Discussion of Veiga & Weyl's "Multidimensional product design."

By Ignacio Esponda
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Efficient allocation – standard problem

primitives

$v(K,\theta) = \alpha K + \theta$
$\theta \sim U[0,1]$
$MC = 3/4$
$\alpha = 1/4$
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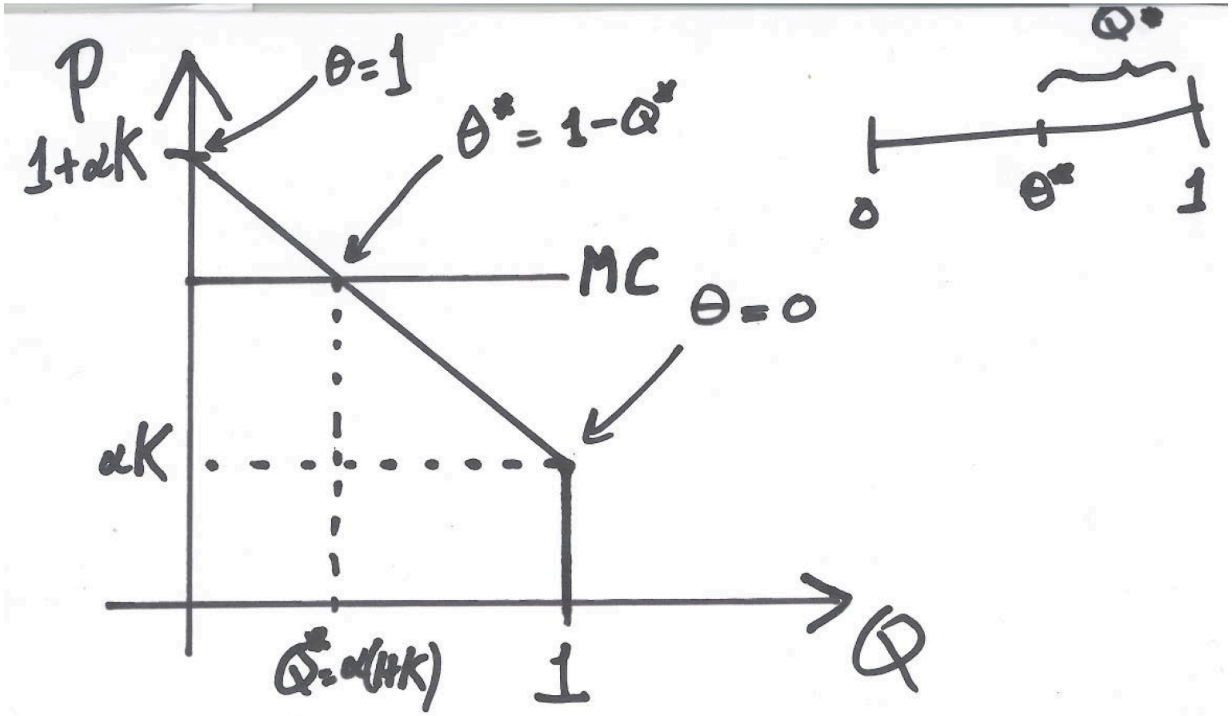

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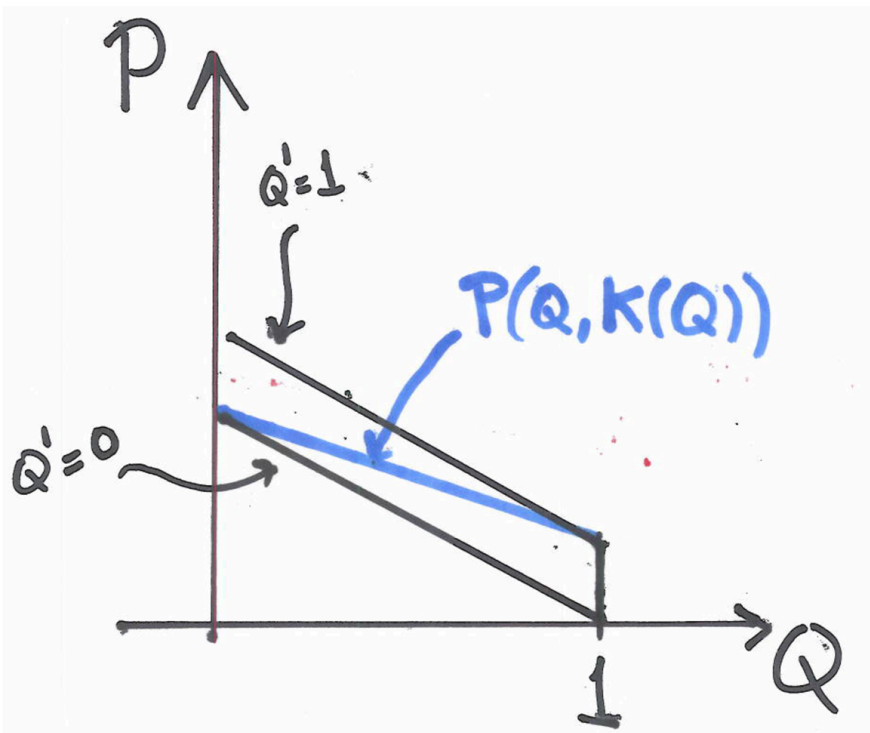

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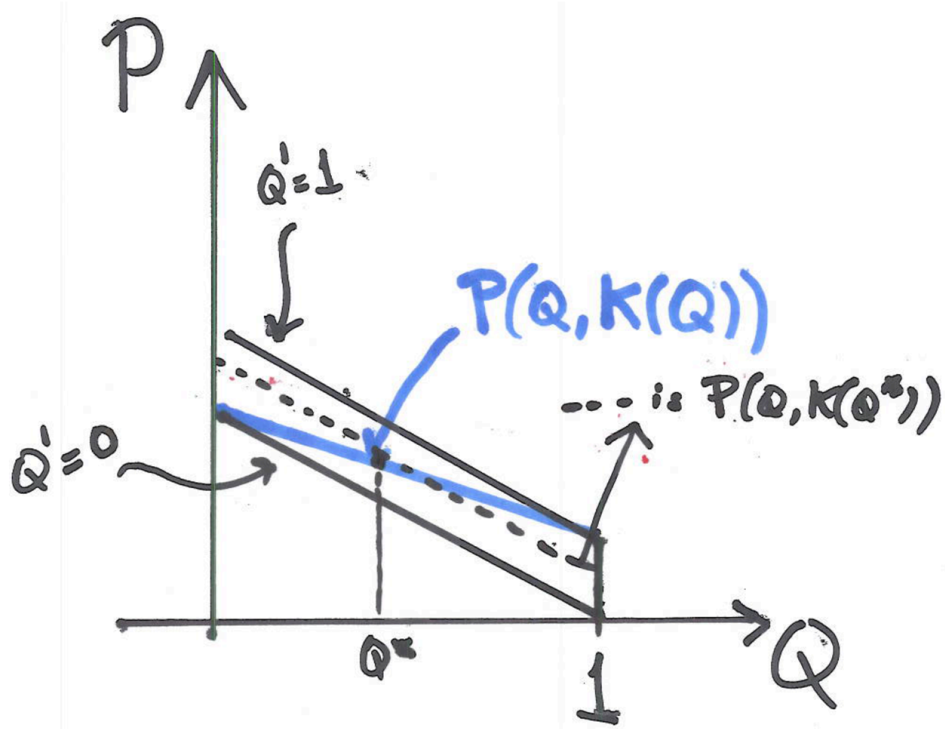

Positive externality of Q*th unit

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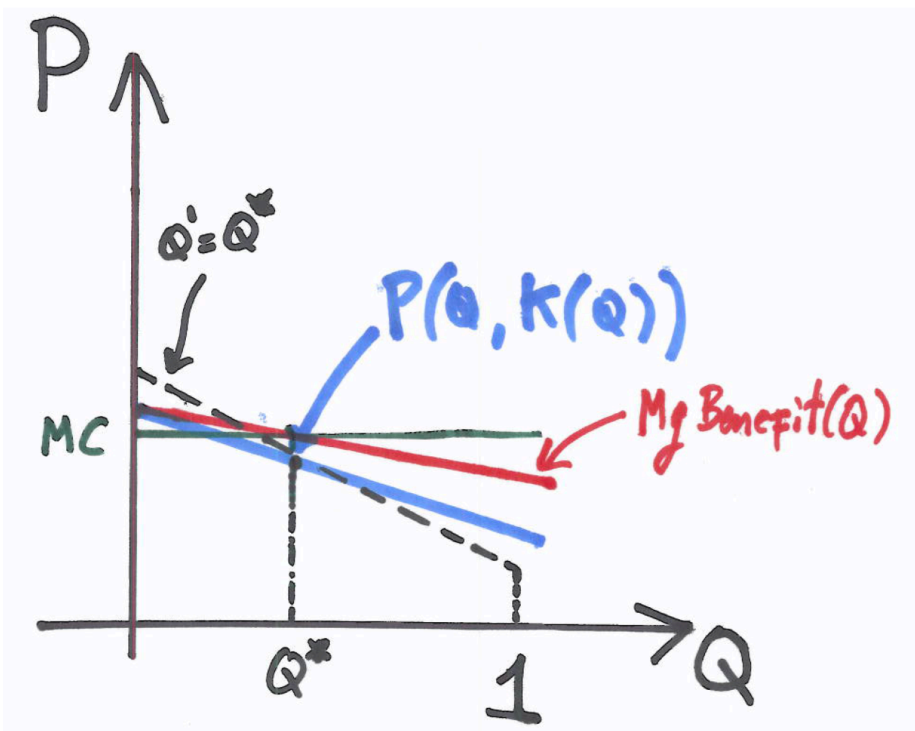

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Mg benefit Q*th unit

$MgB(Q^*) = \boldsymbol{1-(1-\alpha)Q^*} + \boldsymbol{\alpha Q^*}$

$= 1-(1-2\alpha)Q^*$

Mg benefit = MC

$1-(1-2\alpha)Q^*=MC$

$Q^*=(1-MC)/(1-2\alpha)=1/2$

Heterogeneous contribution to externality

Homogeneous: K = $\int_{\theta^*}^{1} d\theta$ = 1-θ* …… **K(Q)=Q**

Heterogeneous: K = $\frac{4}{3}\int_{\theta^*}^{1} \theta d\theta$ = $\frac{2}{3}(1-\theta*^2)$ …… **K(Q)=$\frac{2}{3}$ (2Q-Q^2)**

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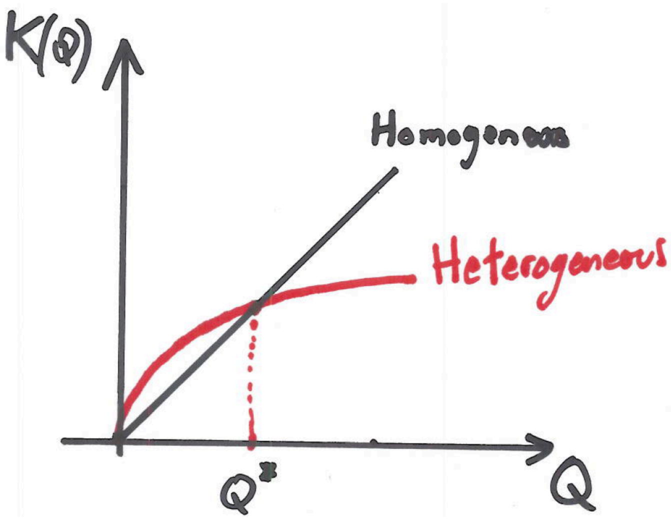

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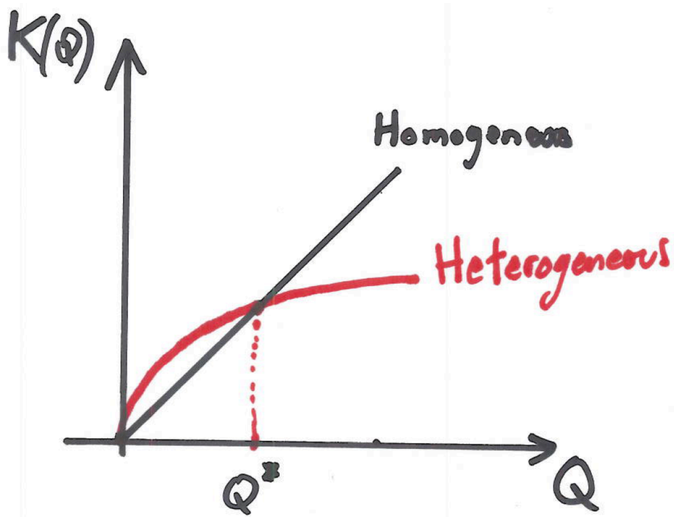

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More generally:

Given P(Q,K) and K(Q):

Mg Benefit = P(Q,K(Q))+ $\int_0^Q \left.\frac{\partial P(\tilde{Q},K(Q'))}{\partial Q'}\right|_Q d\tilde{Q}$

vs MC(Q)

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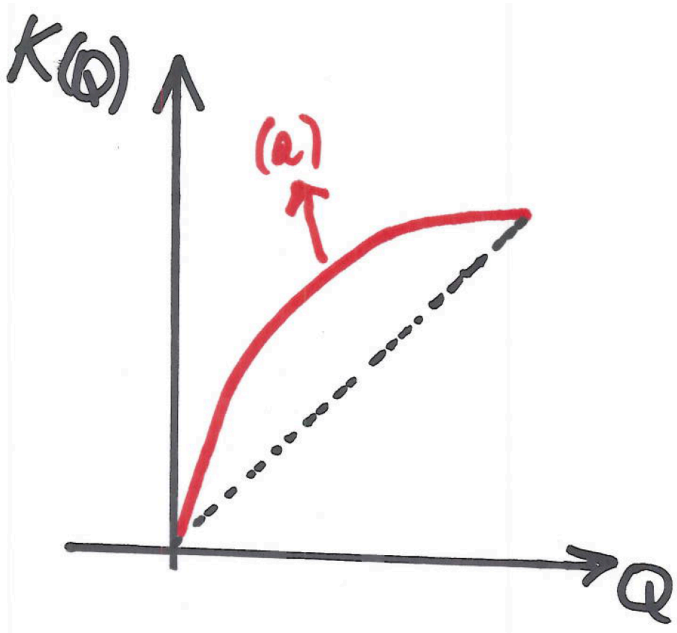

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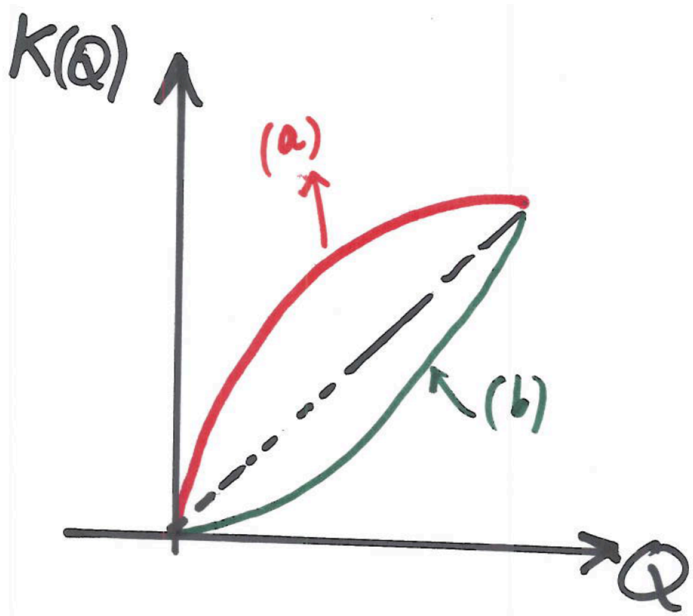

Case (b): **Negative correlation**

$\Pr(\omega=1|\theta) = 1-\theta$

$K = \int_{\theta^*}^{1} (1-\theta) d\theta$

$K(Q) = \frac{1}{2}Q^2$

André & Glen's paper is much more general:

- •Welfare & profit maximization
- •Heterogenous valuations & contribution to externality
- •Multiple instruments (e.g., price, quality)
- •Multi-sided platforms
- •etc.

Theory:

- •Conceptual contribution. Refreshing: back to basics
- •Can "K(Q)-approach" help more generally?
- •Validity of first-order approach: non-monotonicities corner solutions, non-existence of fixed point?

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