opposite effects mitigate each other, so $S_1$ dominates both $S_2$ and $S_3$. The horizontal externality in $S_3$ is moderate because the downstream competition (Cournot as opposed to Bertrand) is mild, while the vertical externality in $S_2$ is severe because it leads to successive monopoly, so $S_3$ dominate $S_2$.

Armed with these three profit ranking, we are now ready to discuss the intuition behind the equilibria. First, it cannot be stable to have only one merger because the remaining two independent firms will be better off by merging. They gain if the originally merged firm remains merged ($R1$), and will gain even more if the merged firm breaks up ($R2$). Second, having two horizontal mergers is unstable, as an upstream firm and a downstream firm would rather merge vertically. Such a deviation is profitable if the other two firms also merge vertically ($R3$), and is even more profitable if the other two firms do not merge ($R2$).

That leaves us with two configurations: all firms remain independent ($S_1$) or carry out two vertical mergers ($S_3$). Both are stable. Consider first the no-merger case. An exogenous merger would have been profitable ($R1$). However, if the other two firms also merge, participants of the first merger would be worse off ($R3$). Therefore, no deviation (in the form of a merger) will be carried out. Next consider the case of two vertical mergers. It is unprofitable to either break up (if the other merged firm remains merged—$R1$) or switch to a horizontal merger (if the other two firms also merge horizontally—$R3$).

### 2.3 Discussion

Four conclusions can be drawn from Proposition 1. First, mergers occur in waves. As revealed in Proposition 1, one of the equilibria is a merger wave consisting of two vertical mergers ($S_3$). Furthermore, it is never an equilibrium to have a single merger ($S_4$, $S_5$, and $S_6$); whenever a merger occurs in an equilibrium, it must be accompanied by another one. This is because a single merger would hurt the other two firms, which will then find it profitable to merge between themselves regardless of what the first pair does (dissolves or remains merged).

So the driving force for mergers to cluster is the negative impact of a merger on
other firms. This finding complements Qiu and Zhou’s (2007) discovery that a positive impact may also account for merger waves: Horizontal mergers (in the same industry) are strategic complements because a merger increases other firms’ profits. In the sequential game that Qiu and Zhou (2007) consider, the positive impact of a merger ensures that a pair of firms have both the incentive and the ability to precipitate another merger through their own merger. In this model with simultaneous game, by contrast, firms worry about the most unfavorable configuration among the other firms. The negative impact of a merger generates a tendency for mergers to follow one another, as the profitability of a merger is sufficient for its occurrence if other firms have merged.

Second, no-merger ($S_1$) is also an equilibrium. All four firms may remain independent even though any individual merger would have been profitable. When a given merger is said to be profitable, the configuration among the remaining two firms is implicitly assumed to be fixed, i.e., they remain independent. If these two firms also merge, however, the first pair will be hurt. The concern for a second merger’s negative impact therefore prevents the first one from taking place. That is, the profitability of a merger is insufficient for its occurrence if other firms have not merged.

Firms in this model refrain from carrying out profitable mergers because of negative interactions between potential mergers. In other models where the interaction is positive, by contrast, there is a stronger tendency for firms to merge. For example, when mergers are strategic complements, even an unprofitable merger may be carried out, let alone a profitable one, so firms remain independent only when the economic conditions (in terms of demand or cost, for example) are extremely unfavorable to mergers (Qiu and Zhou, 2007).

Third, a merger wave may take place without any fundamental change in the underlying conditions. Proposition 1 predicts multiple equilibria in this game. The firms may coordinate on $S_1$ (or $4I$, meaning four independent firms, for a more informative notation) and all remain independent. Alternatively, they may carry out two vertical mergers and end up with $S_3$ (or $2V$). Such a change of market structure duly endogenizes merger waves, as the situations before and after the change are both equilibria. Because the two equilibria are under the same conditions, shifting from $4I$ to $2V$ does not require any change in the economic fundamentals such as demand or technology. The trigger may be something trivial or totally unrelated: mood, expectation, rumor, etc.

Although $4I$ and $2V$ are both equilibria, they are not on an equal footing. Switching from $4I$ to $2V$ is possible or even likely because, if for whatever reason, a pair of firms expect the other pair to merge, they will surely follow suit. The reverse process of jumping from $2V$ to $4I$, interpreted as a divestiture wave, is more difficult. If for some reason a pair breaks up, it is in the best interest of the second pair not to follow. Such an asymmetry between the two equilibria may explain why in real life, merger waves are much more common than divestiture waves.

Fourth, a merger wave may be vertical or horizontal depending on the tradeoff between the vertical and the horizontal externalities. In our $2 \times 2$ setting, a merger wave may consist of two vertical mergers ($2V$) or two horizontal mergers ($S_2$ or $2H$), referred to respectively as vertical and horizontal merger waves. When a firm makes merger

\[ \text{Here the horizontal wave consists of a horizontal merger in each of the two industries, which is slightly different from the usual meaning of multiple horizontal mergers in the same industry.} \]
decisions, it chooses between vertical and horizontal mergers taking into accounts other firms’ possible actions, so ultimately it is a choice between 2V and 2H.

As mentioned earlier, 2V alleviates double markup but intensifies horizontal competition; 2H is the opposite. In the present model, the damage of double markup is greater than that of horizontal competition, so firms end up taking two vertical mergers. In other settings, however, the comparison may be reversed and horizontal mergers may emerge in equilibrium. For example, if the competition is à la Bertrand rather than Cournot, a merger wave could be horizontal. We are going to explore more in the next section about the conditions for a merger wave to be vertical or horizontal.

3 Generalization

The conclusions discussed so far are derived from a highly stylized model with symmetric 2×2 firms, constant marginal costs, homogeneous products, linear demand and Cournot competition. One may wonder whether they still hold under more general conditions. In this section we will investigate five variations of the baseline model by relaxing its assumptions, one at a time, so that products are differentiated, the competition is in price rather than in quantity, the marginal cost is increasing, or the demand is non-linear. All conclusions reached in the baseline model are found to be robust. More importantly, since the baseline model is a special case of some of the variation models, we are able to put the conclusions in perspective and understand better their reason, conditions and implications. In particular, we will discuss the conditions for a merger wave to be vertical or horizontal, and identify a second trigger of merger waves.

3.1 Product differentiation

Suppose that the final products are differentiated (the inputs are still homogeneous) so that the demand for firm i’s product is \( p_i = \alpha - q_i - \beta q_j \), where \( i, j \in \{1, 2\} \) with \( i \neq j \), and \( \beta \in [0, 1] \) represents the degree of product differentiation. Furthermore, the competition in each industry may be in either quantity or price, which gives rise to the following three combinations: Cournot competition in both industries (Cournot-Cournot); upstream Cournot competition and downstream Bertrand competition (Cournot-Bertrand); and Bertrand competition in both industries (Bertrand-Bertrand). All other aspects of the model remain the same as in the baseline model. For each of the three cases, a payoff table similar to Table 1 can be constructed (the appendix shows one such table), where each payoff is a function of \( \beta \) only.\(^9\)

**Proposition 2.**

1. In Cournot-Cournot, \( S_1 \) is stable for \( \beta > 0.29 \) and \( S_3 \) is stable for any \( \beta \).
2. In Cournot-Bertrand, \( S_1 \) is stable for \( 0.22 < \beta < 0.8 \), \( S_2 \) is stable for \( \beta > 0.66 \), and \( S_3 \) is stable for \( \beta < 0.66 \).
3. In Bertrand-Bertrand, \( S_3 \) is stable for \( \beta < 0.66 \) and \( S_2 \) is stable for any \( \beta \).

**Proof:** See the appendix.

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8 Furthermore, a 3×3 case is analyzed in the appendix.
9 Again \( \alpha - c \) can be normalized without losing any generality.
As Proposition 2 suggests, a single merger ($S_4$, $S_5$ and $S_6$) can never be stable. The reason has been discussed in the baseline model and will be further elaborated later. The remaining three configurations, no-merger ($S_1$) and horizontal and vertical merger waves ($S_2$ and $S_3$), can all be stable under certain conditions. A general pattern is that as competition intensifies (moving from Cournot to Bertrand), horizontal mergers become more likely, while vertical mergers and no-merger become less likely. Below we explain why and under what conditions each of the three configurations is stable.

To facilitate our discussion, we summarize the equilibrium results in Table 2, which also contains the results from Proposition 3. The parameter range indicates the condition for a particular configuration to be stable, and the parenthesis below indicates the profitable deviation that makes the configuration unstable when the condition is violated. For example, in the Cournot-Cournot case, $S_1$ is stable for $\beta > 0.29$. If $\beta < 0.29$, $S_1$ becomes unstable because $A + B$ is a profitable deviation.

<table>
<thead>
<tr>
<th>Cases</th>
<th>$S_1$ ($4I$)</th>
<th>$S_2$ ($2H$)</th>
<th>$S_3$ ($2V$)</th>
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<tbody>
<tr>
<td>Cournot-Cournot</td>
<td>$\beta &gt; 0.29$</td>
<td>Never</td>
<td>Always</td>
</tr>
<tr>
<td></td>
<td>$(A + B)$</td>
<td>$(A + 1)$</td>
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<tr>
<td>Cournot-Bertrand</td>
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<td>$\beta &gt; 0.66$</td>
<td>$\beta &lt; 0.66$</td>
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<td></td>
<td>$(A + B)$ $(1 + 2)$</td>
<td>$(A + 1)$</td>
<td>$(A + B)$</td>
</tr>
<tr>
<td>Bertrand-Bertrand</td>
<td>Never</td>
<td>Always</td>
<td>$\beta &lt; 0.66$</td>
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<td></td>
<td>$(A + B)$</td>
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<td>$(A + B)$</td>
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<tr>
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<tr>
<td></td>
<td>$(1 + 2)$</td>
<td>$(A + 1)$</td>
<td>$(1 + 2)$</td>
</tr>
<tr>
<td>General demand</td>
<td>Always</td>
<td>$\sigma &gt; 1$</td>
<td>$\sigma \leq 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A + 1)$</td>
<td>$(A + B)$</td>
</tr>
</tbody>
</table>

**Cournot-Cournot**

The setting is the same as the baseline model except that the final products are differentiated, so the baseline model is a special case with $\beta = 1$. Greater differentiation between the final products weakens the downstream competition. If products are sufficiently differentiated ($\beta$ is small), a downstream merger in $4I$ will not reduce by much the quantity demanded for inputs, and therefore will do little damage to the upstream firms. Recall that upstream firms refrain from an otherwise profitable merger for fear of the negative impact from a downstream merger. If the impact is expected to be small, upstream firms will merge and, as a result, $4I$ becomes unstable.

In the baseline model when products are homogeneous, $2V$ preempts $2H$. Product differentiation will further strengthen the dominance of $2V$ over $2H$ because weaker downstream competition reduces $2V$’s disadvantage (of facing downstream competition) and $2H$’s advantage (of eliminating downstream competition). As a result, $2V$ is always stable while $2H$ is never stable for any $\beta$.

**Cournot-Bertrand**

When downstream firms compete in price rather than in quantity, the downstream competition is intensified. $4I$ is stable only if products are modestly differentiated (intermediate $\beta$). As in the previous case, when products are highly differentiated, the
upstream firms do not need to worry about the negative impact from a downstream merger, so they merge. When products are close substitutes, on the other hand, the downstream competition is fierce, so downstream firms will merge to avoid this competition.

As explained earlier, product differentiation favors 2V over 2H, so 2V continues to be stable when products are sufficiently differentiated (small $\beta$). Conversely, when products are close substitutes (large $\beta$), fierce competition in the downstream industry raises both the benefits of horizontal mergers and the drawbacks of vertical mergers, so 2H becomes stable.

Bertrand-Bertrand 10

The upstream firms also compete in price now, so the upstream competition is greatly intensified. In fact it is intensified to the greatest extent as inputs are homogeneous. 4I is never stable because upstream firms earn zero profits in 4I and they will always attempt to merge. As in the previous case, 2V is stable when products are sufficiently differentiated.

2H is stable for any $\beta$. Starting from 2H, if an upstream and a downstream firm deviate to form a vertical merger, one scenario they may face is that the other two firms do not merge. Then, the upstream firm’s profit share in the merger will be very low because it is calculated based on the situation when the merger does not take place, in which case its profit will be zero due to upstream price competition.

3.2 Cost and demand

We now move back to the setting with homogeneous final product and Cournot competition in both industries. In the baseline model, the cost of input production and the demand for the final product are both linear: $C(q) = cq$ and $p = \alpha - Q$. On the cost side, the model can be generalized to allow increasing marginal cost ($mc$): $C(q) = \gamma q^2$ with $\gamma \geq 0$. Note that the baseline model is a special case of increasing $mc$ with $\gamma = 0$.11 On the demand side, it can also become more general in the form of $p = \alpha - Q^\sigma$ with $\alpha > 0$ and $\sigma > 0$. Again the baseline model is a special case with $\sigma = 1$. Note a peculiar property of this general demand function: its concavity, defined as $\frac{\partial^2 p(Q)}{\partial Q^2}$, is constant.12

Proposition 3.

(1) In the increasing marginal cost case with $C(q) = \gamma q^2$, $S_1$ and $S_3$ are stable for $\gamma < 2$, and $S_2$ is stable for $\gamma > 2$.

(2) In the general demand case with $p = \alpha - Q^\sigma$, $S_1$ is stable for any $\sigma$, $S_2$ is stable for $\sigma > 1$, and $S_3$ is stable when $\sigma \leq 1$.

10 Colangelo (1995) has studied the tradeoff between vertical and horizontal mergers. His game 3 is similar to our Bertrand-Bertrand game except that merger decisions are made sequentially with exogenous target for the initial acquisition. Despite the difference in game rules, predictions from the two models are largely consistent. This indicates that our game is able to capture the essential interaction between merger incentives without specifying the details of the merger process.

11 In the baseline model, the exact value of $c$ is inconsequential, as all profits are proportional to $(\alpha - c)^2$ and consequently $\alpha - c$ has been normalized without any loss of generality. We may as well think that $c = 0$.

12 In fact, $\frac{\partial^2 p(Q)}{\partial Q^2} = \sigma - 1$. This function has been used by Greenhut and Ohta (1976) and other researchers in studying merger incentives and/or vertically related industries.
Proof: See the appendix.

The results of Proposition 3 are summarized in Table 2. Again, a single merger (S₄, S₅ and S₆) is never stable. Below are the explanations for the stable configurations.

- Increasing marginal cost

Producing the inputs at increasing marginal costs will reduce the upstream competition.¹³ When the $mc$ curve is steep (γ is large), 4I is unstable. This is because an upstream merger will not raise the input price by much and therefore will do little damage to downstream firms. As a result, the downstream firms in 4I will find it profitable to merge.

An increasing marginal cost favors 2H over 2V. The benefit of a vertical merger comes from output expansion due to the elimination of double markup. When the marginal cost of input increases with quantity, expansion is increasingly costly, so the benefits of vertical mergers decrease. When the $mc$ curve is flat (γ is small), such a decrease is small, so 2V continues to dominate 2H, meaning that 2V is stable while 2H is not.

- General demand

An increase in the demand concavity lessens the severity of the double markup problem.¹⁴ As will be argued below, 4I is always threatened by horizontal mergers rather than vertical ones. But demand concavity affects mainly double markup, which is present in both 4I and 2H, so 2H is never a real threat and consequently 4I is stable for any value of the demand concavity. When the demand is convex ($\sigma < 1$), the double markup problem is very severe, so firms merge vertically to avoid the damage, leading to 2V. Conversely, when the demand is concave ($\sigma > 1$), double markup is less a problem. Avoiding horizontal competition is the major concern, so 2H becomes stable.

### 3.3 Discussion

As has been seen in the baseline model and will be further discussed below, the stability of a configuration depends crucially on the intensity of double markup and horizontal competition. Each of the five variation models considered above introduces a parameter that affects either intensity. Compared to the baseline model, production differentiation in the final products (Cournot-Cournot) weakens the downstream competition; Bertrand competition in the downstream (Cournot-Bertrand) intensifies the downstream competition; Bertrand competition in the upstream (Bertrand-Bertrand) intensifies the upstream competition. In addition, increasing marginal cost for input production weakens the upstream competition, and greater demand concavity lessens double markup.

In this subsection, we will summarize the common patterns of stable configurations and relate the equilibria to the underlying economic conditions. To facilitate the discus-

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¹³In terms of oligopoly behavior, product homogeneity with increasing marginal costs is mathematically equivalent to product differentiation with constant marginal cost (Vives, 1999). In a sense, the increasing marginal cost considered here introduces differentiation to the inputs.

¹⁴A single-level monopolist facing constant marginal cost $c$ and demand $p = \alpha - Q^\sigma$ will choose quantity $Q_1 = \left(\frac{\alpha-c}{1+\sigma}\right)^{\frac{1}{\sigma}}$. In successive monopoly with the same cost and demand, the quantity chosen will be $Q_2 = \left[\frac{\alpha-c}{(1+\sigma)^2}\right]^{\frac{1}{\sigma}} < Q_1$. As a measure of the severity of double markup, $\frac{Q_1}{Q_2} \equiv (1+\sigma)^{\frac{1}{\sigma}}$ declines with $\sigma$, indicating that double markup is less a problem when $\sigma$ increases. Basically, when $\sigma$ is larger, the demand for input is closer to the demand for the final product.
sion, we demonstrate the equilibrium outcomes of the five variation models in Figure 1, where the bullet point represents the baseline model’s result as a special case.

### 3.3.1 Mergers come in waves

In all cases at any parameter value, there is always an equilibrium with merger wave, either horizontal or vertical. In other words, if a merger takes place in any equilibrium, it is always accompanied by another one. Conversely, single merger ($S_4$, $S_5$ and $S_6$) is never an equilibrium. So mergers always come in waves. The driving force for mergers to cluster is still the two profitability ranking that we have seen in the baseline model: an exogenous merger between any two firms is always profitable ($R1$), and a merger always hurts other firms ($R2$). Below we explain why these two ranking are still valid in generalized models. Once they are established, our game rule immediately implies that no configuration can be stable with any single merger.

First consider the configuration with only a vertical merger ($S_4$). The merged firm becomes more competitive in the downstream competition, so the merger hurts the other two firms. Fixing the first merger, a merger between the remaining two firms is

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15 A minor exception is when firms compete in prices in both industries (Bertrand-Bertrand). A vertical merger removes a competitor from the upstream industry and therefore benefits the remaining independent upstream firm (its profit increases from zero to positive). But this will not affect the equilibrium results.
profitable because it internalizes the vertical externality generated by double markup.

Next consider the configuration with only an upstream horizontal merger \((S_5)\). This merger raises the input price and thereby hurts downstream firms. Fixing the upstream merger, the downstream firms can merge from duopoly to monopoly, which should always be profitable. But the complication is the presence of the upstream industry. In particular, upstream firms may try to share the benefit of the downstream merger by raising the input price, which will reduce the profitability of the downstream merger. However, it can be verified that in all models we have considered so far, including the baseline model and the five variations, a downstream merger never changes the equilibrium input price.\(^{16}\) As a result, a downstream merger continues to be profitable when there is an upstream industry.

Finally consider the configuration with only a downstream horizontal merger \((S_6)\). This merger reduces the quantity demanded for inputs without changing the input price, so it hurts upstream firms. Fixing the downstream merger, the upstream firms may merge from duopoly into monopoly, which again should always be profitable. And again the complication is the presence of a vertically related industry, the downstream. However, the demand that upstream firms face is derived from the downstream competition. As long as the downstream configuration is fixed, that demand does not change. Then an upstream merger continues to be profitable when there is a downstream industry.

### 3.3.2 No-merger as an equilibrium

In all cases except Bertrand-Bertrand, no-merger \((4I)\) is an equilibrium (under certain conditions). Firms in \(4I\) may deviate by having either a horizontal merger or a vertical merger. Given our game rule, what really matters is how the deviators do in a situation when the other two firms also merge. That is, the alternative market structure to \(4I\) is essentially a merger wave, either \(2H\) or \(2V\). It turns out that the stability of \(4I\) depends on both the four firms’ total profit and its distribution among them.

In terms of the four firms’ total profit, \(4I\) tends to outperform both \(2H\) and \(2V\). As in the baseline model, here in all cases except Bertrand-Bertrand, \(2V\) eliminates double markup but retains horizontal competition, so the firms over-produce from the viewpoint of their joint profits, i.e., a horizontal externality. By contrast, \(2H\) removes horizontal competition but has double markup problem, so the firms under-produce, which can be viewed as a vertical externality. \(4I\) has both externalities, but since the two externalities move in opposite directions, one mitigates the other, so the firms’ total profits tend to increase.

The stability of \(4I\) depends not only on the firms’ total profit, but also on its distrib-

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\(^{16}\)In the baseline model, for example, the equilibrium input price is \(t = c + \frac{\alpha - c}{(u+1)(v+1)}\), which does not depend on \(d\). So if there is a downstream merger (i.e., \(d\) decreases by 1), the equilibrium input price will not change. A downstream merger rotates the demand for inputs clockwise around the intercept. This rotation has two opposite effects on \(t\): For any given \(t\), the quantity demanded is smaller, so \(t\) tends to drop. At the same time, the demand becomes less elastic, so \(t\) tends to rise. In the case of linear demand for the final product, the two forces exactly cancel out each other so the equilibrium input price will not depend on the number of downstream firms. For non-linear demand, Greenhut and Ohta (1976) and Ziss (2005) have shown the same invariance as long as the demand has constant concavity, which is satisfied by the general demand we adopt here.
bution among the four firms. For a deviating vertical merger to be profitable in 4I, the vertical pair will have to earn more. Because the two vertical pairs are symmetric, the four firms’ total profits must therefore be higher in 2V than in 4I, which in general does not hold. By contrast, 4I can be easily disrupted by a horizontal merger; all it needs is that such deviation benefits one horizontal pair, not both pairs. This is possible because the two industries are inherently asymmetric and so the two pairs’ profits may be very different. Therefore, the stability of 4I is always threatened by a horizontal merger, not a vertical one.

Whether or not the threat materializes depends on the relative intensity of horizontal competitions in the upstream and downstream industries, and no-merger can be an equilibrium if the two competitions are more or less balanced. For example, in Bertrand-Bertrand, the upstream competition is very strong, so 4I is never stable at any $\beta$. In Cournot-Bertrand, the balance between the two industries depends on $\beta$. When the final products are close substitutes, the downstream horizontal competition is strong, and the downstream firms will merge. When the final products are very poor substitutes, the downstream competition is weak. But then the upstream competition is relatively strong, and firms there would like to merge. As a result, 4I is stable only for moderate differentiation in the final products.

### 3.3.3 Vertical and horizontal merger waves

As mentioned earlier, in all cases at any parameter value, there is always an equilibrium with merger wave, either horizontal (2H) or vertical (2V). Furthermore, except in Bertrand-Bertrand, the two waves are mutually exclusive: When a horizontal wave is stable, a vertical wave is unstable, and vice versa. The reason is the following. For 2H, the threat to its stability comes mainly from a vertical merger. Given our game rule, the deviating firms are mainly concerned about the situation in which the other two firms also merge vertically, i.e., the alternative market structure to 2H is essentially 2V. Similarly, the alternative to 2V is 2H. Therefore, 2H and 2V will be the only market structures that the firms consider when they decide to merge. At any given parameter, either 2H dominates 2V, or 2V dominates 2H, so one and only one of the two configurations is stable.

Between 2H and 2V, which one is stable depends crucially on how the total profit distributes among the four firms, which in turn depends on the relative intensity of competitions in the upstream and the downstream industries. Because firms are symmetric within an industry, the two vertical pairs are symmetric, while the two horizontal pairs are not. For 2H to be stable, 2H needs to dominate 2V for only one horizontal pair, either upstream or downstream. By contrast, for 2V to be stable, 2V has to dominate 2H for all four firms. When the competitions in the two industries are balanced, no industry gains advantage over the other, so 2V tends to be stable. Conversely, when an industry’s competition is greatly intensified or weakened, 2H tends to be stable. For example, price (rather than quantity) competition or closer substitutability between final products will lead to horizontal merger waves. A greatly weakened competition in the upstream industry, due to increase marginal costs of producing the inputs, will have similar effect because downstream competition is now relatively intensified, and firms
there would like to merge horizontally.

So far we have discussed the conditions for the stability of 4I, 2H and 2V. It has been shown that both 4I and 2V are disrupted by 2H, while 2H is disrupted by 2V. Essentially firms are choosing among these three market structures, so the equilibrium depends on both the four firms’ total profit and its distribution among them. In terms of total profit, 4I tends to outperform both 2H and 2V. Between 2H and 2V, the comparison of total profit depends on the tradeoff between double markup and downstream horizontal competition. For example, when the demand for the final products is convex, double markup is severe, so 2V prevails. When the downstream competition is Bertrand and the final products are close substitutes, the downstream competition is intense, so firms merge horizontally. These effects are straightforward. More interesting and somewhat surprising is the role of relative intensity of competitions in the two industries. When the competitions are more or less balanced, both 4I and 2V tend to be stable. When the competition in one industry is greatly intensified or weakened, 2H tends to be the equilibrium.

3.3.4 Causes of merger waves

So far we have been attempting to relate the equilibrium market structures to the underlying economic conditions, and have shown that firms either withhold or cluster their mergers. The next question is, which of the two will be the final market structure? A model of endogenous merger waves must explain not only why a merger wave takes place, but also why it did not take place earlier. In other words, no-merger and merger wave must both be justified as equilibria, possibly under different conditions. A comparison between the conditions for 4I and those for 2V or 2H will help us identify the triggers for endogenous merger waves.

As Figure 1 indicates, in all five cases at any parameter value, there is at most two stable configurations and at least one. For some ranges of the parameters, there is a unique stable configuration, which is invariably a merger wave, either 2V or 2H. If the parameter falls into that range, the merger wave should take place immediately, so the only explanation why the mergers did not take place earlier is that the parameter value was outside the range, where all firms remaining independent was an equilibrium. Such a change of parameter value can be interpreted as an economic shock. For example, in Cournot-Cournot, 2V is the unique equilibrium for $\beta < 0.29$, and 4I is an equilibrium (though not unique) for $\beta > 0.29$. If $\beta$ changes from a value greater than 0.29 to a value below it, the market will undergo a wave of vertical mergers. Therefore, greater differentiation in the final products (due to, say, more investment in R&D or advertising) may trigger a vertical merger wave. Likewise, a more convex demand may also trigger a vertical merger wave. Similarly, a horizontal merger wave may be triggered by greater substitutability between the final products (Cournot-Bertrand), higher marginal cost in input production, or greater concavity of demand.

For some other ranges of the parameters, there are exactly two equilibria and, in most cases, one is 4I and the other is either 2V or 2H. When that is the case, it is possible for a merger wave to take place without any change in the underlying economic conditions, as we have seen in the baseline model. For example, this may happen for a vertical merger
wave when products are close substitutes (Cournot-Cournot), marginal cost rises slowly, or the demand is convex, and for a horizontal merger wave when product differentiation is moderate (Cournot-Bertrand) or the demand is concave.

So there are two types of causes that may trigger a merger wave, one is tangible and the other is not. As explained above, endogenous mergers take place when the economy switches from one equilibrium in which all firms remain independent, to another equilibrium in which all firms merge. The switch can be brought in two ways. The first is a change in the underlying economic conditions, which is interpreted as an economic shock. This is a tangible trigger for merger waves. The second is a shift between multiple equilibria without any change in the underlying economic conditions. This is an intangible trigger, corresponding to rumors or a sudden change in expectation or mood.

So far we have been talking about equilibrium switching from 4I to 2H or 2V. The equilibrium may also switch between 2H and 2V. In a sense this can also be interpreted as an endogenous merger wave, although it involves more restructuring because firms have to disintegrate along one dimension and then integrate along the orthogonal dimension. And both tangible and intangible triggers can be found for such merger waves. For example, in Bertrand-Bertrand, the economy may switch from 2V to 2H, which remind us of the structural changes in the semiconductor industry mentioned in the Introduction. Furthermore, such a wave of vertical disintegration followed by a wave of horizontal integration can have an intangible trigger when products are poor substitutes, and a tangible trigger when the final products become closer substitutes.

4 Conclusion

We have studied endogenous merger waves in vertically related industries where firms can engage in both vertical and horizontal mergers. A negative impact of mergers on other firms’ payoffs causes firms to either withhold or cluster their mergers. A merger wave may be vertical or horizontal depending on the relative intensity of double markup and horizontal competitions in the upstream and downstream industries, and the wave may happen with or without any fundamental change in the underlying economic conditions. The research has also generated many empirically testable predictions relating merger waves to conditions such as product differentiation, cost structure, and demand concavity.

In this model, mergers are carried out for competition purposes: In the absence of merger-specific cost or benefit and the intervention from antitrust authorities, firms merge in order to reduce competition and/or gain competitive advantage. In real life, a merger may have both anti-competitive and merger-specific effects (in the form of cost savings or synergies). By focusing on the first effect, this research helps us understand how interactions between mergers shape the equilibrium market structure, and how the structure may change with or without any change in the underlying economic conditions. Such an analytical framework are useful in understanding results where the second effect is also present. For example, cost or demand changes that are peculiar to a particular merger may precipitate the merger, which in turn may trigger a dramatic change in the overall market structure involving several related industries. The anticipation of subsequent changes in market structure will in turn alter the incentive of the first merger.
That is, the second effect can be added to the analytical framework, and the interaction between the two effects can be analyzed simultaneously.

The model adopts the simplest possible setting with two upstream and two downstream firms. In addition to simplicity, it provides a natural setting where an exogenous merger always benefits the merging firms and hurts non-merging firms. These effects conform to common understanding of what a merger means, are particularly fitting for vertically related industries, and are robust regardless of whether firms compete in price or quantity. As a result of the negative impact, firms do not suffer from “merger paradox” by which a merger always hurts merging firms and benefits non-merging firms in the same industry (Salant et al, 1983). Needless to say, this simplistic setting has its drawbacks. When a horizontal merger takes place, there is no possibility for a vertical merger. So horizontal and vertical merger waves are mutually exclusive, and there is always a tradeoff: Between the damages of double markup and horizontal competition, avoiding one will always exacerbate the other. Also, merging from duopoly into monopoly is special. Surprising as it may appear, the merger paradox captures the important competition effect of mergers within an industry (in the absence of merger-specific benefit and/or cost) and therefore cannot be ignored. So it is important to go beyond the 2×2 setting.17

In the appendix, we have analyzed the 3×3 setting and shown that there are two equilibria, one in which all firms remain independent and the other in which three vertical mergers take place. Similar patterns have been found in settings of 3×4, 4×3 and 4×4 (the analysis have been omitted here for space limitation), so Proposition 1 is indeed robust to the number of firms in either industry. A technical difficulty when there are more than two firms in an industry is that the equilibrium set tends to be large which contains some unreasonable market structures. Refinements are needed to prune the set. More work needs to be done along this direction.

Merger decisions in this model are made simultaneously. A sequential game would have the advantage of generating strategic incentive for mergers and may therefore account as another explanation for merger waves (Qiu and Zhou, 2007). But the disadvantage is that the equilibrium will be sensitive to the order of move and other details of the game,18 which a researcher has no compelling reason or sufficient information to specify exogenously. To deal with this, we have adopted a simultaneous game and an equilibrium concept borrowed from the one-to-one matching game with externalities. Such a setting enable us to capture the essential interdependence of merger incentives without having to specifies the details, and therefore may prove fruitful in studying other merger-related issues.

17 An interesting industry dynamics involving both horizontal and vertical mergers can be seen in the vertically related iron ore mining and steelmaking industries. In November 2007, the world’s two largest iron ore producers, BHP Billiton and Rio Tinto, announced a plan for a horizontal merger. Concerned that the concentration of the iron ore market would raise iron ore prices and jeopardize the downstream steel industry, one of Rio Tinto’s biggest customers, Chinese state-owned Aluminum Corporation of China (Chinalco) launched a preemptive move in February 2009 to purchase 14.9% of Rio Tinto’s shares. Such a (partial) vertical merger was apparently meant to preempt a horizontal merger in the upstream industry.

18 For example, Colangelo’s (1995) sequential game leads to very different predictions of the equilibrium market structure depending on the identity of the first acquisition target, which is exogenously given.
People may have concerns about the sharing rule and the assumption that the two industries do business by arm’s length transaction. We have assumed that merging firms split the merger surplus equally fixing the configuration among remaining firms, which seems to contradict our equilibrium concept where the configuration is variable when firms contemplate deviations. However, if configurations are allowed to vary in calculating profit sharing, the profits will be ill defined because there are multiple configurations. Arm’s length transaction may seem problematic when an industry has only one firm. But what is really needed is double markup and the fact that input price is affected by the market structure. Such a setting is relevant in many situations. An alternative to arm’s length transaction may be two-part tariff. But our Bertrand-Bertrand game removes double markup and may therefore have already captured what will happen when two-part tariff is used between upstream and downstream firms.

The present work is the first step in studying endogenous vertical and horizontal merger waves in vertically related industries. Many interesting extensions remain to be explored, such as asymmetric firms, dynamic game rule, and more sophisticated settings and sharing rules.

5 Appendix

Procedures of proving Propositions 2 and 3

The calculation and proofs are quite involving. Here we only provide the procedures. In each case, we construct the payoff table similar to Table 1. Each payoff will be a function containing a single parameter (it turns out that $\alpha - c$ can be normalized to 1 without any loss of generality). For example, for the Cournot-Cournot case, the following payoff table can be constructed. Other tables are omitted due to space limitation and are available upon request.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\pi_A$</th>
<th>$\pi_B$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>Total profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 = {A, B; 1, 2}$</td>
<td>$\frac{2}{9(2+\beta)}$</td>
<td>$\frac{2}{9(2+\beta)}$</td>
<td>$\frac{4}{9(2+\beta)^2}$</td>
<td>$\frac{4}{9(2+\beta)^2}$</td>
<td>$\frac{4(1+\beta)}{9(2+\beta)^2}$</td>
</tr>
<tr>
<td>$S_2 = {AB; 12}$</td>
<td>$\frac{1}{10(2+\beta)}$</td>
<td>$\frac{1}{10(2+\beta)}$</td>
<td>$\frac{1}{16(1+\beta)}$</td>
<td>$\frac{1}{16(1+\beta)}$</td>
<td>$\frac{1}{2(2+\beta)^2}$</td>
</tr>
<tr>
<td>$S_3 = {A1, B2}$</td>
<td>$\frac{10-\beta^2}{10(2+\beta)}$</td>
<td>$\frac{10-\beta^2}{10(2+\beta)}$</td>
<td>$\frac{6+\beta^2}{10(2+\beta)}$</td>
<td>$\frac{6+\beta^2}{10(2+\beta)}$</td>
<td>$\frac{1}{2(2+\beta)^2}$</td>
</tr>
<tr>
<td>$S_4 = {A1, B, 2}$</td>
<td>$\frac{144+104\beta+9\beta^2}{288(2+\beta)^2}$</td>
<td>$\frac{2-\beta}{8(2+\beta)}$</td>
<td>$\frac{144+40\beta+9\beta^2}{288(2+\beta)^2}$</td>
<td>$\frac{1}{2(2+\beta)^2}$</td>
<td>$\frac{28+8\beta-\beta^2}{16(2+\beta)^2}$</td>
</tr>
<tr>
<td>$S_5 = {AB; 1, 2}$</td>
<td>$\frac{1}{2(2+\beta)}$</td>
<td>$\frac{1}{2(2+\beta)}$</td>
<td>$\frac{1}{2(2+\beta)}$</td>
<td>$\frac{1}{2(2+\beta)}$</td>
<td>$\frac{3}{2(2+\beta)}$</td>
</tr>
<tr>
<td>$S_6 = {A, B; 12}$</td>
<td>$\frac{1}{9(1+\beta)}$</td>
<td>$\frac{1}{9(1+\beta)}$</td>
<td>$\frac{1}{9(1+\beta)}$</td>
<td>$\frac{1}{9(1+\beta)}$</td>
<td>$\frac{1}{9(1+\beta)}$</td>
</tr>
</tbody>
</table>

Given the payoffs, the stability of each configuration is examined. For a configuration to be stable, we have to check every possible deviation and show that it is unprofitable for one firm (it does not have to be profitable for both firms). For a configuration to be unstable, we only need to find one deviation that is profitable for both deviators (when it is a joint deviation by two firms).
For $S_1 = \{A, B; 1, 2\}$ to be stable: (i) $A + B$ is unprofitable: $\pi_{A}^{S_1} > \min\{\pi_{A}^{S_2}, \pi_{A}^{S_3}\}$. (ii) $1 + 2$ is unprofitable: $\pi_{1}^{S_1} > \min\{\pi_{1}^{S_2}, \pi_{1}^{S_3}\}$. (iii) $A + 1$ is unprofitable: $\pi_{i}^{S_1} > \min\{\pi_{i}^{S_2}, \pi_{i}^{S_3}\}$ for $i = A$ or 1.

For $S_2 = \{AB; 12\}$ to be stable: (i) Breaking $AB$ is unprofitable: $\pi_{A}^{S_2} > \min\{\pi_{A}^{S_1}, \pi_{A}^{S_6}\}$. (ii) Breaking 12 is unprofitable: $\pi_{1}^{S_2} > \min\{\pi_{1}^{S_1}, \pi_{1}^{S_5}\}$. (iii) $A + 1$ is unprofitable: $\pi_{i}^{S_2} > \min\{\pi_{i}^{S_1}, \pi_{i}^{S_4}\}$ for $i = A$ or 1.

For $S_3 = \{A1, B2\}$ to be stable: (i) Breaking $B2$ is unprofitable: $\pi_{i}^{S_3} > \min\{\pi_{i}^{S_1}, \pi_{i}^{S_4}\}$ for $i = B$ and 2. (ii) $A + B$ is unprofitable: $\pi_{A}^{S_3} > \min\{\pi_{A}^{S_2}, \pi_{A}^{S_5}\}$. (iii) $1 + 2$ is unprofitable: $\pi_{i}^{S_3} > \min\{\pi_{i}^{S_2}, \pi_{i}^{S_6}\}$.

For $S_4 = \{B; A1, 2\}$ to be unstable, we only need one of following cases to hold: (i) $B + 2$ is profitable: $\pi_{i}^{S_4} < \min\{\pi_{i}^{S_3}, \pi_{j}^{S_4}\}$ for both $i = B, j = A$ and $i = 2, j = 1$. (ii) $A + B$ is profitable: $\pi_{A}^{S_4} < \min\{\pi_{A}^{S_3}, \pi_{A}^{S_5}\}$. (iii) $1 + 2$ is profitable: $\pi_{1}^{S_4} < \min\{\pi_{1}^{S_3}, \pi_{1}^{S_6}\}$. (iv) Breaking $A1$ is profitable: $\pi_{i}^{S_4} < \min\{\pi_{i}^{S_1}, \pi_{j}^{S_4}\}$ for either $i = A, j = B$ or $i = 1, j = 2$. (v) $A + 2$ is profitable: $\pi_{i}^{S_4} < \min\{\pi_{i}^{S_1}, \pi_{j}^{S_4}\}$ for both $i = j = A$ and $i = 2, j = 1$. (vi) $B + 1$ is profitable: $\pi_{i}^{S_4} < \min\{\pi_{i}^{S_3}, \pi_{j}^{S_4}\}$ for both $i = B, j = A$ and $i = j = 1$.

For $S_5 = \{AB; 1, 2\}$ to be unstable: $1 + 2$ is profitable: $\pi_{1}^{S_5} < \min\{\pi_{1}^{S_2}, \pi_{1}^{S_6}\}$. For $S_6 = \{A, B; 12\}$ to be unstable: $A + B$ is profitable: $\pi_{A}^{S_6} < \min\{\pi_{A}^{S_2}, \pi_{A}^{S_5}\}$. Q.E.D.

The $3 \times 3$ case

Suppose there are three identical upstream firms ($A, B, C$) and three identical downstream firms (1, 2, 3). Other aspects remain the same as in the baseline model. Then, there are 10 possible market configurations as shown in the following payoff table. As the next proposition reveals, the firms will either remain all independent or carry out three vertical mergers, which is consistent with the result of the baseline model.

Table 4: Payoffs in the $3 \times 3$ case

<table>
<thead>
<tr>
<th>Configurations</th>
<th>$\pi_A$</th>
<th>$\pi_B$</th>
<th>$\pi_C$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
<th>Total profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 = {A, B, C; 1, 2, 3}$</td>
<td>47</td>
<td>47</td>
<td>47</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>246</td>
</tr>
<tr>
<td>$S_2 = {A1, B2, C3}$</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>28.5</td>
<td>28.5</td>
<td>28.5</td>
<td>187.5</td>
</tr>
<tr>
<td>$S_3 = {C; A1, B2, 3}$</td>
<td>42.5</td>
<td>42.5</td>
<td>21</td>
<td>42.5</td>
<td>42.5</td>
<td>15.5</td>
<td>206.5</td>
</tr>
<tr>
<td>$S_4 = {B, C; A1, 2, 3}$</td>
<td>61.5</td>
<td>28</td>
<td>28</td>
<td>49.5</td>
<td>28</td>
<td>28</td>
<td>222</td>
</tr>
<tr>
<td>$S_5 = {BC; A1, 23}$</td>
<td>99</td>
<td>21</td>
<td>21</td>
<td>74.5</td>
<td>14</td>
<td>14</td>
<td>243</td>
</tr>
<tr>
<td>$S_6 = {BC; A1, 2, 3}$</td>
<td>98</td>
<td>31</td>
<td>31</td>
<td>42.5</td>
<td>15.5</td>
<td>15.5</td>
<td>234.5</td>
</tr>
<tr>
<td>$S_7 = {B, C; A1, 23}$</td>
<td>65</td>
<td>18.5</td>
<td>18.5</td>
<td>86</td>
<td>24.5</td>
<td>24.5</td>
<td>237.5</td>
</tr>
<tr>
<td>$S_8 = {A, BC; 1, 23}$</td>
<td>74</td>
<td>37</td>
<td>37</td>
<td>49.5</td>
<td>25</td>
<td>25</td>
<td>247</td>
</tr>
<tr>
<td>$S_9 = {A, BC; 1, 2, 3}$</td>
<td>83.5</td>
<td>41.5</td>
<td>41.5</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>250</td>
</tr>
<tr>
<td>$S_{10} = {A, B, C; 1, 23}$</td>
<td>41.5</td>
<td>41.5</td>
<td>41.5</td>
<td>62.5</td>
<td>31</td>
<td>31</td>
<td>250</td>
</tr>
</tbody>
</table>

Proposition 4. In the three by three case, only $S_1$ and $S_2$ are stable.
Proof: $S_1$ is stable because there is no profitable deviation. There are three possible collective deviations excluding symmetric ones: $B + C$, $2 + 3$, and $A + 1$. $B + C$ is not
profitable since both $B$ and $C$ are worse off in $S_5$. $2 + 3$ is not profitable since both firms are worse off in $S_5$. $A + 1$ is not profitable because both $A$ and $1$ are worse off in $S_2$.

$S_2$ is stable. First, there is no profitable unilateral deviation, as no firm wants to break up with its merger partner. For example, if $C3$ breaks up, both $C$ and $3$ are worse off in $S_3$ as compared to $S_2$. Second, there is no profitable collective deviation. There are two possible collective deviations: $B + C$ and $2 + 3$. $B + C$ is not profitable since both of them are worse off in $S_6$ as compared to $S_2$. $2 + 3$ is not profitable either since both of them are worse off in $S_7$ as compared to $S_2$.

$S_3$ is not stable since $C + 3$ is a profitable deviation. $S_4$, $S_5$, $S_6$ and $S_7$ are not stable since $B + 2$ is a profitable deviation in any of the four configurations. $S_8$, $S_9$ and $S_{10}$ would have been stable using our stability criterion as specified in the baseline model, but they are unreasonable equilibrium market structures. $BC$ prefers to break up in both $S_8$ and $S_9$, while $23$ prefers to break up in $S_{10}$. Those firms won’t merge horizontally due to the merger paradox argument (Salant et al., 1983).

**Q.E.D.**

### References


